Flow Through Porous Media
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Lecture - 04
Mass Continuity (Introduction)

I welcome you all again, to this course of Flow through Porous Media. We have given you some background of you know the basic characterization of porous media, basic characteristic parameters of porous media. And what all applications; you know we can have for; where this flow through porous media plays a very important role. What mixed I will do is I will gradually get into the theories of flow through porous media.

So, the first step there is to understand mass continuity in porous media because as I pointed out in earlier lectures, that we are working with a continuum assumption. We do not take into account that discrete changes in every nook and cranny of this porous media, rather we treat this porous media as more of a know homogeneous set, where the porosity, if I say porosity is 0.25 that 0.25 porosity is distributed uniformly. And so, I can consider the velocity to be a continuous field, velocity is not discretely changing from point to point. It is it is changing from point to point, but in a continuous mode.

So, our sampling volume is greater than the threshold volume that we referred in the last class. So, our continuum assumption is valid. So now, once we and once we assume that continuum is valid, then we can play with this continuum and try to see how what all we can what all we can gain in these; what all understanding we can gain by applying continuum in porous media flow.

So, the next step that I am trying to do here is I am going to discuss in today's lecture; obviously, Darcy's law I need to work on it a little bit more. And then mass continuity in Cartesian and cylindrical coordinate systems and pressure equations that we derive out of this.

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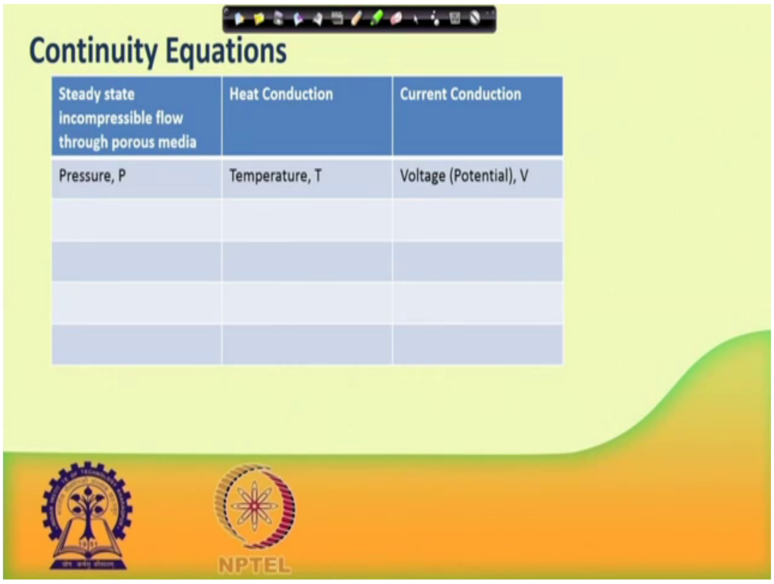

Now, regarding this Darcy’s law, now what we want to, what we want to show here in continuity equation that they equate that the Darcy’s law that I mentioned, which is which I am going to work on now to develop further equations with continuity. There it is very synonymous, it is very similar to heat conduction or current conduction and that cause and effect relationship that I mentioned in the previous lectures, so that how that plays out.

So, but before we get to this; let us dwell a bit more on these Darcy’s law. I mean what exactly we have learnt in the earlier lecture let us dwell a bit more on this. Let me dwell a bit on this Darcy’s law. Let me see what are we understood earlier, we thought we worked with a porous media.

