Flow through Porous Media
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Lecture - 43
Immiscible Flow (Contd.)

Welcome to this module of Flow through Porous Media. We may discussing about Immiscible Flow.

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We were particularly talking about this Buckley Leverett front and how it progresses through a porous medium.

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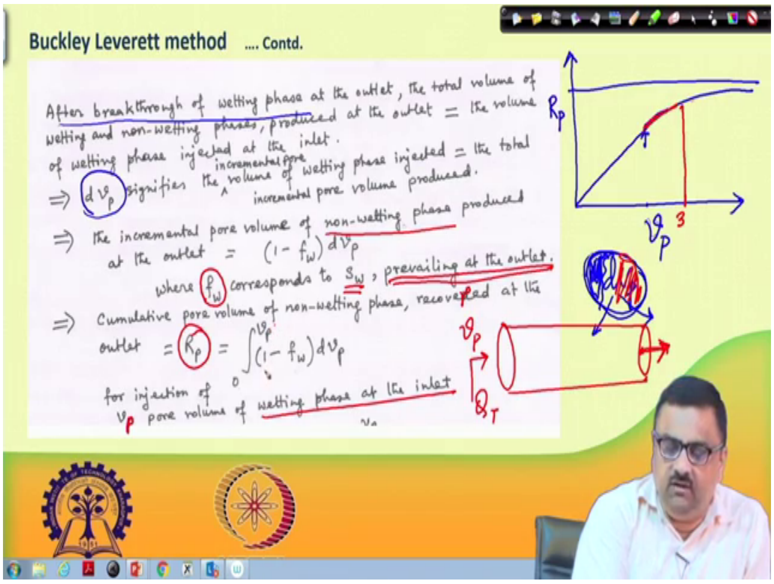

So, what we discussed so far is about this Buckley Leverett front here. We said that there are two regimes if you will off. On x-axis, we have we can directly now we can start writing it as v p that is the pore volumes of injected fluid which we know in this case is water. So, v p is the pore volumes of water injected into this porous medium. And on the y-axis, we write these as R p the cumulative. Now, you note here this is not a flow rate this is cumulative value.

So, it is continuously increasing with time. So, at any time this much of pore volume has been injected that means, over this time period over this t. So, this is a cumulative v p. And we can call it now the cumulative oil produced is now we call this as R p. And we said that initially this would be a 45 degree line until the Buckley Leverett front breaks through at the outlet. And beyond this point this will take a turn and asymptotically reach a limit which is already set based on the residual oil saturation.

So, what we note here is after breakthrough of wetting phase that means, after breakthrough of wetting phase means we are talking about this part beyond this part where it is no longer linear. The total volume of wetting and non-wetting phases produced at the outlet is equal to the volume of wetting phase injected at the inlet, so that means, beyond this point whatever volume is produced and that is that is comprising of both oil as well as water.

But till this point it was the produced is only oil, no water because Buckley Leverett front has not reached the outlet end till that till this point. So, here d v p signifies the incremental pore volume of wetting phase injected that is equal to total incremental pore volume produced. So, d v p signifies the incremental pore volume of what. So, at any point let us say I am looking at d v p. So, d v p injection means d v p production, but these d v p beyond this point is these d v p is comprising of a part which is oil and a part which is water.

So, now, if there is a fraction means the total d v p I have injected d v p and I produced d v p it has to happen whatever I inject I have to produce only injected, I inject only water, but produce when comes to production produce I produce water and oil ok. So, d v p as a fraction, if I if this is my d v p part, then I can say a part of this d v p is a part of this d v p is you have as water, and the other part of the d v p is let us say oil. So, this is the d v p, d v p is comprising of oil and water.

So, how much would be the non-wetting phase in this case, how much would be the oil, how much would be the water in this in this produce d v p that is f w into d v p, because f w is q w divided by q t. So, f w into d v p would be the this thing what are produced. So, 1 minus f w into d v p would be the oil produced at this time. So, here now mind it we already noted that this f w corresponds to sum S w, f w they we have looked at f w is w curve.

