

Flow through Porous Media
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Lecture - 41
Immiscible Flow (Contd.)

I welcome you to this lecture of Flow through Porous Media, we were discussing about Immiscible Flow. In particular we just briefly introduced these Buckley-Leverett formulation, where the porous medium is at interstitial saturation and then the wetting phase would be injected into that porous medium and we want to study how saturation changes, how the wetting and non-wetting phases, how they to they come out how they how they how they are collected, how they how they at what rate they come out at the effluent.

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So, what we did is we had earlier we had discussed about relative permeability and two phase flow. The model that we have in mind we have this (Refer Time: 01:09) cooler and channel model, channel mode by which the flow can take place and what would be these the this capillary pressure, surface tension, contact angle other rock properties and how they affect this relative permeability. We have already gotten some idea of this by now.

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Multi-phase Flow of immiscible fluids ... Contd.

Buckley-Leverett Method

$f_w = \frac{Q_w}{Q_T} = \text{fraction of flowing stream, comprising of displacing fluid (wetting phase 'water').}$

Material balance over an infinitesimal element

$$\left(\frac{\partial S_w}{\partial t}\right)_x = - \frac{Q_T}{\phi A} \left(\frac{\partial f_w}{\partial x}\right)_x$$

f_w can be approximated as fractional flow function = $\left(\frac{k_r}{\mu}\right)_{\text{wetting}} + \left(\frac{k_r}{\mu}\right)_{\text{non-wetting}}$

So, we were discussing we in fact, just briefly introduced in the last lecture this Buckley-Leverett formulation, where we said that if we have a porous plug encased in a in in in a in a core holder or encased in some kind of wrapping. So, that the flow can cannot take place through the site, flow can only take place unidirectionally through this in this direction and this is at interstitial wetting phase saturation.

And, it is what is injected to this is flow over this is only the wetting phase is injected. So, it is at interstitial saturation what; that means, is that if there is if interstitial saturation is 10 percent that is this is the limit; that means, 0.1 is the limit. Water can be or wetting phase can be in this porous medium by a fraction of 0.1. So, what that means, is that rest 0.9 part of the void volume is occupied by non-wetting phase.

So, the by let us say oil so, 0.1 is the wetting phase and 0.9 is the non-wetting phase. So, that was the case in these in this porous medium, in this porous plug and then we are injecting wetting phase which is water, at a rate the total flow rate is Q T. So, this is this is what we are injecting at the inlet here. So, we said that we if we if we pick up at differential length at a distance x from the inlet, if we pick up a differential length delta x then if we can draw a quickly a volumetric balance.

And, we can do volumetric balance because all densities they are incompressible wetting and non-wetting phase. So, density is not density is constant. So, we do a volumetric balance, volumetric balance in the sense these Q T is the flow rate, total flow rate and we

have defined already the this f_w which is a fractional flow function and f_w is given by the volumetric flow ratio at any point, say at any point we pick up this this is the ratio Q_w by Q_T .

So, basically Q_T is comprising of Q_w and Q_{nw} . So, Q_w is wetting phase Q and Q_{nw} non-wetting phase; so, that is being injected. So, Q_T is injected and we at every grid we have some value of Q_T , if I break this into 100 such grids or 1000 such grids over the length then at every grid we will have value of Q_w , some value of Q_{nw} . So, some of them would be Q_T and we will have some value of S_w which is the wetting phase saturation, some value of S_{nw} which is non-wetting phase saturation.

And, once again sum S_w plus S_{nw} has to be equal to 1. So, to start with we assume that this entire core is having let us say 0.1 is the wetting phase saturation and 0.9 is a non-wetting phase saturation. So, that the sum becomes 1 that is the total void volume. So now, this is there is a flow going in. So, we said that the volume of this differential element I mean the void volume of this differential element, first of all the overall volume if area of cross section is A .

