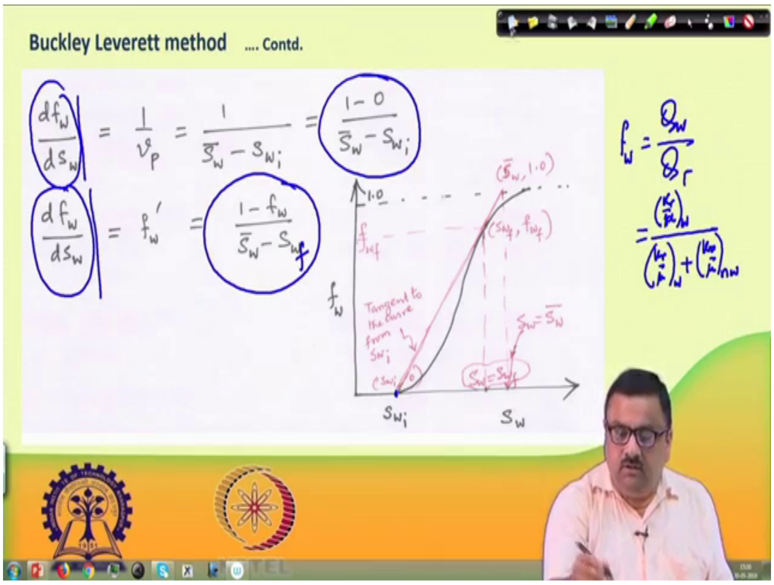
Flow through Porous Media
Prof. Somenath Ganguly
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 45
Immiscible Flow (Contd.)

Welcome you all to this lecturer of Flow Through Porous Media. We were discussing about Immiscible Flow and we had talked about this movement of Buckley Leverett Front in our last lecture. So, I will conclude this discussion on Buckley Leverett front theory and then I will move to other topics where we have gas phase present and there is a vapour liquid interaction term.

(Refer Slide Time: 00:55)



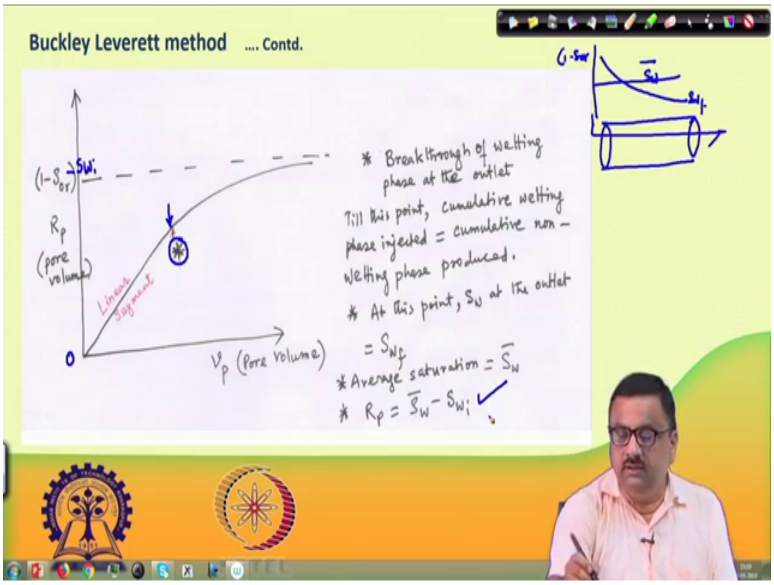
Buckley Leverett method .... Contd.

So, what we were discussing at the end of what we discuss before about this Buckley Leverett method is that we saw that this d f W, d S W on one hand is this quantity. And same similarly d f W d, S W is also this quantity and these two have to be satisfied when Buckley Leverett front reaches the outlet.

So, this we just put this as S W f and so, we made a geometric construction where we had f W versus S W curve already known to us because f W is basically Q W divided by Q t volumetric flow of water divided by total volumetric flow where Q W was k r by mu for wetting face divided by k r y mu for wetting plus k r by mu for non wetting.

