Flow through Porous Media
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Lecture – 36
Immiscible Flow (Contd.)

I welcome you to this lecture of Flow through Porous Media. What we were discussing was Immiscible Flow; that means we have two different phases they are not mixing with each other and they are flowing together side by side in porous media. So, how to handle this? So, to do to get to get better insight to this we were discussing about the capillary pressure.

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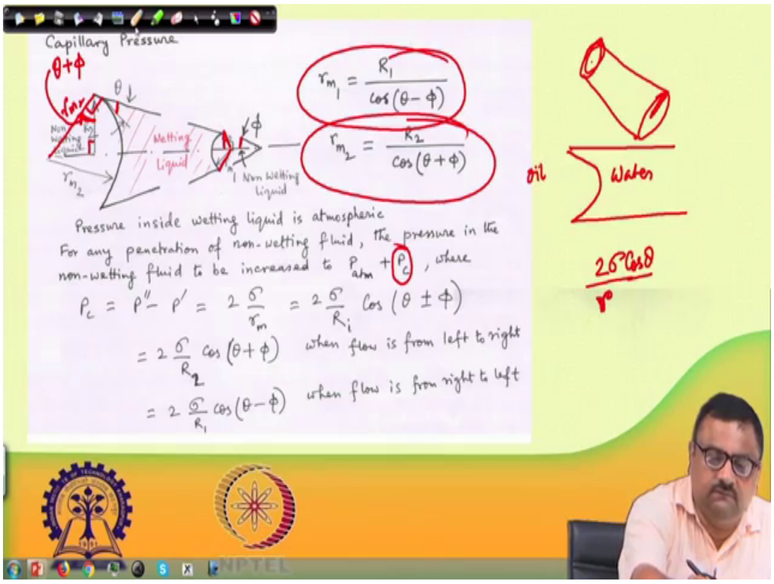

Now, capillary pressure, we said that for a cylindrical system we already said that if you have a cylindrical pore if you have a cylindrical pore and inside it is water the substrate is water wet type; that means, it is like glass and water, the contact angle water mix here is less than 90 degree.

So, but it could be different for example, there could be a hydrophobic surface, some polymer which is let us say hydrophobic and if you place water, so, water will be sitting there as a drop and sitting there as an isolated drop water will not sprayed and contact angle could be greater than 90 degree. So, here in this case the substrate is made of glass let us say and water makes a contact angle which is less than 90 degree. So, naturally this water will make a meniscus like this outside there is oil.

So, now we said that if oil has to penetrate into these pore; that means, had it been water inside water outside, moment there is a pressure gradient we expect that a flow will take place. So, minimum I mean miniscule amount of pressure gradient we will expect there would be a flow, but if oil is outside and water is inside to establish the flow one has to overcome this capillary pressure; and that capillary pressure in this case we found was 2 sigma by r where r is the radius of the capillary and it depends one can put this cos theta term; so, 2 sigma cos theta divided by r.

If theta is very theta is let us say 10 degree for glass water system you may check what is the value of theta is very small. So, if one ignores this they consider theta to be closes to 0, then whether we put cos theta or you don’t it does not matter because cos theta tending to 0 would be one. So, this is the capillary pressure side it is more like as if there is a there is a there is a valve you can and one can think of that once you exceed this pressure then only there could be flow then only oil will enter into the pore otherwise not.

So, now suppose instead of a cylindrical pore, instead of a cylindrical pore suppose we are working with a conical pore and this is how this drawing is given here. The wetting it is inside it is the wetting liquid and outside there is this non wetting liquid. So, you can see here the angle for this cone here is given as phi and the radius of this cone means, it is it is something like this right this is the cone so, this is the cone.

So radius of this here is the smaller one is r 1 and the larger one is r 2 that is the radius. So, radius means this is this is this is the radius ok. So, this radius we are referring as r 2 and smaller one is r 1. And there are two other parameters here, one is this r m 2 and r m 1 r m 2 is the mean radius of curvature as far as this side is concerned and this r m 1 is the mean radius of curvature as far as the as far as this side is concerned.

So, now, one can note here, I mean it is it can be shown by geometry if this angle is theta if this angle this is the contact angle so this angle is theta right, so and if you if you draw a line here since this is the radius; so this is making an angle 90 degree here to this to this line. So, you can you can you can do this by the geometry one can say that these r m 2 will be equal to r 2 r 2 divided by cos of theta minus phi ok. What is phi? Phi is this particular angle is phi this angle is phi and this angle is theta this angle is theta.