If this area is A so, then this A multiplied by the Δx that gives me the volume of this particular, section cross sectional area perpendicular to the screen is area A and this length is Δx . So, $A \Delta x$ is the volume, this multiplied by the porosity ϕ . So, this gives me the void volume and out of this void volume, how much was the saturation is there initially? Let us say at some point of time the saturation is S_w at time t and at time $t + \Delta t$ these saturation had become $S_w + \Delta S_w$; that means, over duration Δt the saturation increases by ΔS_w amount.

So that means, once again the what is the amount of so, so this quantity defines what? $A \Delta x$ is the volume multiplied by ϕ that gives me the pore volume, pore volume multiplied by the saturation of wetting phase. So, this gives me the volume of wetting phase in these differential element. And here this gives me what? $S_w + \Delta S_w$ and multiplied by $A \Delta x$ into ϕ , this gives me the volume of this wetting phase after $t + \Delta t$ time.

So, this is at T time saturation was S_w , this is at $t + \Delta t$ time; we are assuming that over Δt time the change in saturation was by the amount to ΔS_w . So, it was S_w , now it is this $S_w + \Delta S_w$. So, what is the change in volume of wetting phase in

this case? Change in volume and volume of wetting phase in this case would be $A \Delta x$ is the total volume multiplied by ϕ . So, these term remains as it is and multiplied by $\Delta S W$ because, change means this and this their difference.

So, $S W$ and $S W$ will cancel out only you are left with $\Delta S W$. So, this is the change in volume of wetting phase in as far as this differential element is concerned and why this change has taken place? First of all these changes taking place because or that that is that is the only reason this change has taken place, because there is a flow in and there is a flow out and this in minus out they are different these in minus out is different.

So, how much is in? In was $Q W$ at x multiplied by Δt , Q is the flow rate; flow rate means meter cube per second into second. So, this gives me the meter cube of water that has gone from inlet and what has gone out is $Q W$ at $x + \Delta x$ into Δt , that is the amount that is living from the right phase. So, we take the difference between these two. So, in minus out is equal to accumulation. So, in minus out is equal to accumulation. So, these two we have to link and moment we will link this two, we say that we take out this Δt out and we take this Δt up here.

And, we take out Δx out, we take this Δx here and further $Q W$ instead of $Q W$ we replace this $Q W$ by, we replace this $Q W$ by $f W Q T$ and $Q T$ is a constant. So, $f W$ is varying over length. So, $f W$ into $Q T$ why because $f W$ into $Q T$ here is equal to $Q W$ we know, that is how the $f W$ is defined and similarly here also we do the same thing, here we write $f W$ at $x + \Delta x$ into $Q t$. So now, we take out this $Q T$ outside and what we can and if we if we insist for a limit Δx tending to 0 here and on this side limit Δt tending to 0 here.

So, then we end up with here $\Delta S W \Delta t$ term and here $\Delta f W \Delta x$ term and that is exactly what we arrived at here in this material balance, that $\Delta S W \Delta t$ at x is equal to minus $Q T$ by $\phi A \Delta f W \Delta x$ ok. On top of that what we did? We have we have this $f W$ is the ratio of the water flow rate divided by total flow rate, wetting fraction flow rate divided by total flow rate. And, that was written as K_r by μ for K_r by μ relative permeability because bass permeability K will cancel out.

So, relative permeability of so, basically we are looking at the volumetric flow as per Darcy's law, a wetting phase divided by volumetric flux of Darcy's law over the wetting phase and the non-wetting phase. So, K by K and Δp by l those were canceled and it

is written as K_r by μ_w wetting divided by K_r by μ_w wetting plus K_r by μ_w non-wetting. So, that is how f_w is defined.

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Buckley Leverett method Contd.