So, this so, since k r is the function of saturation. So, f W would be a function of saturation and if someone takes relation for relative permeability that is k r versus saturation for water phase and oil phase and put this in there. So, f W versus S W curve takes the shape and to satisfy this two conditions of d f W, d S W at the when Buckley Leverett front reaches the outlet one has to draw tangent to this curve f W versus S W curve from the point here. Which is S W i and wherever it hits this f W is equal to 1.0 line we call that S W bar.

(Refer Slide Time: 02:56)


Buckley Leverett method .... Contd.

* Breakthrough of wetting phase at the outlet

Till this point, cumulative wetting phase injected = cumulative non-wetting phase produced.

* At this point, S_w at the outlet $= S_{wf}$

* Average saturation $= \bar{S}_w$

* $R_p = \bar{S}_w - S_{wi}$ ✓

So, this simplified the process much and so, beyond this point what we said is that production of oil from this porous medium would be. So, this is R p as a function of pore volume this is if you are starting from 0 here R p production would be then 1 minus S o r minus S W i because originally the oil that was placed oil was in place is 1 minus S W i and out of that S or fraction has to be left out at the end.

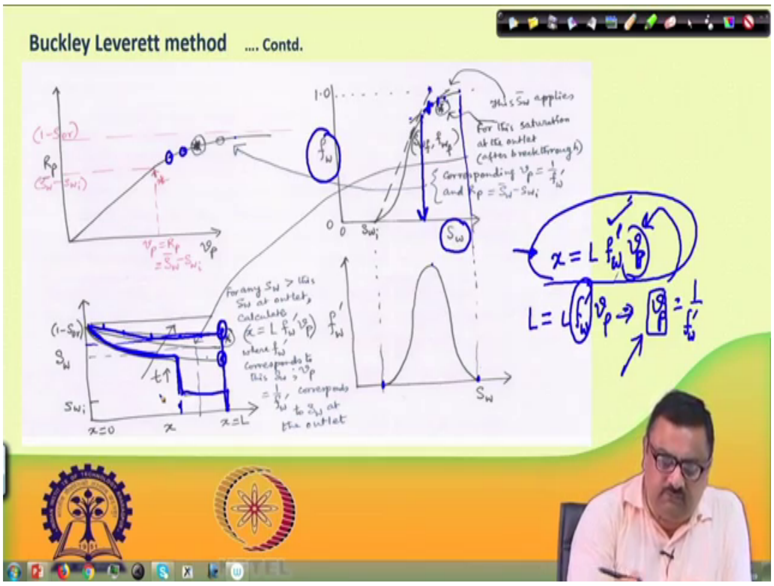
So, this is the end point and then we find that here the V p is the pore volume total pore volume produced that is water plus oil which is equal to total pore volume of water injected. And total pore volume produced would be only oil, until some point which we referred here as star. So, till this point this curve would be a linear straight line. So, this star refers to break through a wetting phase at the outlet breakthrough a wetting phase at the outlet.

So, beyond this point both water and oil will be produced. So, that is why the curve is it is deviating from the straight line. And so still this point cumulative wetting phase injected equal to Q cumulative non wetting phase produced at this point S W at the outlet is S W f average saturation at this point; that means, I have the plug porous plug and the front has reached.

So, this is the this is 1 minus S o r and this is S W f this is how the S W versus x curve looks like. So, then the average of these all this saturations which is let us say S W bar. So, that S W bar refers to average saturation here and the oil produced at any time would be the S W bar existing in the porous medium minus the original S W i that was in place. So, that is the change in saturation average saturation to start with was S W i the interstitial saturation.

And after some flooding the average situation became S W bar and these difference in saturation in water happened because the corresponding amount of oil was expelled from the porous media so from there one can find out what is the total oil produced using this relation of S W bar minus S W i.

(Refer Slide Time: 05:47)



Buckley Leverett method .... Contd.