And each value of f w corresponds to some value of S w right that they have some relation, and that relation comes from definition of f w which is k r by mu for wetting phase divided by k r by mu from for wetting plus k r by mu non-wetting right that is what q w by Q t is. And then k r depends on saturation there are correlations available so we bring in those and we know that f w corresponds to some S w.

So, now, when it comes to this at the time of breakthrough or beyond this period, we are looking at S w that is prevailing at the outlet, S w that is prevailing at the outlet. Now, the cumulative pore volume of non-wetting phase recovered; recovered at the outlet, the cumulative pore volume would be R p is equal to integration also this, this is basically incremental pore volume of oil produced. So, if you if you go for instead of an differential form, it would be 0 to v p because this is run over d v p. So, 0 to v p 1 minus f w into d v p, this is the cumulative pore volume of oil produced for injection of v p pore volume of wetting phase at the outlet. So, v p pore volume of wetting phase at the sorry at the inlet.

So, what essentially this whole thing says is this is the porous medium. We are injecting Q t which is only water and then we are producing. So, we are producing here initially it was only oil and when Buckley Leverett front breaks through at the outlet beyond that point, it would curve, it would reproducing both oil. And water and cumulative pore volume of oil at any source, so I suppose I have injected v p pore volume of water.

So, if somebody wants to know how much of oil I have produced against v p say I am talking about v p which is let us say three pore volume. So, after injection of three pore volume of water how much of oil I have produced, so that has to be integration 0 to v p 1 minus f w d v p. So, that would be the volume of oil produced. So, this would be this would be the equation for cumulative pore volume of non-wetting phase recovered.

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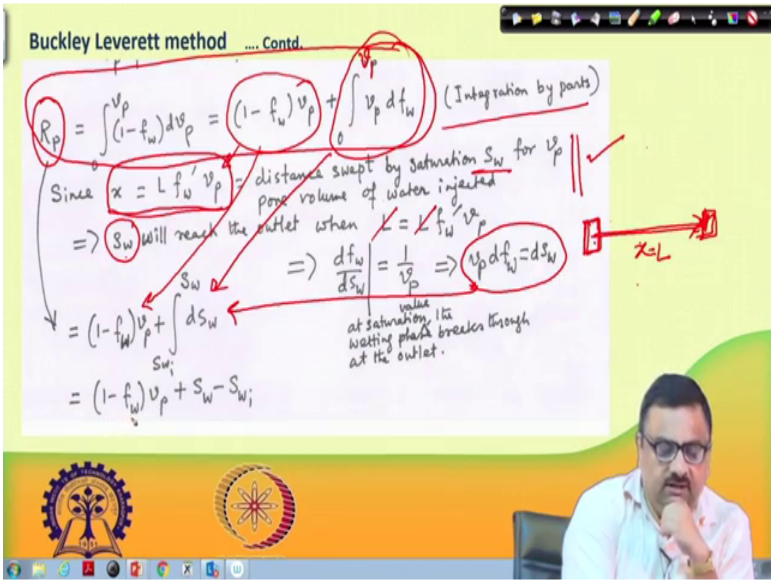

Now, if we look at further this expression of R p, we know that R p is equal to we just now said 0 integral 0 to v p 1 minus f w d v p ok. So, now, we can integrate it by parts. So, integration by parts take this is the expression that we already have written that x is equal to this, this we already have shown. And these x is basically the distance swept by saturation S w for v p pore volume of water injected, this, this we already have demonstrated in earlier lecture.

So, if this is so, if this equation is to be valid, then S w, a front will reach the outlet when L is equal to L multiplied by so that means, you are looking at x is equal to L f w prime v p what does this mean I have a saturation packet, and that saturation packet will travel by a distance x which is equal to L f w prime v p because of injection of v p pore volume. So, now, we are interested to know when a particular saturation front will reach the outlet. So, when it reaches the outlet, then x becomes equals to L, so that is why we are saying when this saturation packet will reach the outlet that (Refer Time: 09:14) at that time x will be equal to L.