From the material balance,

$$\left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{Q_T}{\phi A} \left(\frac{\partial f_w}{\partial S_w}\right)_t$$

Upon integration,

$$x_{S_w} = \frac{Q_T}{\phi A} t \left(\frac{\partial f_w}{\partial S_w}\right) + x_0$$

- * Assumed that at $t=0$, the water saturation in the entire porous media is uniform and equal to irreducible water saturation.
- * All water saturations from S_{wi} to $(1-S_{nr})$ are represented at $x = x_0$ (inlet).
- * Each saturation will advance into the system at a rate that is directly proportional to $\frac{df_w}{dS_w}$.

So, next what is done here is; so, this is there has there is some readjustment in this material balance in the form of this equation and then this equation was integrated to find out; if we integrate this. So, $\frac{dx}{dt}$ at constant S_w so, x at S_w is equal to this quantity, this is this is basically this t goes to that side. So, $\frac{df_w}{dS_w}$ remains as it is, t goes there so, dt goes there. So, when you integrate this dt became t and the one add one some x_0 term stating that at t is equal to 0 x is equal to x_0 .

Now, what is the meaning of this x_{S_w} ? What does this mean? So, let us let us first see what we have written here and then I try to make some meaning out of this expression. Because what exactly is the x ? Is it position of the front? But, then I do not think it is just the position of the front. So, we need to define what we mean by x carefully because, x within a with a subscript S_w , it is not just any x ok. So, here we actually let us see what we are written here; assumed that at t is equal to 0 the water saturation in the entire porous media is uniform and equal to irreducible water saturation.

This we already know, this we already know in fact, this we or this we already know that if we this was the porous medium we have and this porous medium is contained; this wetting and non-wetting phase initially the wetting phase was at S_{wi} irreducible water saturation. So, that is exactly what it says that at t equal to 0 the water saturation in the

entire porous media is uniform. So, everywhere it is S_{wi} because, we cannot go below S_{wi} by not by any means. So, this is this is the energies uniform and equal to irreducible water saturation. All water saturations from S_{wi} to $1 - S_{or}$, represented at x equal to x_0 .

So, we are assuming as if I have some packets now we started injecting water. So, when I started injecting or wetting phase, when I state started injecting wetting phase from this side. This is primarily 90 percent of the void space is non-wetting phase and 10 percent is wetting phase and I am injecting the wetting phase. So, what could be then what could be the range of saturation possible here? The minimum saturation possible here is S_{wi} and maximum what a saturation minimum would be a S_{wi} you cannot go below that.

And, similarly the maximum possible saturation is $1 - S_{nwr}$ which is which is given here as S_{or} , this is same as S_{nwr} non-wetting residual or here o is o stands for oil; so, S_{or} or a S_{nwr} . So, you have a limit between S_{wi} and $1 - S_{nwr}$. So, water saturation in this porous medium has to remain within these range ok. To start with at x equal to 0 everywhere it is S_{wi} .

When you start injecting wetting phase saturation, then the at the at the inlet this saturation would be $1 - S_{nwr}$ you cannot go anything beyond, because you have to leave a S_{nwr} . The residual saturation in the porous medium even after 20 or 1000 pore volumes of non 20 or 1000 pore volumes of wetting phase injection; still there would be non-wetting phase remaining and that amount that saturation is a S_{nwr} . So, this is the limit within which the water saturation can be changed.

So, what they are saying is that all these water saturations I mean, I can put some numbers I can put these S_{wi} as 0.1 let us say and S_{nwr} as on 1 second 0.1, let us say. I mean it could it is generally 0.2 0.25 like this and at that range let us say we assume 0.1 there and 0.1 there. So that means so, $1 - S_{nw}$ so 0.1 is basically 0.1 is basically S_{nwr} . So, $1 - S_{nwr}$ would be 0.9 so, these within this range the S_{wi} can vary. So, what it assumes is as if I have packets of saturations, I have packets of saturations; some belonging some 0.1, some 0.2, some 0.3, some 0.5, 0.65, 0.75.