So, this whole exercise refers to I think these plots here. First of all one has to generate; first of all one has to generate this f W versus S W curve. Based on the relative permeability versus saturation information. Relative permittivity versus saturation was the there would be correlation available one such correlation was coris relation, otherwise this correlation has to developed for an unknown porous medium and for an unknown wetting and non wetting phases.

So, this curve has to be developed and then one has to draw from S W i and find out what is W bar, but this S W bar is at the time in Buckley Leverett front has reached the outlet ok. So, that is this point so, this at this point how much oil was produced would be S W this S W bar minus S W i. So; that means this distance if you put draw the vertical here this is W bar from here and this S W bar minus S W i that would be the amount of oil produced till that time and till that time it will be linear 45 degree line.

So, we can easily locate this point and draw this line. Beyond this point one has to find out one has to pick up another point here and consider that to be S W f at that time so, that using that as S W f one has to draw a tangent find out where it hits, find out what is f W bar. And find out where once you have S W bar what is that S W bar minus S W i gives me the corresponding R p and the slope of this line at that time, this is f W prime and that f W prime is essentially 1 by v p ok. This corresponds to the outlet saturation such S W value at the outlet.

So, from this f W prime from looking at this slope one can find out what is the v p and that v p would be here, and the corresponding R p would be S W bar minus S W i and that R p would be there. So, one can start locating these points one by one take another value, draw a tangent, find out what is f W prime, find out what is v p and based on this see where it hits, get S W bar, find out S W bar minus S W i that you started with, and find out corresponding oil produced. So, v p and R p, one can always find out from this exercise and locate these points one by one.

Finding out f W prime would maybe little difficult, because always; see the focus here though you are drawing f W S W curve like this, but the focus here is beyond S W f. So, this part of the curve, this part is not you are not interested in. So, this part, so, this part is there is too much of clutter so, what one can do is.

Get the correlation of f W versus S W and analytically take the derivative. Take analytically differentiate it and then get f W prime versus S W curve and one can generate this curve of f W prime versus S W and they put one just below the other so, that this is W i corresponds to here this S W i and this 1 minus S o r corresponding to this here; so, 1 minus S o r. So, end point corresponds to here.

So, and in between this is the curve of f W prime versus S W. So, any point we are looking at let us say I pick up this point here so, automatically we can drop it here and find out what is the corresponding f W prime ok. Similarly any point there you can find out what is the corresponding f W prime. So, instead of going through this clutter one can use this curve to generate the corresponding f W prime values, because you need f W prime values to calculate the v p ok.

And you need S W bar anyway to calculate the, you one has to find out the S W bar any way to calculate the corresponding oil produced. So, this is one thing and then, if we look at the saturation profile, if we look at the saturation profile with respect to position. So, we can see here this is this is where we started that, this is x equal to 0 this is x equal to l inlet outlet. And then we see that the Buckley Leverett front has travelled up to this location up to this location and beyond this it was all in at interstitial water saturation because, to start with entire porous flag was at interstitial saturation.

So, now this since the Buckley Leverett front is travelled in so, water has gone in and increased the water saturation, but beyond this Buckley Leverett front where water as not gone in any further there the saturation is S W that will remain as S W i. So, now here this we found out from this plot what is this is I mean after you had a breakfast through we can find out what is the corresponding S W f here ok so, this is the value of S W f.

So, since this is the value of S W f so, one can get this curve here and then after that one can find out what S W bar. And then you one can find out what are the corresponding saturations that these values one can get because, we had already written you may recall that x is equal to L f W prime v p so, from there at any location at any location at for any saturation.

Let us say I pick up this is the front saturation; this is the front situation. Any other saturation which is higher the corresponding f W prime I need to find out, corresponding f W prime we have to find out and for the same v p, v P is fixed. See once we have if this is the Buckley Leverett front Buckley Leverett front reaches the outlet; that means, this is S W f so, that S W f is fixed. So, all saturations in the upstream side would be all the all saturations would be the greater than S W f.