So, one can one can say that this angle is 90 degree this particular angle is ninety degree. So, because this side it is r m 2 and here this angle is 90 degree once again. So, one can one can find out from here, that this r m 2 is nothing, but r 2 divided by cos of theta plus phi. So, if it has to be cos of theta plus phi, so what that means, is r m 2 into cos of theta plus phi is equal to r 2. So; that means, r m 2 into cos theta plus phi, so cos of theta plus phi is equal to r 2 divided by r m 2 so; that means, this particular angle this particular angle is defined as theta plus phi this particular angle, this angle is theta plus phi that is what it is showing.

So, one can show by geometry that this angle is theta plus phi and similarly for this side also you will get you will you will construct a triangle construct a triangle in the sense. If this is r m 1 if this is r m 1 this is this is r m 1 this is r 1. So, one can find out what is the angle between these two and one can show that is equal to theta minus phi. So, basically this r m 1 and r m 2 can be written by these two expression one is larger other one is smaller I can see by looking at that drawing so that is geometry.

So, now if somebody assumes pressure inside the wetting liquid is atmospheric, then for any penetration of non wetting fluid the pressure in the non wetting fluid is to be increased to atmospheric plus the capillary pressure. So, this is the capillary pressure PC, we have already said for a cylindrical system this is the capillary pressure. So, now, in this case in case of a conical one what is the capillary pressure? It is again by the same logic P double prime minus P prime which is 2 sigma by r m. Now we have two different values in this case when the flow is from left to right I have to sigma by r 2 cos of theta plus phi whereas, when it is from right to left it is 2 sigma by r 1 cos of theta minus phi.

So, this is what we this is what we understood, but this creates different I mean we now have a different situation all together, let us let us talk about it quickly.

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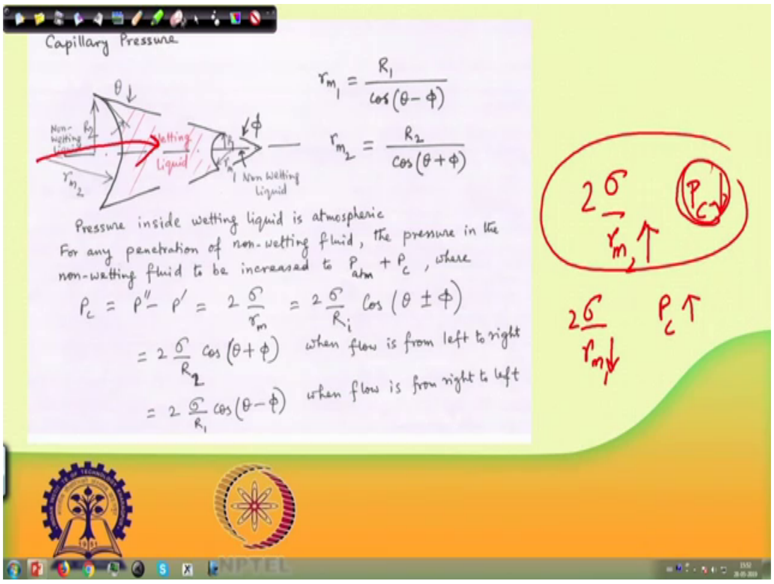

Suppose a fluid let us say oil enters into this phase, I want oil to go into this phase from left to right. So, then in that case since the radius is more, so the capillary pressure would be less right. Here I can see the finally, it is the r m and I can see in this case the r m 2 is more. So, if we talk about 2 sigma divided by r m 2 r m 2 and then this r m 2 is more. So, naturally I have r m 2 is higher when r m 2 is higher the capillary pressure would be less capillary pressure would be less.

So, what; that means, is I have to put less P double prime on this side for the flow to be established. On the other hand if I look at from flow from right to left I need to have in that case it is 2 sigma by r m 1 and I can see r m 1 is small. So, naturally for this side the capillary pressure is higher. So, naturally I mean suppose I am increasing the pressure slowly, the pressure inside water is atmospheric and outside I am raising from atmospheric to higher value. So, I would see the first first the oil will enter in this system is from left right because first this pressure will cross the capillary pressure for this case because here the PC is small.

So, that smaller pressure would be overcome first. So, there would be fluid flowing from left to right and then probably as we continue to pressure at some threshold pressure we found that the this part also the capillary pressure as exceeded. So, if it still some feeling needs to be done, if oil has not filled this pore completely we can expect something going there, but that is at a higher pressure. Now think of it that we reverse this situation we reduce the pressure. So, once we reduce the pressure suppose also that now the situation is that we have let us say now the situation is that we have the oil inside and outside I put water. And now I want this oil to of this water to push.