So, L is equal to L f w prime v p. So, in that case, d f w d S w, d f w d S w at saturation value when the wetting phase breaks through at the outlet that would be equal to 1 by v p because, here the L and L they will cancel out. So, basically f w prime is 1 by v p, or in other words v p d f w is equal to d S w. So, one can show from this expression that when this saturation front breaks through at the outlet at that time d f w d S w at that saturation value when the wetting phase breaks through at the outlet is equal to 1 by v p. So, this is one thing we must keep in mind. And further we note here that this is R p is equal to this, this expression right.

So, out of that 1 minus f w into v p this remains as it is here and this part v p d f w integration 0 to v p. So, v p d f w is equal to d S w we have already seen here. So, these goes here, so instead of v p d f w, instead of v p d f w this goes here. So, instead of v p d f w, we are writing d S w. So, this will change from S w i which is the interstitial saturation to the given value of S w. So, this becomes equal to 1 minus f w v p plus S w minus is S w i. So, this is the pore volumes of oil produced. So, this is given by this quantity 1 minus f w v p plus S w minus S w i, this is the cumulative oil produced.

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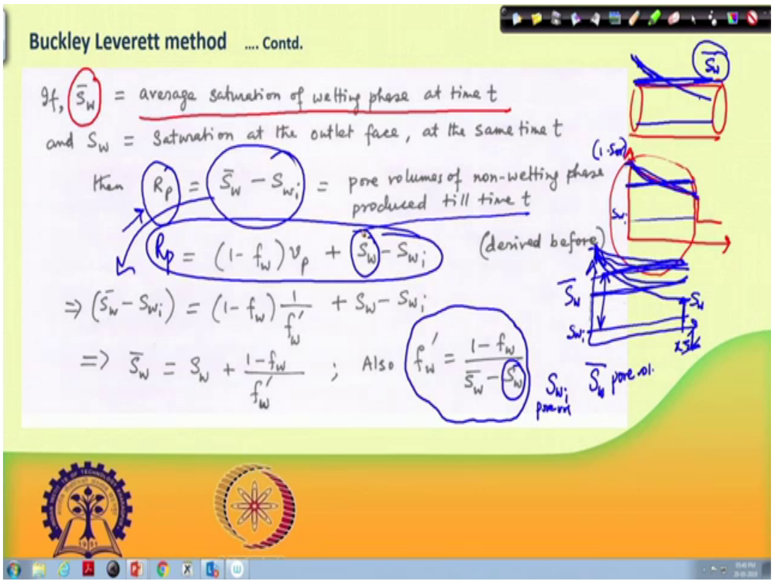

Now, if we look at quickly define S w bar that is average saturation of wetting phase at time t, what does this mean, we said that this is the core and when we plotted the saturation we know if Buckley Leverett front is Buckling Leverett front is here. So, then this is the this is this part if somebody wants to find out what is the S w bar in this case what is the average saturation of water as far as this particular part, where Buckley Leverett front has already traveled, if somebody wants to know what is the S w bar what is the average saturation so that value is here. So, that value is most possibly this value will come to something like these, because at this end it is 1 minus S o r and here it is something less and here it is S w i.

So, this particular level, I can think of which is an integral average of this saturation which is changing from 1 minus s what to something below. So, this value is S w bar. So, S w bar is the saturation on the Buckley Leverett front. So, when the front reaches the outlet when the front. So, this is x equal to L. So, this is x equal to L the front has reached the outlet; the front has reached outlet. So, at that time what is S w bar, let us say this is the value of S w bar. Here we have S w I somewhere here we have 1 minus S o r.

So, now an S w is the saturation at the outlet face at the same time, so these value is let us say S w. The saturation at the outlet face, because here it is S w and here it is one minus S o r. So, in between the saturation changes, so we can have an average saturation which is given by this. So, these S w bar minus, so, so, now we look at here the r p cumulative volume of oil produced they are writing it as S w bar minus S w i, why because initially how much of water was present in the system that is the interstitial water saturation.

