Flow Through Porous Media
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Lecture – 40
Immiscible Flow (Contd.)

I welcome you to this lecture of Flow Through Porous Media.

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We were discussing about Immiscible Flow more importantly we are talking about relative permeability in case of two phase flow.

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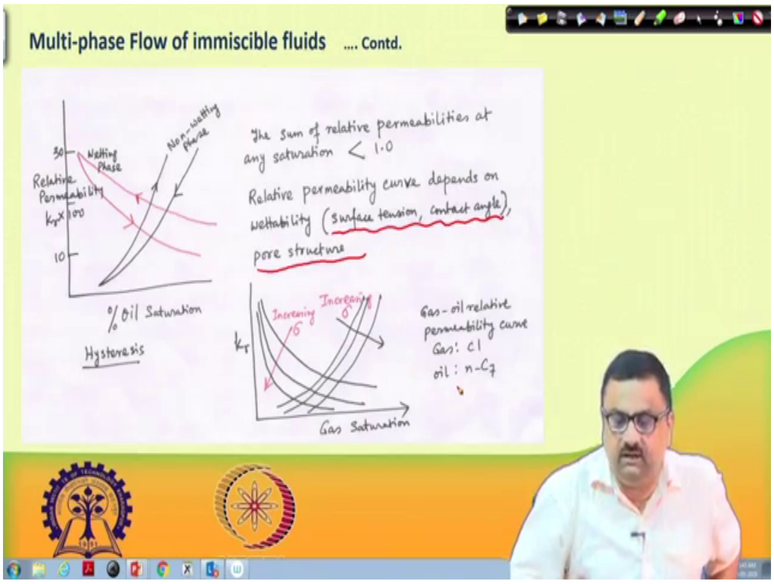

What we did in the last lecture was we said that the relative permeability would be a function of saturation and we said that relative permeability ideally is a function of surface tension, contact angle and pore structure and the other fundamental work done, but and one can see if they change this interfacial tension. So, relative permeability would be the curves will shift for a gas oil system as I have reported here. However, this general trend is to do experiments and get an empirical fit of this relative permeability versus saturation curve.

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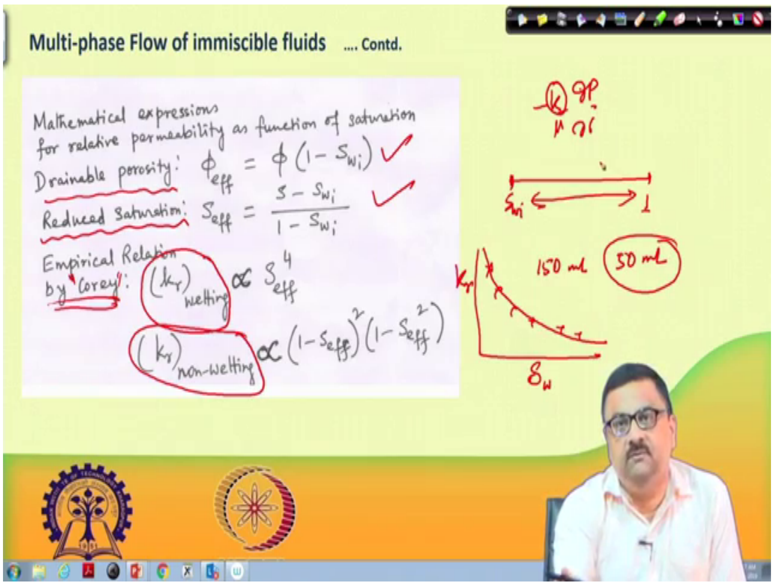

So, what we see here is these are typical mathematical expressions for relative permeability for wetting and non wetting phase, you can see this is the relative permeability of the wetting phase and relative permeability of the non wetting phase and you can see there is empirical relation, this is considered proportional to s effective to the power 4 S is S effective is called reduced saturation and this reduced saturation is defined as S minus S wi divided by 1 minus S w i.

So, what essentially they are saying is that the saturation has a window we said between 0 to 1, but what they are saying here is that it is not exactly the window of 0 to 1 rather its a window of S wi and 1. This part anyway you can never achieve you can never achieve this part because it these this you can never go below S wi for a two phase system. So, actually the window is between S wi and 1. So, the that is why one got these reduced saturation and further they have conducted this something called a drainable porosity; that means, porosity which can be which can be drained.