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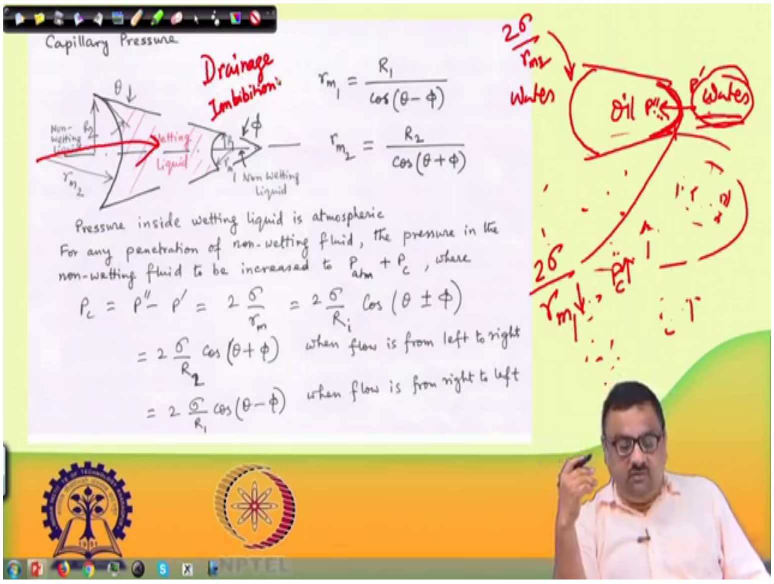

So; that means, this was the it was filled with oil now and we want so, oil and outside it is water here outside it is water here and so now, I can see think of water making a meniscus so; that means, water will make a meniscus like this and water in this side would make a meniscus like this. And so water will tend to now go into the oil. So, I mean I expect now water will. So, I am what I am doing is earlier let us say I had a capillary pressure which was higher, but now I reversed it first of all I filled it outside it is water.

And now what I would see in this case is there would be there would be an automatic section. Automatic section in the sense see you can see that the capillary pressure the pressure of oil, these this pressure here these difference in pressure I can I can think of here the water will be when I reduce this pressure see once again this is 2 sigma divided by r m, in this case this side it is r m 1 and this side it is r m 2. So, we can see here the r m 1 is small, r m 1 is small, so, the surface tension force in this case would be the capillary pressure would be more in this case. So, in this case capillary pressure is between the convex side and the so we you remembered it was P double prime here and P prime here.

So, if we look at this P double prime and P prime in this come in this cem meniscus and P double prime P prime in this meniscus we will find that water has to overcome less pressure drop here in this case. And so one can one can or in other words water is having an upper hand, so water will tend to flow through this side of the meniscus. So, though we where we put the oil from left to right, but when it comes to the feeling of the same pore by water we would see that the flow from right to left will be favored for water. Because the obvious reason is this substrate is water wet.

So, water will be pulled the pulling of water would be more here because here the r m is less. So, the pulling action would be more because pulling action would be 2 sigma by r m r m is it is inversely proportional to r m. So, here the r m is less. So, there pulling action would be more. So, water will tend to flow from this side I mean it is if it is outside we are increasing the pressure we will see that the water will tend to flow from this side because there is a sucking action on the water as the substrate is water wet.

So, we so it generally when we when we think of a pore containing let us say water and we are putting the oil, to displace the wetting fluid or if in a. In a general term we have a wetting phase present in the pore and at non wetting phase is pushed into the pore the process is known as drainage. On the other hand when the non wetting phase is sitting inside and the wetting phase is sucked into the system that process is known as imbibitions, we will we will define in the rigorous terms this drainage process and imbibition process and how what are their impacts.

But grossly this is what it is and you can see that this could be I mean if the pore is a conical pore, it this can this can change the change the situation quite a bit.

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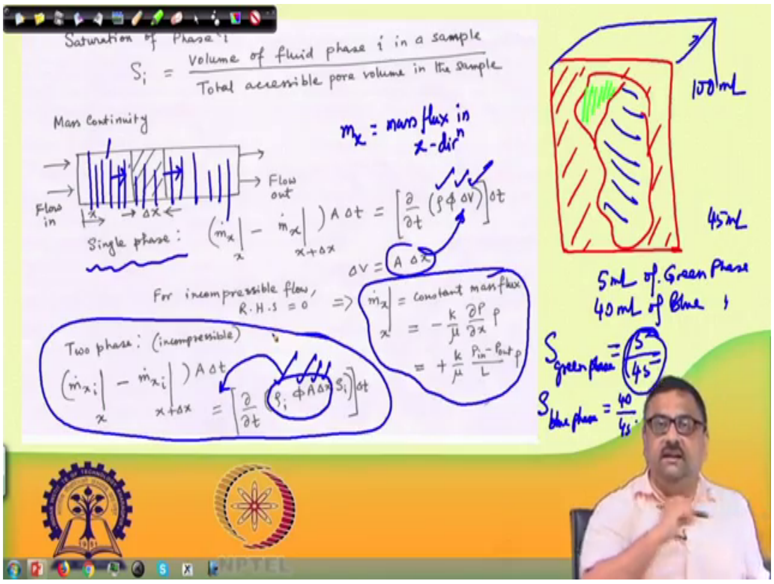