So, all kinds of saturation packets are present here at the inlet and now I since I am injecting from this side, these saturation packets are traveling ok. So, when we write $d x$

$\frac{dx}{dt}$ at S_w , this term implies at what velocity that saturation packet S_w , this is a constant S_w . So, that particular S_w will be traveling at this velocity. So, so that let us say I am focusing on a saturation packet which has saturation in a packet whose water saturation is 0.3752.

So, 0.3752 would be traveling at a velocity which is given by this so, but let us say we look at the right hand side. So, how would I get Q_T is already known, that is the total flow rate that we are injecting, that is some meter cube per second is known. Porosity we know to start with what is the porosity of this medium, A the cross sectional overall cross sectional area that is known and $f_w \frac{dS_w}{dt}$ so, somehow we have to find out what is $\frac{df_w}{dS_w}$. Now, I have some hook there because already we have written f_w as equal to you may recall K_r by μ for wetting phase divided by K_r by μ for wetting plus K_r by μ for non-wetting.

So, if and we know for these K_r this so, this is K_r wetting this is K_r wetting this K_r non-wetting, μ is constant, μ of wetting phase if it is water it is close to 1 centipoise and one can figure that out. Similarly if it is oil, oil has this viscosity which is known. So, this μ can be just directly plugged in, but K_r would be we already saw Corey's relation we already saw, this K_r would be a function of saturation. In fact, we have already seen those plots of relative permeability, relative permeability versus saturation. We have found that if we look at K_r versus water saturation then which one would be going down with increase in water saturation?

Definitely, this curve is K_{ro} because K_{ro} will decrease with increase in water saturation whereas, K_{rw} is expected to increase with increase in order situation, because more part of the pathways will now be available, because more part of the pathways would be occupied by water. So, they will be that pathway would be available; so, relative permeability would be high. So now, we have already we have some expression. We have some empirical relation, we have K_{ro} as a function of S_w and K_{rw} as a function of S_w ; we already know these. So, though we can put those equations here and we can find out what is f_w versus S_w , what is the relation.

So, f_w is basically f_w is only the function of S_w . So, then we have some way of finding out the f_w as a function of S_w . So, by using these using these using these relations using these correlations. So now, suppose i is at the in fact, in fact one can draw

a plot of f_w versus S_w and this this a plot can be drawn. A plot can be drawn which takes shape something like this where, these value of f_w is 1.0 and this plot will start here from S_{wi} and this will reach 1 at $1 - S_{nr}$ or S_o .

So, this again within this range f_w can f_w will take a shape like this, where f_w will vary from 0 at S_{wi} , at S_{wi} if everywhere it is S_{wi} and so, so I can expect that add S_{wi} only the non-wetting phase will flow, wetting phase will be 0. The fraction of wetting phase will be 0, if the saturation is interstitial water saturation. So, this is 0 whereas, if the saturation in the per plug is $1 - S_{nr}$ or water saturation is $1 - S_{nr}$; or in other words the non-wetting phase saturation is at the at their limit there. So, then it would be only the non-wetting phase flow and no wetting phase.

I mean so, there they there you have these or rather it will be only wetting phase flow and no non-wetting phase flow. So, this is the limit f_w is between 0 to 1 and if you use this relative permeability curve these correlations, one can come up with and come up with a curve for f_w versus S_w and, these curve so, this curve now at any point let us say at these S_{wi} I am interested to know what is $\frac{df_w}{dS_w}$ that is very simple. I have to draw tangent at this point and find the slope of this.

So, that gives me this $\frac{df_w}{dS_w}$ ok. So, so what does this what does this what does this equation mean in that case? These equation means that these packets, let us say this packet that such a saturation we are looking at is let us say 0.4137 that water saturation and 0.4137 I want to know at what rate this is traveling or over time t over time t how far this packet will travel over time t or at what rate this will be traveling? See if we want to know so, for that 0.4137 I will go up, find out what which point we are looking at.