So, any is any packet you pick up saturation packet how far that will travel. So, we have to find out what is the corresponding S W f and then one has to draw a tangent or use this curve to find out what S f W prime. So, corresponding f W prime one has to find out. But v p has to be; how do you get v p? V p is 1 by f W prime corresponding to this outlet saturation ok.

So, that is the v p, but f W prime is corresponding to any other saturation any other points you look at here. So, that is how one would look at because, when we freeze when we freeze this particular situation let us say I am freezing this saturation profile here so, that means this is the S W f ok. So, to achieve this S W f what is the v p? V p would be because then you would put x equal to L is equal to L f W prime v p and these f W prime corresponds to this particular saturation packet which has reached the outlet; that means, this particular situation packet for which x equal to L.

So, basically this f W prime in this case so, now from these I am calculating v p is equal to 1 by f W prime. So that means these S W prime is at the saturation at the outlet. So, first of all you have to pick up what is the saturation at the outlet, let us say this is saturation in the outlet. So, corresponding f W prime gives me this f W prime.

But v p give v p is obtained from sorry what I said is this first you are first you; let us say this is the saturation at the outlet at any time. This is the saturation let us say at the outlet. So, for this particular saturation one has to find out what is the corresponding f W prime. So, that brought that from that a f W prime one has to find out what is the corresponding v p so, that gives you the v p; so, and that v p goes in here.

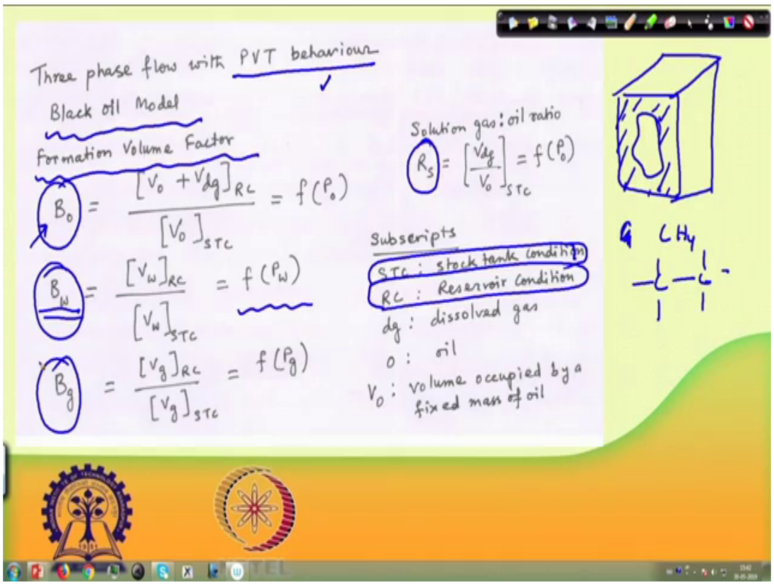
Now, if you pick up any other situation so, that is the outlet saturation and the v p is frozen. Now if you pick up any other saturation intermediate saturation at intermediate location so, any other saturation you try to find out how much x that situation packet will travel. You have to use this expression and there the v p is already known and f W prime you have to find out whatever saturation you are picking up, let us say I am picking up this situation the outlet situation is this much.

But I am picking up the saturation and I am trying to find out how far it will travel so, for that I have to then draw a f W prime here or obtain f W prime from the graph for that particular situation and that will give me the f W prime. And once I know this f W prime for the saturation and v p for the outlet saturation at the outlet. Then I take this multiply this by L and I get x. So, I know that this situation will travel as per this equation this saturation will travel up to so and so distance. Similarly pickup another saturation find out how far it will travel and one can create this profile.

So, this is essentially this is exercise one can take up from Buckley Leverett form. So, what I what is written here is for any S W; for any S W greater than this S W at outlet. This S W we are referring to here for any S W greater than this S W at out let calculate x equal to l f W prime v p where, f W prime corresponds to S W v p is equal to 1 by f W prime corresponds to S W at the outlet.