So, that is called drainable porosity the porosity which can be through which oil could be injected the entire say let us say I talk about last time you remember 150 m l out of these 150 m l 50 m l is the pore volume. So, entire 50 m l is not drainable entire 50 m l we cannot fill this entire 50 m l by oil apart there has to be there would always we have to keep the provision for interstitial water saturation right. So, interstitial water would remain in some place rest of it is drainable rest of it can be filled by oil.

So, then that is drained. So, that is why we have a drainable porosity which is the actual porosity into 1 minus S wi. So, we basically we are working with this part of the porosity the other part is practically I mean closed for us and similarly this saturation we are working with this effective saturation because below S wi we cannot go ever. So, with these definition of phi effective S effective phi effective and s effective the empirical relation that is suggested by the researcher Corey is k r wetting is proportional to S effective to the power 4 and k r non wetting is proportional to this function of s effective. So, generally what that the trend here is 1 gets a plot like this. So, generate these data points by experiment where this is k r and here it is let us say S w or S oil and then fit a line through this keeping these correlations in mind ok.

So, these are these correlations are I mean from empirically available. So, keeping these correlations in mind these lines are fitted and these lines are now considered further for predicting what would be the flow now we are talking about only the linear flow right in one unidirectional flow, but we may work with a radial system. So, there these then this they are also with this we have for a radial system we have what minus k del P del k by mu del P del r was the v r right.

So, here also this only this k would be changed by k into k relative permeability and this pressure would change to that pressure of that particular phase. So, this all these extensions will be done, but first we need to know what is this k; k is k k r k is known I mean k is the base permeability, but what is this k r? Relative permeability. So, this k r one must know. So, generally it is an experiment is performed on that porous medium with the same liquid system and then one can find out what is the relative permeability.

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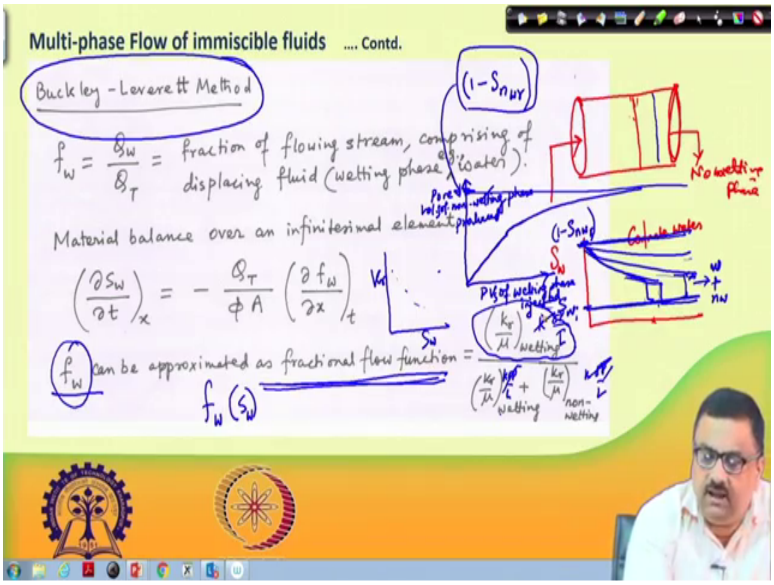

Next what we get into is suppose we have a two phase; suppose we have a two phase flow and suppose we have a situation and which is very common for a reservoir is that there is let us say I have we talked about a unidirectional flow. So, let us say we are talking about a core plug. And these core plug is at interstitial water saturation interstitial saturation; that means, it is basically oil with some amount of what they call there is in reservoir language they call it connate water. So, there is small amount of water inside which is the minimum amount of water that has to remain and rest is are all oil and then suppose the water is injected into this core ok.

This Buckley Leverette method is developed for flow of wetting phase into a core which is at the limiting saturation of non wetting phase; that means, it has highest amount of non wetting phase present with bare minimum wetting phase that has to be there. So, that is how the entire core is that is the status and then what is injected is the wetting phase only and then there would.

So, there would be a production. So, we expect first it would be what it would produce, it cannot produce any wetting phase because wetting phase inside it is all non wetting phase with bare minimum of wetting phase which can never be displaced right that is what is called interstitial saturation. So, initially there they will produce all non wetting phase. So, if I inject non wetting phase.

So, if we inject 100 milliliter per hour of water, I will produce 100 milliliter per hour of oil from the outlet. So, this will continue and the water the I am injecting a water. So, water front will travel through this. So, then at one point this water front will reach the outlet. So, then after the water after water front reaches the outlet from that point on, we will be they will be producing water plus oil.

