

**Flow through Porous Media**  
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**Lecture - 37**  
**Immiscible Flow (Contd.)**

I welcome you all to this lecture of Flow through Porous Media, we were discussing about this Immiscible Flow. And we have talked about this concept of saturation and capillary pressure and we are going to build on this further and see what insight we can draw from these theories.

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of saturation

- Capillary pressure is inversely proportional to pore radius.  $P_c \propto \frac{1}{r_m}$
- As the pressure is increased, there will be penetration into some more smaller size pores.
- Increase in non-wetting phase saturation (i.e., decrease in wetting phase saturation) happens at higher capillary pressure.
- Irreducible Saturation ( $S_{wi}$ ): The lowest wetting phase saturation, which is independent of further increase of externally measured capillary pressure.
- Residual Saturation: The lowest non-wetting phase saturation arising from entrapment when the externally measured capillary pressure is decreased from high value to zero.
- Imbibition Curve: The relationship characteristics of the displacement of non-wetting phase from the irreducible saturation to the residual saturation.
- Drainage Curve: The relationship characteristics of the displacement of wetting phase from residual saturation to the irreducible saturation.

Handwritten notes on the slide include:  $P_c \propto \frac{1}{r_m}$ ,  $S_d + S_w = 1$ , and a diagram of a pore with a meniscus. The graphs show  $P_c$  vs  $S_w$  and fraction of saturation vs radius of pore.

Now, this capillary pressure we had talked about; capillary pressure we have talked about is, basically inversely proportional to pore radius this we already have seen that  $P_c$  is inversely proportional to  $1/r_m$  which is the mean radius of curvature which is basically the mean radius of curvature as far as the; as far as that meniscus is concerned. And then that is related to the radius of the capillary at that time at that point and if it is a cylindrical capillary; obviously, that is same as the radius of that capillary or radius. If it is cylindrical it is it is the radius of the pore that matters.

So, capillary pressure is inversely proportional to pore radius. So, what; that means, is that if there is a wetting phase inside the pore and non wetting phase pressure is gradually increased in the system. So, I have a system of pore several pores and let us

say to begin with they are all filled with wetting phase and then non wetting phase pressure is increased gradually into these in these in the system.

So, then as the pressure is increased there will be penetration into some more smaller size pores; that means, we have already seen that the penetration will first the penetration will take place in the largest pore. So, let us say I think of a porous medium having a pore size distribution; that means, we are having a pore size distribution. Let us say we have a pore size distribution with the mean is about 1 micrometer and the y axis. So, this x axis is the radius of the pore and radius of pore and y axis is fraction of total pores; fraction of total pores total number of pores with given radius.

So, that means; so that means, I am talking when I am talking about these radius, this is the corresponding fraction of total pores; that means, if there are 100 pores and if this number shows 45, then I would say that this particular radius which is maybe 0.5 micron there would be 45 percent of the pores are having 0.5 micron So, so we can have a pore size distribution like this.

So, naturally the pores are pores that are larger I mean I since  $p_c$  is inversely proportional to  $r_m$ , so; that means, when we are talking about larger pores; that means, where the  $r_m$  is large, then the capillary pressure would be small. So, that would be easier that would be a first place I mean as I am increasing the pressure from the water from atmospheric pressure as I am increasing the pressure, the first we will cross that pressure because that is a low pressure  $r_m$  is higher; that means, capillary measure is lower. So, this is a lower threshold and this lower threshold will be crossed first.

So, low lower threshold of capillary pressure is given by larger portion. So, that is why as the pressure is increased there will be penetration and this penetration into the first would be the largest pore should be penetrated and then gradually as more and more pressure as pressure is increased more and more there would be some more small smaller size pores, there would be penetration of non wetting phase into some more smaller sized pores. So, essentially what we see here is that increase in non wetting face saturation. So, what does this penetration of non wetting phase into the pore means? Penetration of non wetting phase into the pore means the non wetting phase saturation is increasing into the pore that is what the definition of saturation.

So, what; that means, is increase in non wetting phase saturation or in other words decrease in wetting phase saturation these are just reverse things. So, that will happen at higher capillary pressure. So; that means, as the capillary pressure is increased that non wetting phase saturation will start to increase and as the capillary I mean by capillary pressure means I mean to say the pressure of the system, as the pressure of the system is increased.