At that location I have to draw a tangent, find out the slope that slope gives me $\frac{df_w}{dS_w}$ and we put this time, we put this $\frac{df_w}{dS_w}$ all these this is basically $Q T$. So, $Q T$ is known, ϕ is known, A is known and x_0 we can assume to be 0 because at time t equal to 0 all saturation packets are near the inlet. So, now we can find out these saturation packet we are referring to 0.4137 how far that will travel over time t because some saturation packets will travel far, some saturation packets for example, if we look at saturation packet which is 0.9.

This 0.9 would not travel anywhere, 0.9 would be almost here itself whereas, you will find 0.1 that is the lowest, that has to that can travel all along because the already these

entire porous medium is to start with this was the saturation and in between some packet will travel up to this much, some packet will be this much, some will be this much. So, that is why you recall what was the curve we got? We got this type of curve saturation S_w as a function of distance and this endpoint was these endpoint was $1 - S_{nr}$ or S_o or the same thing.

So, this is the limit that is that is what the saturation would be at the face, that would be least traveled whereas, this would be S_{wi} ; that means, this is at the interstitial water saturation. So, the front has come up to this point. So, this is this this part is still at S_{wi} , front as front has come up to this point, so rest of it is all still S_{wi} and here this saturation packet this has travelled up to this distance, this saturation packet this is traveled level this distance whereas, saturation packet which is for the lower, it has traveled up to this distance. So, how far they have traveled over time t ?

So, this these this curve that we are referring to these curve is frozen at a particular time. So, this curve is frozen at a particular time. So now, this at this time how far the saturation front has traveled? How far the saturation front has travel? So, each packet how far they have travelled we can find this out; so, this is what is x_{S_w} . So now, if we could see how it has been articulated here, all water saturations from S_{wi} to $1 - S_{nr}$ or $1 - S_o$; I mean N_w is a non-meeting phase, o is the oil. I mean they are used synonymously here, these are represented at x equal to x_0 which is inlet out rather x equal to 0 for our purposes.

And, each saturation will advance into the system at a rate that is directly proportional to $d f_w / d S_w$. Each saturation is traveling at a rate, this is the rate dx/dt that is the speed at which each saturation packet is traveling and that is proportional to df_w / dS_w and how we get df_w / dS_w ? We got these f_w this expression was there with us and these f_w expression there we already have relative permeability curves. We have, I mean empirical relation if one wants one can write these relative permeability as a function of surface tension contact angle and other rock properties that is also welcome.

So, somehow you can find you at the end you have to come up with some expression either or some kind of curve between f_w and S_w and this curve will very much look like these because, I can see that f_w has to be 0 when saturation need to S_{wi} whereas,

when the saturation is at the other limit; f_w has to be equal to 1; $f_w = 1$ means only wetting phase is flowing no non-wetting phase, $f_w = 0$ means only non-wetting phase is flowing no wetting phase. In fact, you can see when this saturation here is S_{wi} as saturation S_{wi} what is flowing here?

Only the non-wetting phase right, wetting phases is retaining its own saturation it is not changing. So, what you are producing at the outlet is non-wetting phase; you are injecting wetting phase, but producing non-wetting phase because that is the case you are operating your point is here f_w equal to 0 whereas, this entire place when it would be all flooded there is this is this this you will be here at this point and so, it would be only producing water because, water will be flowing through the system and oil will be simply trapped inside the pore as residual situation at that time.

And, at that time the saturation profile would be this, that is the final thing; I mean after injecting 20, 30, 100 pore volumes of water; 100 pore volumes of wetting phase this would be the saturation and but still it is $1 - S_{nwr}$ because, S_{nwr} you have to leave, you have to leave the little amount of oil inside the system as residual saturation.

I will build on this further on the on this in a you know in our next lecture. So, this is all I have as far as this lecture module is concerned.

Thank you very much.