So, till this time only oil will be produced. So, till this say let us say this water is injected. So, there would be water front travelling, but it we need not have to assume that this would be water front means say let us say if I look at the saturation at an in between time ok. So, I am talking about saturation of wetting phase ok. So, let us say the water that I have injected the front that I have injected it has come up to up to this location ok. So, the saturation that we will have here is, the saturation here it is all water at this phase. So, what would be the saturation here? It is all water being injected.

So, only leaving out the residual oil which can never be pushed out from the core, there is a residual oil right our concept was that if this is the pore there would be some amount of oil always will be sitting at the center which we can never remove and I mean when we inject water and when we start injecting oil only, then oil will flow this pathway and this blob will grow and everything, but there is some amount of water; there is some amount of water which will be remaining here which we can never remove.

So, this water is we are referring as interstitial saturation and this oil we are talking about is the residual saturation. So, now and any in between we are having either we can switch from all water flow to all oil flow, but always this phase remains.

So, now let us say this front has traveled up to this point. So, here at this location the saturation would be this we have to we have to leave this part out. So, it would be 1 minus S nwr; nw S non wetting residual means this has to be there minimum. So, let us say we write 0.15. So, 0.15 saturation; that means, 15 percent of the total pore volume will be remain the non wetting phase will remain in there which can never be pushed out. So, that is a limiting value. So, this 0.15 non wetting phase saturation; that means, the water phase saturation would be 1 minus 0.15; that means, 0.85. So, water saturation at this location at the inlet phase we are injecting only water no oil is injected from this phase.

So, the phase will have the saturation which is 0.85; that means, 1 minus 0.15. We cannot go beyond 0.85 that is a limit because 0.15 of 0.15 of non wetting phase has to remain inside the porous plug. So, this is the limit. On the other hand what is the limit on this side? Let us say the front has traveled up to this point. So, beyond that what do we have? Beyond this there has to be this saturation of what is that? That is interstitial water saturation that is the minimum water that one has to have inside the plug the you cannot go below this saturation. So, that is another limit ok. So, in between there would be some. So, the in between water has traveled. So, it pushed out some amount of oil.

So, now one can expect this kind of a plot like this. So, this is the typical saturation profile I can think of when the front has traveled up to this point. Beyond this point this is interstitial water saturation that is; that is; that is the situation when we started that is the saturation everywhere, but already I have produced from this part whatever is the oil that has already been produced at the outlet.

So, this would be the saturation profile in this case. If I look at the next instance when the front has traveled up to this place. So, this would probably look like; this would probably look like this. Then when the front reaches the outlet lead this probably look like this, the front has already reached in the water saturation we are plotting water saturation.

Then gradually the water saturation will go up at further time as time increases and then at if after I have flowing 24 volumes or a 30 I mean we after flowing several pore volumes, we will reach this entire plug will have the 1 minus S nwr; that means, we say that this entire plug is now at residual saturation residual non wetting phase saturation.

So, that is it is primarily water and the minimum amount of non wetting phase has to be there. So, it is flat it is all constant. So, that would be the that would be after injecting 20 or 30 pore volumes. So, initially it develops like this initially it was to start with it was like this, it was all everywhere this was the saturation S wi and then started injecting water.

So, front has traveled up to this position, front has traveled up to this position, front has traveled till the outlet this is gradually growing and at the end this would be the saturation. So, this is something this is a conceptual understanding we have as far as the flow of two phases are concerned. Now if somebody tries to plot cumulative flow at this time, if somebody tries to plot a cumulative flow here in that case one notes that.

If I plot pore volume of non wetting phase non wetting; non wetting phase produced and this axis is pore volume I write PV pore volume of wetting phase injected. So, pore volume of wetting pore volume I should say P pore volumes pore volumes pore volumes means let us say I have I said that 150 m l is the size of the core are from outside out of that pore volume at the void volume is 50 m l. So, rest 100 volume is the solid volume.

So, 50 m l is the void volume and so, porosity is one third. So, now, out of that 50 m l now 50 m l is the pore volume, it is called pore volume of that particular plug. Now suppose I inject 300 milliliter so; that means, I have injected 300 divided by 50 which is 6. So, I have injected 6 pore volumes through the core. So, instead of calling it in milliliter pore volume PV becomes a unit of volume if somebody says I have injected let us say 25 milliliter. So, immediately the implication would be that I have injected 25 divided by 50 which is half pore volume. So, immediately it will occur to me that half pore volume if I injected I would have must have gone halfway down this plug.

If I have injected three fourth pore volume, then I must have traveled three fourth down the plug. So, instead of calling it in milliliter we will be the referring it in the pore volume would be the unit of volume in this case. So, in this case; so, if this is if we look at pore volume of non wetting phase produced and pore volume wetting phase injected, I must note here that how much of pore volume of non wetting phase can we produced there is a limit to it.