Why we are calling a pressure of the system? We thought of thought this as an idealized situation where all the pores are filled with wetting phase; that means, water let us say. And then outside I have dipped this entire porous medium into a bigger full of non wetting phase which is let us say oil and then gradually I am increasing the oil phase pressure and; obviously, leaving some pathway for water to leave the system.

So, if we could get into this hypothetical situation then I would expect that as pressure is increased more and more, then more smaller and smaller pores will be penetrated by oil and what; that means, is more and more the oil saturation that is non wetting phase saturation will increase. So, the pressure outside that that we are originally the pressure inside the porous medium was originally the porous medium contained water only and atmospheric pressure nothing happened outside oil inside water they will coexist.

But now, if we increase the pressure of the oil I expect some of the water will be moving out of the core and in that place some of the oil will be getting in and that differential pressure that is that we are referring as capillary pressure. So, now, this capillary pressure if somebody plots with saturation then this would appear something like this see mind it here this x axis is wetting phase saturation not non wetting for saturation. So, as non wetting for saturation increases; that means, more and more oil get into the system and that will happen only at a much higher pressure.

So, that is why as  $S_w$  decreases; that means, wetting for saturation decreases in this direction. So, in this direction in this direction wetting for saturation decreases non wetting for saturation increases and that is reflected here capillary pressure also increases. In these direction non wetting phase saturation decreases; that means, now we have less oil, so now, capillary pressure also decreases. Now, there are two limits to it and these are the two limits we have put here though  $S_w$  starts from 0 here and possibly go all the way to  $S_w$  could be 1 right.

So, 1 is the limit its a basically limit between 0 to 1 right because  $S_w 0.4$  means  $S$  in  $W$ ; that means,  $S$  wetting phase point four means  $S$  non wetting phase would be 1 minus 0.4 that gives 0.6 sum  $S$  wetting plus  $S$  non wetting has to be equal to 1. So, now, we have we are plotting with  $S$  wetting. So,  $S$  wetting the minimum can be 0 theoretically and maximum can be 1 theoretically, but there is one small issue which is the definition of something called irreducible saturation and residual saturation.

Irreducible saturation which is referred as  $SWI$  which is given here this is the irreducible saturation. What irreducible saturation means is the lowest wetting phase saturation which is independent of further increase of externally measured capillary pressure. That means, if we continue to increase the capillary pressure we will see that this will become a straight line like this, but saturation is not decreasing any further. What does this mean? This means that I have a porous medium filled with water ok. So, water is what does the saturation at that time  $S_w$  is equal to 100 percent water saturation single phase present in the system.

Now, we are introducing non wetting phase we are calling it oil and this oil is being pushed into the system and I am increasing the injection side pressure. And oil is going into all the pores first the larger pores then as I increase the pressure into smaller pores and pushing the water out from the system, but whatever pressure you impose there would be some water will always remain in the system oil can never push this out from the system or non wetting phase can never put this out.

This is a property of a random porous structure; that means, this is this is I mean one can think of these as the let us say these are the pores this is a pore. And let us say water if it is wetting phase or any wetting phase wetting phase tends to flow like these through this way. Wetting phase because, wetting phase prefers to stay near the wall there is an attraction of wetting phase with the wall because they go together.

On the other hand when it comes to the non wetting phase, non wetting phase is sitting here at the center of the pore as a block and when you inject non wetting phase. So, non wetting phase will create a pathway through the center of the pore, but if you expect this non wetting phase to go all the way and displace the wetting phase from everywhere 100 percent displacement that does not happen. So that means, there would be as you increase the pressure maybe this part would be filled this is this is probably grow, but

there will be some remnants of water always remain there and that is referred as the irreducible saturation.

On the other hand, now if we look at the other side of it; that means, when these now when you now you go back and then you start pushing water through the system, try to push the oil out from all the pore at that time this will shrink this will shrink, this oil will shrink. This oil will shrink means there would be more water traveling here like this like this, but there may be a small speck of oil sitting at the center which may not be one may not be able to displace that is called residual non wetting phase or residual saturation.