So, this is something which I mentioned just now. So, this is overall Buckley Leverett form so, if somebody gives you the relative permeability versus saturation curve, or if you have the instrument I already explain explained the experimental procedure. So, if one does those one can find out what are the relative permeability versus saturation curve. So, one can do this exercise one can repeat this plots and get a meaningful result.

(Refer Slide Time: 17:23)



Three phase flow with PVT behaviour ✓

Black oil Model

Formation Volume Factor

$$B_o = \frac{[V_o + V_{dg}]_{RC}}{[V_o]_{STC}} = f(P_o)$$

$$B_w = \frac{[V_w]_{RC}}{[V_w]_{STC}} = f(P_w)$$

$$B_g = \frac{[V_g]_{RC}}{[V_g]_{STC}} = f(P_g)$$

Solution gas : oil ratio

$$R_s = \left[\frac{V_{dg}}{V_o}\right]_{STC} = f(P_o)$$

Subscripts

STC : stock tank condition

RC : Reservoir Condition

dg : dissolved gas

o : oil

V_o : volume occupied by a fixed mass of oil

So, this is all about these Buckley Leverett form. Now as I said that if we go to three phase flow, there would be another consideration which is known as PVT behaviour ok. PVT behaviour in the sense that if you have; if you have; let us say this is the; this is the differential volume we are talking about and out of that this is the pore space and within this void, this is the pore space and rest is solid and within this pores phase now we are sharing this pores space between oil water and gas three phases ok.

So, generally water would be at the at the near the wall, it is most wetting and then oil and then gas so, they would be aligned in between, but they have good amount of mixing one can expect that there would be an equilibrium between those phases. Why do you need an equilibrium there? Because this oil consists of; let us say if I look at the crude oil in a reservoir underground this oil consists of several alkenes, which starts from I mean is there will be some amount of dissolved gases so, depending on the pressure.

So, it is alkene means methane is CH 4. So then ethane 2 carbon and rest hydrogen; so, then you will have a 3 carbon like this when you go to 3 4 5 6 10 20 30 40 like this you will go to higher alkenes. So, crude oil means it is a mixture of all these alkenes, and then there would be alkenes there would be other components.

So, now if you have so, these are now suppose I have a (Refer Time: 00:00) oil is there is a natural gas also existing in that pore, then there would be; for example, natural gas should be primarily methane with some amount to ethane. So, there would be an equilibrium it let be a sharing so, amount of ethane; for example, how much it goes to the way gas phase and how much it will go to the liquid phase.

So, there a so there is a; there is a sharing ok and it depends on pressure, if you reduce the pressure then you find that lot of oil will go to the vapour phase, lot of components go to the vapour phase, but you increase the pressure you will find those components again we go back to the liquid phase.

So, this is there is an equilibrium and it depends on pressure and temperature. So, that is why this there is this PVT behaviour is important and for this one needs to go to phase equilibria thermodynamics. This is basically you have a mixture here and which one will go to the vapour phase by what amount and which one will remain so, there would be a distribution co efficient of sort and then that distribution co efficient depends on molecular interactions and all other in a few gas it is an other term.

So, basically it is a PVT relation and it is basically it comes under the purview of phase equilibria thermodynamics. But when people are working in reservoir flow through a porous media in connection with reservoir modelling, there is a simpler way to handle things and I thought I briefly touch upon on that at least just to give you the give you a glimpse of what is what is followed there.

It is basically the one can instead of going to phase equilibria thermodynamics and then one has to use thermodynamic relationships. For example, there are several some several once available, say unique, vac, univac and those are various activity coefficient models available, which can be utilized in this regard to find out how much go to the vapour phase and how much remains in the liquid phase.

But instead of that there is a more often more of empirical way of doing things and which is in terms of formation volume factor ok. And this is the way this model is handled I mean these are also there is a name attached to it ok. So, now formation volume factor the way it is defined is, it is written here we can see V let us say oil, see there are two conditions specified, one is known as reservoir condition.

Reservoir condition means: we are talking about let us say 50 atmosphere pressure 85 90 degree centigrade, I mean pressure and temperature they are higher. And then we are talking about stock tank condition, which is more of like our standard condition 25 degree centigrade 1 atmosphere, I mean that can be decided I mean what stock condition one uses.