First of all it has to be less than 1 because I have to start with the non wetting phase was there I mean non wetting phase I am not injecting anything non wetting phase is what is there in the plug. So, that only that is something which we are extracting first and second thing is that is the first thing. Second thing is that there is a limiting saturation which goes by the name residual saturation. So, we have to leave something called a residual saturation.

So, that has to remain inside the core that cannot be pushed out. So, pore volume of non wetting phase this limit upper limit here is simply 1 minus S nwr 1 minus S nwr right. So, because that is the residual non wetting saturation we have to keep. So, initially I had 1 I mean the out of one that the out of the oil that was present I can produce I mean I can produce oil, but that there is a limit to it, it has to be less than it has to be a 1 minus it has to be you one has to keep this residual phase saturation. So, this is; this is; this is there is a limit to it and as a matter of fact I think it should be not only 1 minus S nwr to start with it was not fully one pore volume to start with this was this was to start with we had a S w i. So, S wi.

So, initially the oil that was present was 1 minus S wi pore volume of oil was present and S wi of wetting phase was present 1 minus S wi of non wetting phase was present and out of that so, that was present and at the end we have to leave. No it is not necessarily this we can simply we have to leave this much of oil at the end. So, this much of oil has to be.

So, the pore volume if I look at it is 1 minus S nwr. So, that is the limit ok. So, now, if I look at initial part when this front has not reach the outlet till this front has not reached the outlet till that time whatever wetting phase was injected, the same pore volume of non wetting phase will be produced whatever wetting phase is injected same non wetting phase would be produced. So, it would be a 45 degree line.

After the breakthrough; after the breakthrough now you would be producing both wetting and non wetting phase together. So, it will not be the amount of wetting phase injected and amount of wetting has produced they will not be equal. So, this will start turning like this and it will reach asymptotically to this limit. So, if we look at the pore volume of non wetting phase produced and versus pore volume of wetting phase injected.

Now, you can inject 24 volumes, here wetting phase there is no limit, but non wetting phase there is a limit because non wetting phase is what is sitting there in the plug that is only produced and it will follow a line like this follow a trend like this. Now suppose somebody wants to predict this kind this behavior, somebody wants to predict this behavior of wetting and non wetting phase, this is this would be the starting point. Obviously, if this is a simple situation in the sense only we assume that it is we are injecting only the wetting phase.

If we inject a mixture, there could be extension of these and there are ways to handle additional complications, but this would be the starting point of two phase flow. So, here in this case in this in this treatment we have a very well known theory which goes by the name the Buckley Leverett method. In case of Buckley Leverett method there is an important term here we must define which is called f w; f w is fraction of flowing stream comprising of displacing fluid which is the wetting phase or in this case water. So, fraction of flowing stream comprising of displacing fluid. So, this is defined as Q w by Q T. So, if the total flow is Q T and Q w is the out of that the Q w is the fraction of water.

So, you can see this f w can be approximated as fractional flow function; fractional flow function is k relative permeability by mu for the wetting phase divided by k relative by mu wetting plus k relative by mu non wetting why is it happening this way? You k say what are the other terms you have? K delta P by L they are assuming same delta P here and here also this would be k k r delta P by L k k r delta P by L.

So, these with this assumption canceling these terms all together one come one can come up with f w the fractional flow function which is its this this gives me when I multiply this k r mu k delta P by L this gives me the superficial velocity or volumetric flux. Volumetric flux of wetting phase divided by volumetric flux of wetting phase plus volumetric flux or non wetting phase. So, this is basically same as Q w by Q T Q w by Q w plus q n w. So, that is Q w by Q t.

So, this f w can be approximated by fractional flow function which is k r by mu wetting by k r by mu wetting and k r by mu non wetting. And k r as a function of saturation we have already said that we can do some experiment there plot k r versus saturation and then we can generate data points and we have some correlation for example, coreys expression is there how relative permeability depends on saturation.

So, those kind of expressions can be fitted to find out and an expression for relative permeability. So, if we have. So, basically this for a f w will depend on, see these are all constants. So, mu of wetting phase mu of non wetting phase these are constant and if I say that k r for a particular system depends on saturation S w or I mean k r is a function of saturation.