So, I have two limits here it is not going all the way to one, but before that there is a. So, this is given as  $1 - S$ . So, if you call this residual saturation as  $S_{nw,r}$  non wetting residual if I write this  $S$  this if this is SWI and  $1 - S_{nw,r}$  non wetting residual if I write, then you can see here the residual saturation becomes the lowest non wetting phase saturation arising from entrapment. So, you call this the entrapment, these non wetting phase is entrapped by the wetting phase so entrapment when the externally measured capillary pressure is decreased from high value to zero. So; that means, externally measured capillary pressure is decreased from high value to zero, that there is no capillary pressure you are not injecting any oil to the system out.

So, from outside it is all water. So, there is no capillary pressure no oil pressure externally imposed so that is 0 so at that time. So, at this other end there would still be some lowest non wetting phase saturation existing. So, non that that lowest non wetting phase saturation is  $S_{nw,r}$  and this irreducible saturation is already we said it is SWI. So, since we are drawing it on  $S_w$  scale. So, that is why I am writing it as the limit for is  $w$  is  $1 - S_{nw,r}$  because the sum of  $S_w$  and  $S_{nw,r}$  basically  $S_w + S_{nw,r}$  that has to be equal to 1. So, here in  $S_w$  since the axis is for  $S_w$ , so I have to write it as  $1 - S_{nw,r}$  so this is the residual saturation.

And there is these imbibition curve, which says that the relationship characteristics of the displacement of non wetting phase from irreducible saturation to the residual situation. So, displacement of non wetting phase from irreducible saturation to the residual saturation irreducible saturation is this SWI to this one. So, these is known as the this is known as imbibition because this here in this case the water saturation is increasing; that means, it is it is water is simply imbibed into the porous framework. Whereas, the

drainage carbon this is exactly what we defined in a rigorous term what is drainage imbibition. So, in rigorous terms it is the drainage curve is the relationship characteristics of a displacement of wetting phase from residual saturation to irreducible saturation, that means from this side to that side.

Generally now here in this context one may see that when this is gone backward, it may take a shape like this. So, this curve is known as drainage and this curve is known as imbibition. So, imbibition means the water saturation is increasing, oil is thrown out of the pores or wetting phase saturation is increasing non wetting phase is thrown out of the pore. Whereas, drainage means now you are increasing the capillary pressure you can see when increasing the capillary pressure and the non wetting phase is gradually getting into the pore first the larger pores and then the gradually into smaller pores at much higher pressure.

. So, this is so this is this is the going backward now ok. So, now, only point to be noted here is that these two curves are different they are not following the same pathway this is when it is drainage it is going this way when it is imbibition it is coming this way. So, one must look at I mean why it happens, but more or less these I mean its not completely unexpected because the way the pore was filled; you can see when oil entered oil entered to larger one when water entered water entered through the other side.

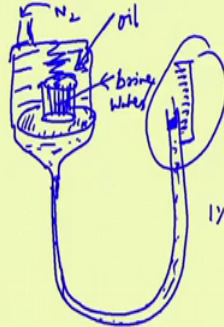
So, naturally the pathways are different I mean the same pore is getting filled, but through the different pathways. So, you can never expect that they would follow the exactly the same line anyway.

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Capillary pressure determination

Porous diaphragm device

- Sample saturated with the wetting phase is placed in a chamber with ultrafine fritted glass disk to prevent passage of oil. The glass disk permits the passage of wetting phase (brine).
- As the pressure on the oil is increased, the wetting phase is displaced from increasing smaller pores of the sample.
- Volume of wetting phase, forced out of the sample by non-wetting fluid is determined in the graduated portion of the tube.



The diagram illustrates the porous diaphragm device. It consists of a chamber containing a sample and a fritted glass disk. Oil is applied to the top, and brine is displaced into a graduated tube. A nitrogen gas inlet is also shown.

So, with this now let us see the way to determine capillary pressure now, we will get to that capillary pressure histories we are referring to capillary pressure histories that we will go to in a later. But let us first talk about what is this capillary pressure how capillary pressure is determined? There are two ways one can do it one is known as porous diaphragm device porous diaphragm device it works something like this that they have on one in porous diaphragm device let us say this is the porous medium.