So, these so there are two conditions and within this these two conditions as we switch from one condition to other. So, naturally when you when it is in reservoir condition there would be a lot more gases, which are which will remain in the dissolved state in the liquid, but moment you bring into stock tank condition those gases will flash out from the liquid ok.

So, to account for how much we will go to a go to what state; for example, if I look at the simplest thing here the formation volume factor for water. So, water here it is written V W. Volume occupied by fixed mass of water under reservoir condition, divided by volume of that same mass of water under stock tank condition. So, if water has any compressibility if water changes its volume ok, because you have taken the pressure off, you have taken the temperature off. So, then this is the; this is the ratio and this is known as formation volume factor for water.

And this is considered function of P W. P W P o we know P is the pressure, we are now differentiating between P o P W P g because we know that they may differ by capillary pressure and all. So, to remain on, so to remain inconsistent there, instead of function of pressure it is in a function of pressure of the wetting phase; a pressure of the water phase here, because water oil gas we are talking about. Similarly here we can see for formation volume factor for the gas is same as V g under reservoir condition to V g under stock tank condition.

And formation volume factor for oil, here we have V o plus V d g; V d g is the, V d g is the dissolved gas in reservoir condition, which will not be present in the stock tank condition ok. So, V o plus V d g at R c divided by V o s t c so, this is how it is and further one goes to solution gas oil ratio that is also another term r S in this regard, which is V d g by V o under stock tank condition, which is considered function of pressure.

So, these are some of the terms one defines, and now they treat these that as if I have information a priori about what would be B o B W B g as a function of pressure, because if one knows all these then one can draw continuity equation quickly. One can use this B o B W B g to draw the continuity equation, without; but otherwise I mean if I if we had not known a priori this B o B W B g then we have to go for the thermodynamics

I mean we have to go to phase equilibria thermodynamics to find out how much would be the what would be the volume of gas at reservoir condition, against what would be the volume of gas under stock tank conditions so, how much would be the expansion. When I am changing the pressure from 50 atmosphere to 1 atmosphere how much would be the expansion. But that I mean we have ways to do it, I mean we have studied it already in physical chemistry how it is to be done.

But instead of relying or instead of going to those correlations, the idea here is to work with these parameters and now try to write continuity equation, because we have to write a continuity equation for oil. Because you remember our original, originally there was a continuity equation then we had, we can, we say that we can do these k r o k r W relative oil, sorry relative permeability to oil and relative permeability to water, and use Darcy's law. So, that can give me I can use the Darcy's law with the relative permeability.

So, relative permeability term so, we can use Darcy's law; but now at we have to; we have to at this time, I mean Buckley Leverett form there we had it was situation was much simpler. There we had only oil and water at interstitial saturation was present and only water was injected, but now in this case we have three phases sitting there in the porous medium, three phases oil water and gas setting there in the porous medium. And now I am having a flow and maybe I have an withdrawal from some point ok.

Just when you when you generalize this; I mean obviously, there would be either injection or production of either oil phase or gas phase water phase either. So, one as to write a general equation and so, we have to write three continuity equation mind it. One for the oil other for the water and other for the gas and obviously, that there would be an accumulation term and accumulation term will only talk about S o S W and for S o S W S gene; in fact, you need only 2 because the other one would be 1 minus.

Say for example, if you do not know S g; S g would be simply 1 minus S o minus W that is one thing. And how much would be the density change, but again the density change we are leaving it to this formation volume factor ok. So, now now it will be challenging to see now it be interesting to see how these continuity equations play out, now we will write this continuity equations only instead of density terms.

Now, we would express it in terms of density at, some say density stock time condition and use formation volume factor. So, by that way we can circumvent this problem of using a phase equilibrium thermodynamics and going for a rigorous solution. So, this is something which I am going to cover in the next lecture this is all I have as far as this module is concerned.

Thank you very much.