So, if the total pore volume is one out of that S w i pore volume of water was present in the system. And act sometime t when this front has reached and this at outlet the value saturation is S w at that time the average saturation in these average know where the saturation profile exists, but I have to draw that integral average. So, if you draw that integral average that average saturation happened to be S w bar, so at that time how much of water is present in the system that is equal to S w bar pore volume right. So, they at this time it is S w bar pore volume of water is present in the system and earlier S w i pore volume of water was present in the system.

So, now, if we subtract S w bar minus S w i, so that gives me that earlier I have much less pore volume of water present now I have is w bar which is a larger pore volume of water present. So, where is this difference coming from, these difference is coming because I have already produced some oil, I have already pushed out some oil by injecting water. So, the average saturation for this core; average situation saturation for this porous plan has gone up earlier to start with at t equal to 0, this was S w i and at this time this is S w bar. So, this difference is arising.

So, this pore volume difference is arising because this part of the pore volume of oil has been displaced. So, then what is the total pore volume of oil produced that is simply S w bar minus S w i. So, pore volume of non wetting phrase produce till time t. So, at any time let us say at some other time I can find this being this part is remain same, let us say this is the curve or this is the curve.

So, then you have to find out what is the corresponding S w bar, what is this difference S w bar minus S w i and that difference arises because that much of oil has been pushed out from the core, so that is why produced oil pore volume of produced oil is S w bar minus S w i.

And we have noted earlier that R R p just in the earlier slide, we have we have shown that R p is equal to 1 minus f w into v p plus S w minus S w i. So, in this case, if we now equate instead of R p the left hand side if we bring in a S w bar minus S w i on the left hand side and right hand side remains as it is, then we get here S w bar is equal to S w plus 1 minus f w by f w prime. Or in other words I mean it is just simple some manipulation, we see here that f w prime is equal to 1 minus f w divided by S w bar minus S w. So, this is this is an expression we get for f w prime.

So, let me recapitulate quickly what we are talking about here number 1. We now defined a concept called S w bar which is the average saturation existing over the pore space. So, initially truly I have S w i everywhere interstitial water saturation, but when I continuously I injected water, I created a saturation profile, but there is so I can always get an integral average of that and I can find out what is the average saturation this represents, some part is above that value, some part is lower the value, but average saturation as far as this entire length is concerned is S w bar.

So, this S w bar is going to be an important term for me. S w bar is the average saturation of water inside the pore. And at the same time we are talking about a term we are having a term which is S w we are carrying it, here in this factor also. What is S w? S w is the saturation at the outlet. If you look at what was the definition, when we arrived at this expression for R p when we arrived at this expression for R p in the earlier slide.

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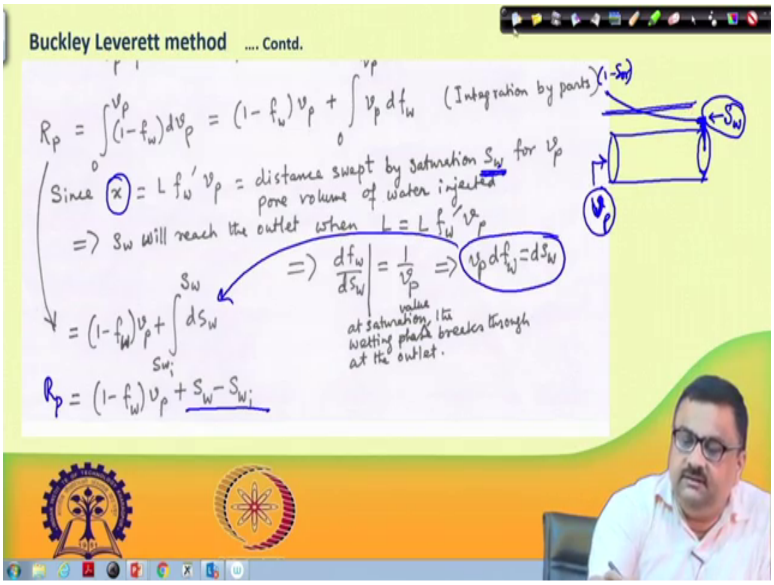

There we had we had used these part v p d f w is equal to d S w this has gone in there to so that is why we have this S w minus S w i for R p. And how do we arrive at this v p d f w d S w that we arrived at by writing x equal to L. Why am I writing x equal to L, what was x, the x was distance swept by saturation S w for v p pore volume of water injected.