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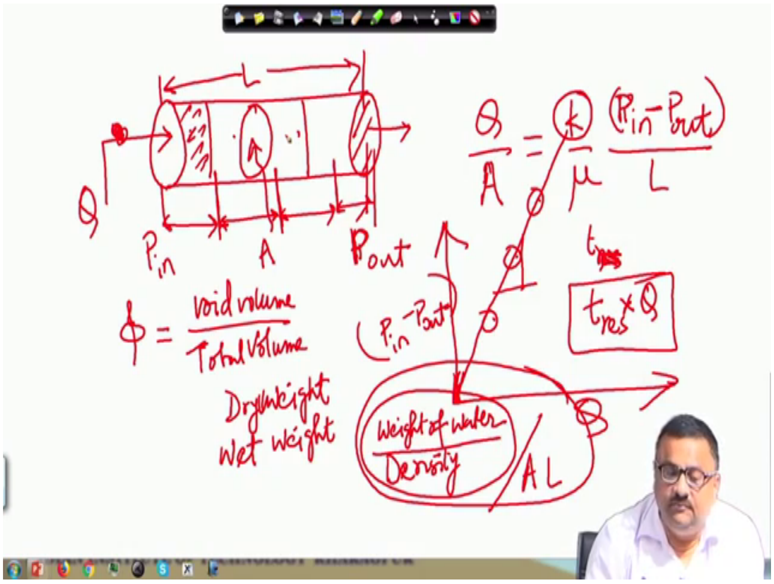

So, let us say I have a cylindrical porous media of cylindrical cross section, through which there could be flow possible. So, what we did at that time is we said that let us say I have a flow rate through this, which is Q and I have a pressure drop across this length of the bed, which is let us say this is P in and this is P out. So, we have what you call delta P which is the difference between these two pressures.

So, we said that Q divided by cross sectional area A; A is the overall cross sectional area this entire cross sectional are A, so this is A. So, this overall for this overall cross sectional area A Q divided by A that is equal to we said k divided by mu and then P in minus P out divided by if the length of the bed is L, so this is L. So, how do we find out the permeability? We said permeability can vary depending on whether it is a gravel pack or granite. So, one can have different levels of permeability and the unit of permeability is Darcy.

So, we can point out here, that I need to find out what is this permeability. So, how do we find the permeability? In that case what we will do is, we will find out the pressure difference; that means, this P in minus P out. And we will plot P in minus P out, P in minus P out; we will plot P in minus P out as a function of flow rate ok. So that means, I put a single flow rate and I get some value of P in minus P out; that means, I am giving 1 meter cube per hour let us see, or 0.1 meter cube per hour flow rate, I am putting in there.

And, I am getting a pressure drop to establish that flow through the porous medium. We get a pressure drop let us say of, so may, so and so much of Pascal.

So, then this forms a point on this plot ok. Generally I mean I can directly find out what is the permeability out of this because if I know P in minus P out if I know Q, I know the area cross sectional area available, I know the length L. And I know the viscosity let us say I am flowing water through this, so it is close to 1 centipoise. So, everything being known I can find out what is the permeability, but the common way of doing it is, we generally do we generally take 4-5 such data points because there could be experimental error.

So, you change the flow rate because after all flow is established through a pump or through hydrostatic head. So, flow rate is changed, so depending on these, we take some we take the different flow rates. So, as the flow rate is increased I can expect P in minus P out to go up. So, for a higher flow rate I see P in minus P out is higher further higher flow rate P in minus P out is even higher.

So, what we do in that case is then we draw a line through these points. Generally by least square feet; that means the error the line that I draw, the line will have some linear equation and with our linear equation is arrived at in such a way, so that the error incurred the amount of deviation of this line from the actual experimental data points, the sum of square of these errors that is minimum. So, that is how we define this slope.

And we expect this line to pass through 0 0 because there is no intercept to it ok. So, naturally, so we can draw these lines and go for a least square feet and from the slope of this line; from the slope of this line, one can find out what is the permeability. So, you can see here that, instead of relying on one data point, we can rely on multiple data points.

So, that if there is any error in measurement, there is that there is any error instrumentation error or any error in measurements, that gets that error gets even out. So, it is so, you can find out what is the permeability by measuring the slope of this line. So, this is one way of finding out permeability. So, if someone get a porous block; if someone get a porous block then apparently what they do is, suppose somebody has pulled out a core from a reservoir, then naturally it has to be put in inside a core holder using a rubber sleeve so, that no flow can take place through the sides of it.