This is filled with the wetting phase this porous medium is first of all placed in a ultrafine fritted glass disk. So, this is ultrafine fritted glass disk this is placed here then whatever is being collected that is being first of all this is completely filled with water and then there is a graduation graduated scale is there and then there is a small oil plug is present. So, that there is no evaporation can happen through this because if there is evaporation then there could be this change in the height because. So, that to avoid that they put a small oil blob here and then. So, this is filled with this is filled with water.

If it is a reservoir if this porous medium is taken from a subsurface reservoir, generally instead of water they prefer to use brine the reason I already mentioned. So, this brine is basically 1 percent in nacl instead of pure water they put water with a small amount of nacl. So, that this porous medium that there is not any swelling or any other clay material will not come out of the porous medium ok, so to avoid that.

And then this is placed inside a chamber and this chamber is filled with oil and then there is a this chamber is filled with oil, then there is a top there is a spring, by which this porous medium is held in place. And from here there is a from the top there is a nitrogen connection and one can change the pressure nitrogen by putting. So, this is this is all filled with oil, this is oil originally there was brine or water filled brine or water it was inside this porous medium.

So, this is porous medium this is the porous block and then oil pressure is gradually increased by application of nitrogen pressure on top here. So, that pressure will so gradually water will come out now this particular ultrafine fritted glass disk will not allow oil to flow out only it will allow water to flow out or brine to flow out. So, amount of so oil goes into the porous medium and the brine comes out of it and whatever brine comes out through these here the level rises on the right; hand side and that rise is measured by the that skill that is provided.

So, let us just try to articulate what is mentioned here, sample saturated with wetting phase this is the porous medium sample a sample saturated with wetting phase is placed in a chamber with ultrafine fritted glass disk to prevent passage of oil the glass disk permits the passage of wetting phase. So, when oil gets into these pore, then the water comes out and that water can be drained through this ultrafine fritted glass disk. As the pressure on the oil is increased the wetting phase is displaced from increasing smaller pores of the sample; that is as the pressure on the oil is increased the wetting phase is displaced wetting phases water or brine that is displaced from increasing smaller pores of the sample.

So, first when the pressure is increased first the larger pores will be penetrated by oil, but as the pressure is increased further, more increasingly the smaller pores will also be penetrated and the water will come out of those and that will be collected. Now volume of wetting phase forced out of the sample by non wetting fluid is determined in the graduated portion of the tube; that means, here.

So, here as this oil plug as it is rising so from here one can see how much is the water level rising and that water must have come out from this porous medium. So, this is how this porous diaphragm device works. Of course, it the whole process it depends on the effectiveness of this ultra fine fretted glass it should allow water freely, but it should plug



oil completely. So, instead of relying on this there is an alternative way that are suggested and that is something like this.

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**Capillary pressure determination .... Contd.**

**Centrifuge method**

- Small uniform plug is initially saturated with the wetting fluid, and is held in a cup containing non-wetting fluid.
- The cup is rotated at a series of increasing angular velocities in a centrifuge, and the quantity of wetting fluid, removed at each speed of rotation is measured.
- The pressure at the inner radius of rotation ( $r_1$ ), i.e., at the inlet face of the sample is  $P_{c1}$  (Capillary pressure)
- Here,  $\omega$  is angular rate of rotation, and  $r_2$  is the outer radius of rotation, where pressure has same value in both phases = 0.
- Wetting fluid (water) has greater density than non-wetting fluid (oil)
- Average saturation is obtained from the volume of displaced wetting phase.

$$P_c = \frac{1}{2} \rho \omega^2 [r_2^2 - r_1^2]$$

The slide also features a hand-drawn diagram of a cup with a plug inside, and a small video inset of a man speaking in the bottom right corner.

Capillary pressure determination by centrifuge method. So, in this case a small uniform plug is initially saturated with the wetting fluid and is held in a cup containing non wetting fluid. And this cup is rotated at a series of increasingly angular velocities increasing velocities basically. So, the cup this is the cup, the oil is sorry the plug this uniform porous plug that plug is placed inside the cup and the this pore contains this porous medium contains water and outside it is non wetting phase; that means, the inside the inside the cup outside the porous plug is all oil.