So, v p pore volume of water was injected, I have the core v p pore volume of water was injected. And we are trying to locate how far S w front will travel. But now we have insisted S w will reach the outlet when L is equal to L f w prime v p. So, these v p and these f w prime these they are they are just tired, there is another additional condition in place.

What is that condition that after v p pore volume of injection this s the I am assuming that the Buckley-Leverett front has broke through at the outlet. Buckley Leverett front has broke through at the outlet. And this when it breaks through at the outlet these saturation is S w that has to happen here, because these x is the distance swept by saturation is S w and we are setting this x as L. So, this S w has reached x equal to L.

So, we are talking about that condition. So, S so that means, when we are talking about S w bar on one hand which is the average saturation, but at the same time we must remember that the saturation at the outlet that has to be that is basically that S w in this equation this is not any arbitrary saturation value somewhere in between these S w is the S w at the outlet, and S w bar is the average saturation. So, from 1 minus S o r to S w here 1 minus S o r to S w these change this change that happens the average saturation here is S w.

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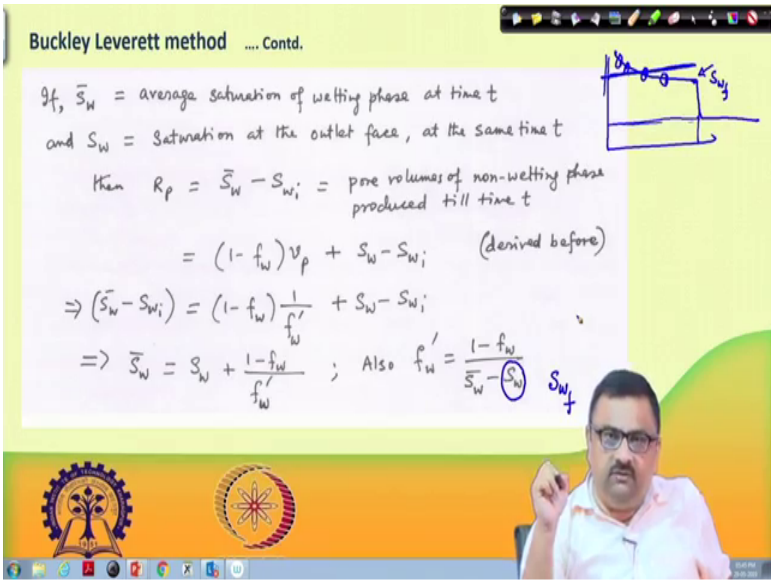

So, in fact this was because of these reason I prefer to we should we should be writing these S w very soon, we will you will see that this S w will be changed to S w f that means, S w of the front. So, Buckley Leverett front has reached x equal to L and that saturation there because we have you can see this saturation profile it takes a shape like this. It is this is S w i, but here this is higher than S w because this is because of the way the f w curve wave is f w curve had it been in some other form I mean you can you can see this, this is the f w curve. If f w curve is this way, so then probably the tangents and all there they were different.

So, these I will show that in a moment do not, do not bother about this part. But here these saturation here when it breaks through here at x equal to L, we call this as S w f. To differentiate this from any other saturation values S w would be any individual saturation values, any individual saturation values. And S w bar would be simply the average of these saturations, so that is something which we are heading to.

So, I will continue this in the next lecture because our original aim is to predict the how the oil produced and what are injected or non-wetting phase produced verses wetting phase injected, how we can predict the part beyond breakthrough. And when the breakthrough takes place and what would be the saturation profile over the length. So, these are some of the answers we need to get out of this exercise, so that is all I have for this lecture.

Thank you.