So, all flow will be directed through the core and this would be placed in a core holder or there could be some kind of wrapping used, where there would be no flow coming of coming through this only you are allowing an unidirectional flow from inlet to outlet. And you establish a different flow rates and one can see what all pressure drops; that means, what P in minus P out.

So, if it if P out is open it is open to atmosphere then P out is atmospheric, but as you increase the flow you find P in is increasing; obviously, there will be more resistance because you are trying to flow try to establish higher flow rate. So, then this accordingly you find you plot these data points and you can find out what is the permeability of this medium.

So, other than permeability we talked about porosity, the porosity is we said that the last time we said that porosity is equal to the void volume divided by total volume; void volume divided by total volume. So, void volume divided by total volume how would you measure? I mean I can understand there is some amount of porosity inside. So, how we measure that porosity?

One way of doing it is if you have a dry; say for example, a reservoir core is provided to you, if it is a completely dry core you can saturate this core with; let us say brine salt water a saline water. So, by flowing it by making sure that generally this if the rock is water weight; if that if the rock is sort of hydrophilic, so all the pores will be filled with water. So, in that case if you measure the weight before introducing water; that means in a dry state and weight after it is completely soaked in water.

So, this difference in weight, the difference in weight gives you the total weight of water residing inside this core ok. So, now, you have the dry weight; you have dry weight; you have dry weight, you have weight 100 percent wet weight ok. So, if you take the difference; that means, wet weight minus dry weight that gives you the weight of water that is in place ok.

So now, if you take this weight of water and divide it so, you get weight of water by subtracting dry weight from the wet weight. So, weight of water, it is not exactly water it is saline water it is a brine. The generally it is brine is preferred in work when you work with this kind of sand stone course or that the reason is that it is the salt is put intentionally. So that the, clay material that are present inside the core that should not swell, that should not change the property. So, generally it is it is saline it in art environment, in many places it is already saline water. So, you try to simulate that you know simulate that water condition by putting some salt in it. It is generally we do not put de mineralized water in this.

So, weight of water; weight of water you get you divide it by density of water. So, if you divide it by density, this gives you density of water, this gives you the volume of; this gives you the volume of water. So, this is the volume of water that is present; that means, this is the volume of void present inside and then you know what is the total volume by looking at this I mean you know this is the area, this is the length. So, length into area is the volume. So, once you have this as the volume of void, these divided by A into L which is the total volume, so this gives you the porosity.

So, this is a one quick gravimetric way of measuring porosity, there are many other ways to measure porosity. For example, one can do some dispersion studies, we will discuss this as we proceed. That if we put a, if I have a flow going on there is some amount of residence time. For example, I introduce the flow here and that suppose I put some tracer here, I some tracer some color; I put here with the flow itself at the same flow rate. So, that color I expect that color to come out after a certain lag that is referred as residence time ok.

So, that residence time and if we know what flow rate we are injecting this into. So, then let us say I see this color to appear at the outlet, at let us say after some time which is known as, the residence time t res. So, color is introduced and you waited for some time, and you see that color is appearing at outlet. So then, you are talking about, so we are talking about residence time t res.

Let us say we are talking about t res, the residence time. Then this residence time if we divide; if we multiply this residence time with the flow rate; flow rate is let us say Q. So, then that gives me the flow rate is in meter cube per second and residence time in second. So, this second and second they cancel, so you are left with only meter cube.

So, this is basically the volume of pore in this porous media we did. So, we have some way of measuring permeability and some way of measuring porosity. See this exercise I want you to make note that these are the parameters which are not completely theoretical, it we have we what with these every day. I mean we can this is these are the parameters that people work with every day ok. And another point is you it may so happen, that this permeability of this core is not uniform. A part of this is of lower permeability, this part is of higher permeability. So, there may be sections of high permeability and low permeability.

So, how to find out what are the permeability's of different sections or to identify which section as lower; which section has higher permeability, for that typically you introduce pressure pores and regular intervals. Instead of relying only on P in and P out you can have pressure pores at intermediate locations and here you measure the pressure drop between this to this points, then pressure drop between this to these points. Similarly pressure drop between these two these points, then pressure drop between these two like this.