So, now this is now this cup is rotated at a series of angular velocities so; that means, this cup is placed in a centrifuge and this cup is rotated. And because of this rotation and at some angular velocity and then next at some other and then some other and gradually increasing angular velocities; and the quantity of wetting fluid that is removed at each speed of rotation. So, at one speed of rotation, so basically the omega which is the speed of rotation is increased step by step. So, at one step whatever omega value is there that is that whatever omega value is there so against that there is some collection quantity of wetting fluid.

So, that is collected then the pressure at the inner radius of rotation that is at the inlet phase of the sample is  $P_{c1}$  which is the capillary pressure. So, here you can see that the

quantity of wetting fluid that is collected that is noted there. Now, one can see that the pressure at this phase, pressure at this phase and pressure at this phase what is the difference between these two pressures? That is pressure at the inner radius of rotation that is at the inlet phase here of the sample is  $PC_1$ , where  $PC_1$  is  $\frac{1}{2} \Delta \rho \omega^2 (r_2^2 - r_1^2)$ , where  $\Delta \rho$  is the density difference between oil and water. Now inside it is water outside it is oil density is lower ok, typically say 80 percent of the density of water.

So, that  $\Delta \rho$  is there  $\omega^2$  that is the rotational velocity and  $r_2^2$  square minus  $r_1^2$  square. So, this is  $r_1$  and this is  $r_2$  so these are the two radii. So,  $PC_1$  is defined as this one and the capillary pressure at the other end is basically the pressure of water and oil at the other end is basically 0. So, on one side this is the pressure, so here  $\omega$  is angular rate of rotation and  $r_2$  is the outer radius of rotation where pressure has same value in both phases. So, pressure in water because it is basically its open.

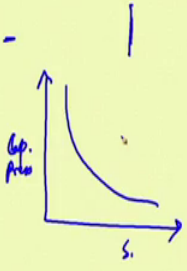

So, now wetting fluid has greater density than non wetting fluid. So, as the  $\omega$  is increased so wetting fluid will be will tend to go out from the we have it will be collected in the cup the cup is already there. But wetting phase will be coming out of the plug out of the porous plug and in that place the non wetting phase will enter and what is that gradient what is the pressure difference? That is  $\frac{1}{2} \Delta \rho \omega^2 (r_2^2 - r_1^2)$ . So, wetting fluid has greater density, so  $\Delta \rho$  would be positive and then the non wetting fluid average saturation is obtained from the volume of displaced wetting phase. So, for a particular  $\omega$  this is rotated this is the capillary pressure and corresponding saturation is measured from the displaced wetting phase.

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**Capillary pressure determination**

**Porous diaphragm device**

- Sample saturated with the wetting phase is placed in a chamber with ultrafine fritted glass disk to prevent passage of oil. The glass disk permits the passage of wetting phase (brine).
- As the pressure on the oil is increased, the wetting phase is displaced from increasing smaller pores of the sample.
- Volume of wetting phase, forced out of the sample by non-wetting fluid is determined in the graduated portion of the tube.

So, if I look at the earlier case we had the diaphragm there what did we measure? We measured at various say pressures we measured the pressure versus saturation. So, I set the pressure at some point I find that this much of water level has gone up. So, from there I can find out what is the change in saturation in these porous medium porous plug. So, that so gradually I can draw a I can draw the capillary pressure in that case capillary pressure as a function of saturation just by just by the same way and we can go reverse and we can we can draw the other side.

So, this is how we can do this here and at the in a similar way if somebody follows the centrifuge method, if somebody follows the centrifuge method in that case also this becomes the capillary pressure and the corresponding saturation would we calculate obtained from the volume of displaced wetting phase that has to be measured. So, either of these two methods can be used to find out capillary pressure versus saturation curve ok, with increasing I mean in with imbibitions as well as and the drainage part of it.

So, this is all I have as far as this particular lecture is concerned we will continue we will continue our aim would be to use these concepts into two phase flow. Because next what we will do is we will we will talk about the capillary pressure hysteresis and then we will try to get to the Darcy's law the flow equation how that will be modified if we have two phases present two phases are together and flowing. So, keeping these capillary pressure and keeping these concepts of saturation surface tension contact angle in mind. So, that is where we are heading to. So, this is all I have as far as this particular module is concerned.

Thank you very much.