So, this k r this k this k r is for wetting phase this k r is for non wetting phase here. So, these k r; so, then entire thing we can write f w for a particular system, f w is a function of saturation let us say what a saturation. So, f w is a function of S w. So, f w can be. So, one can find out by looking at these correlations that one get different data points and fitted with some expression. So, one can come up with some f w expression for f w as a function of saturation. And further what one can do in this regard is one can do a material balance over an infinitesimal element.

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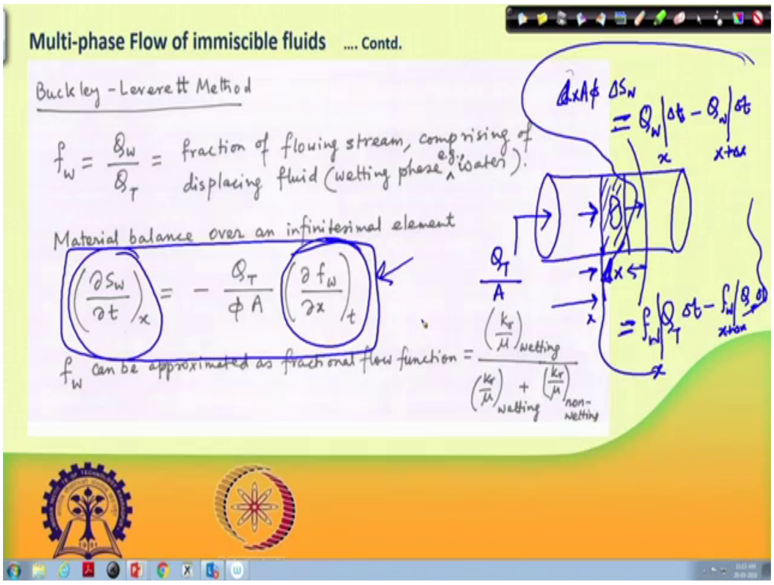

That means, that one can have one can draw a material balance; that means, suppose this is the core if one picks up a differential length d x let us say one picks up a differential length d x and they say that what is the; what is the flow that is taking place here? Here the flow is QT.

So, QT is the volumetric flow rate these divided by area this gives me volumetric flux ok. So, that is the flow of flow QT is equal to Q w because I am only having flow of no this say for example and one particular differential element. So, we have a flow of Q T here, then we have here we can write flow in minus flow out would be the accumulation in this differential element.

So, accumulation in the sense here in this case since it is an incompressible system the only accumulation can take place is the change in water saturation ok. So, a part that was S w now after time delta T we find that there is a change in water saturation which is let us say delta S w. So, delta S w is the change in saturation. So, that is the only how that is how the only accumulation can take place.

So, what is the volume of this element? D x into A is the volume overall volume multiplied by porosity is the pore volume multiplied by delta S w that is the change in saturation ok. So, this would be the change in volume of water as far as this differential element is concerned ok. So, change in volume of water as far as this differential element is concerned and this has to be equal to this is basically the accumulation of water in that case this has to be the Q w volumetric flow of water volumetric flow rate multiplied by. So, volumetric flow rate at x volumetric flow rate. So, let us say this is delta S w this is there actually you should not be writing it as d x you should be writing it as delta x.

So, delta x A phi delta S w that has to be the volumetric flow rate at x this is x. So, at x volumetric flow rate over time delta t over time over duration delta t minus the volumetric flow rate that has left x plus delta x over time delta t. So, when we take this and further this Q w on the right hand side so, this is equal to Q w and the right hand side can be replaced by f w into Q T we can see f w into Q T Q w is equal to f w into Q T at x sorry Q T is constant Q T is not changing.

So, we should be writing it as f w at x, Q T delta t minus f w at x plus delta x and x plus delta x Q T delta t. So, now, if you take this delta t to the left hand side. So, delta S w divided by delta t delta t tending to 0 this gives me del S w del t at constant x and on the right hand side Q w at x minus Q w at x plus delta x divided by this delta x divided by this delta x, this gives me. Now Q w is basically Q T into a Q T comes out Q T is the constant. So, Q T comes out. So, this becomes f w at x minus f w at x plus delta x divided by these delta x limit delta x tending to 0.

So, this gives me del f w del x at constant t and outside you have Q T here and this phi into A goes to the right hand side. So, from these equation; from these equation one can arrive at this material balance. So, this; so, essentially this is a material balance we have, this is the f w expression for f w we have. So, this; so, we will; we will; we will build on these we will build on this Buckley this Buckley Leverett method works with this basic equation with the definition of f w. So, we will build on this in the next lecture how Buckley Leverett method can be used for to predict for example, the cumulative flow that I presented or the saturation profile over the length of the porous plug that I presented in the next lecture.

Thank you very much.