And, then when you apply the same equation, same equation when you apply then you will have not just one line you will have several lines ok. One line for each section and by measuring the slope you can find out the permeability of this section; permeability of this section; permeability of this section separately ok. And also if you work with P in and P out you can measure the overall permeability.

So, here then some permeability's they are in series and then you can; you have a way to measure the overall permeability also. So, this that one can work with to find out this permeability's and porosities. So, I mean take home message here from this particular slide is that you can, these parameters porosity and permeability though they are appearing in a differential form, but these parameters are very much you know working parameter that people work with every day. The other point that we need to discuss, before we go to this our main PPT slide again is about the continuity itself. What we understand by continuity?

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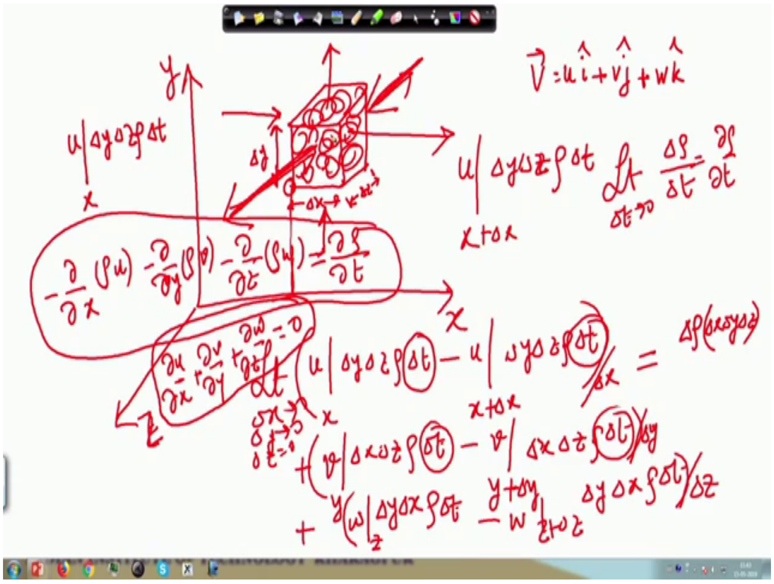

See we have studied, in case of fluid mechanics you may recall we have studied this continuity something like this. Let us say I have this is x y and z. So, what we do here is we pick up a differential element; we pick up a differential element of size delta x, delta y and delta z.

So delta x, delta y and delta z we have these, there is a differential volume we are referring to. Now, suppose I have a flow taking place and let us say I have this velocity filled at this location at this at the center of this the velocity field is, let us say or let us say the velocity at this location at x, y, z at this location.

So, this point refers to x this point refers to x y and z coordinate. So, I am talking about a differential volume of delta x, delta y and delta z. So, if we try to find out, how much of flow is in leaving from this phase and entering into this phase. What we write there is, with the flow that is entering from this phase is basically; if I write this velocity field as u i hat plus v j hat plus w k hat. Then we can see here, that the flow that is going in from the left phase is basically u hat x that is the velocity multiplied by delta y delta z; delta y delta z is the area of this phase, so that side also it is the area of this phase.

So, we are talking about the flow that is entering from this side. So, it is velocity in meter per second into meter into meter. So, meter per second into meter square which gives me meter cube per second, so that is the flow rate that is going in ok. And then if we multiply this by rho that gives me the; I have meter cube per second into kg per meter cube that gives me kg per second. And, this if I multiply over duration delta t then this becomes kg per second into seconds.

So, many kg of some mass that, is going into this system from this phase ok. Similarly, if we try to find out how much is leaving from this side it is u x plus delta x, similarly delta y delta z rho delta t. So, so many kg is leaving the, so many kg is leaving this phase. So, the same thing we can apply for flow that is coming from below and leaving from the top and a flow coming into this phase and leaving from the side. So now, if you take the now if you count all of them together; that means, in minus out plus in minus out; so in minus out plus in minus out.

So, if you take all these 3 ins and 3 outs, so if you account them, then what is left to it is the accumulation as far as this differential volume is concerned. So, in this case you can you can write, and if you expand this it would be u at x delta y delta z rho delta t minus u x plus delta x delta y delta z rho delta t. So, this is as far as the u part is concerned plus if we look at the v part which is y, then v at y this time v is this one, so this time you have the area which is delta x delta z

So, this is delta x delta z rho delta t minus v at y plus delta y that is leaving from the top phase, y plus delta y delta x delta z rho delta t. And similarly you will have another term here with w which is w at z; w at z actually we have drawn it in the reverse direction which should not which you should not have done. These let us say this is in and this is this is out. So, then it is we have some from some z, so that said into delta y delta z. Sorry delta y delta x in this case, rho delta t minus w at z plus delta z delta y delta x rho delta t.

So, this is the in minus out in all three directions and then all these in minus out has to be equal to the any change in change is happening inside. So now, this volume is fixed delta x delta y delta z they cannot change, the only thing that can change here, if at all is the density. Now, if you consider this to be incompressible, then obviously, that density there will not be any change in density, then you will treat this to be equal to 0.

And if you consider this to be compressible, then there would be a change in density which is by the amount delta rho. So, we have let us say change in density is delta rho; delta rho has a unit of kg per meter cube, but I have on the left hand side in minus out these are all in kg.

So, what do we have delta rho is kg per meter cube into delta x, delta y, delta z which is the volume of this differential element. So, this gives me the change in mass of this differential element because of change in density. So, in minus out cause this change in density on the right hand side. So, this is how we look at it and then if we divide this delta x, delta y, delta z to the left hand side, we will get the equation of that is what we refer because refer as continuity.

Basically, if you divide it by delta x delta y delta z you can see the fate of this equation. Here it is u at x this delta y delta z will cancel out when you divide by delta x, delta y, delta z this delta y delta z will cancel out rho will remain there ok. And this delta t is going to right hand side, I am sending these all these delta t is to right hand side. So, if we if we send this, so then we it is going to that denominator. So, that the denominator here the right hand side becomes delta rho by delta t; delta rho by delta t which when we put a limit delta t tending to 0, then this becomes delta rho delta t, the right hand side becomes delta rho delta t.

And this delta x delta y delta z has come to the left hand side, so these, delta x, delta y did out of this delta y, delta z we cancel out. So, you are left with this part of the equation would be left with only divided by delta x, these part of the equation since it is delta x delta z this will be left with divided by delta y. And this part of the equation will be left with divided by delta z. Now, if you say since delta x all are; these is a differential elements, so now, if you set limit delta x tending to 0, limit delta y tending to 0 and delta z tending to 0.

Then what you would get in this case is this would be this delta t is already gone to the right hand side ok. So, you will get and on as a left hand side you will get del minus of del del x of rho u minus of del del minus because, it is u at x minus u at x plus delta x ok. So, when you take limit delta x tending to 0 this time it you will get del del x of rho u minus del del y of rho v minus del del z of rho w that is equal to you have del rho del t.

So, this is typically an equation for mass continuity; this is an equation for mass continuity and when you have these rho is it is incompressible, then rho goes to 0 and these rho will come out from each of these within from first bracket outside.

So, in that case you will have simply del u del x plus del v del y plus del w del z that is equal to 0. So, this is the basic equation of continuity that any flow; any velocity field this is called a velocity field. So, any velocity field has to satisfy these components they have to satisfy this called mass continuity one has to satisfy this equation.

Now, I what I would like to see is that how this mass continuity applies when we have these differential volume as not just a volume, but it is a porous media. So, it is not just about out of this volume there is a porosity inward. So, when you have that kind of a situation; how this situation would how this porous, if you have a porosity term how this equation would be affected? So, I will continue this in the next lecture. Thank you for as far as this lecture is concerned, I will continue this continuity part in the next lecture.

Thank you very much.