Now, we need to define at this time, what is known as the saturation of a phase saturation of a phase when it comes to flow through porous medium. Now we have two phases, so, I have a void space present say let us say this is a differential volume I have taken out of these this is the solid part and this is the this is the void part. And within this void I can have two different phases, we said water and oil or wetting and non wetting phases.

So, we can think of two different phases present in this system, one is this particular phase and the other is let us say this particular phase. So, these are the two different phases. So, now, we talk about something called a saturation; that means, if this void space is 100 or if this void space is 1 out of that 1 what fraction or what percentage is one particular phase what fraction is the other phase it may not have to be a binary system I mean two phase system it could be general n number of phases can be present there. So, what is the saturation of the ith phase that is what is defined here as saturation of phase I is equal to volume of fluid phase I in a sample divided by total accessible pore volume in the sample; that means, let us say I have this total accessible pore volume is let us say 45 ml out of that this green one is let us say 5 ml.

So, a let us say I am talking about total volume of these porous medium is let us say 100 ml and this has a porosity of 0.45 so; that means, the void volume is 45 and the solid volume is 45 out of this 45 I have let us say 5 ml of green phase and 40 ml of blue phase. So, in that case saturation of green phase would be as for this definition green phase would be equal to 5 divided by 45 and if we want to convert this two percentage then you could have to multiply this by 100.

Similarly, saturation of blue phase would be equal to 40 divided by 45. So, that is what it says volume of fluid phase I in a sample divided by total accessible pore volume in the sample. Now we look at this mass continuity at this time because we have always picked up a differential element and then we did mass continuity. So, now, if we look at the mass continuity in this case what did we do at that time particularly when we talked about this single phase, for single phase we had the mass flow of x mass flow in the direction x at location x minus so that is in minus out at x minus the mass flow rate.

Mass flux basically here m x m x is equal to mass flux mass flux in x direction. So, m x is equal to mass flux in x direction and this is the m x at in so in minus out. Mass flux multiplied by A into delta t A is the cross sectional overall cross sectional area and delta t is the duration, over which this is happening moment I multiplied by a it will not be a flux then it becomes a flow rate right meter. So, mass flux would be kg per meter square second multiplied by a is the meter square and delta t is the second.

So, kg per meter square second into meter square into seconds with this left hand side is kg in minus out and that has to be equal to accumulation. So, all along what we did earlier is that we said that density if it is an incompressible system. So, then this would be if it is a compressible system then density can change in that case such term can exists, but if density it if it is incompressible then, rho will come out and then the phi is not changing with time you are assuming porosity is not changing with time. So, in that case this right hand side becomes 0. So, that is why for incompressible flow right hand side is equal to 0 and delta v is considered a delta x and. So, one writes in that case for incompressible flow right hand side equal to 0.

So, mass flow rate at x is equal to mass flow rate at x plus delta x. So, or in other words the mass flow rate at x is equal to constant and that constant is minus k by mu del p del x and since it is a mass sorry mass not mass flow rate mass flux since these are mass flux this becomes the superficial velocity is basically the volumetric flux you are multiplying it by rho the density which is kg per meter cube. So, this gives me the mass flux volumetric flux multiplied by rho and we had another embodiment of the same equation where instead of del P del x.

Now, since it is constant so; that means, del P del x is this is constant, so del P del x has to be equal to constant del P del x equal to constant means you can simply consider the pressure to be linearly changing with x. So, del P del x is removed and del P del x is replaced by P in minus pi minus of del P del x is replaced by minus of del P del x is replaced by P in minus P out divided by L. So, that is why there is a plus sign. So, this is something which we had done earlier already we are familiar with this, but now that we have a new ballgame here is that saturation.

So, right hand side if we if we write the same thing here for a two phase system, but incompressible. So, it would be now we are instead of writing only one phase that is m x we have another extra subscript which is i. So, we are looking at the ith phase; so, m x dot for the ith phase at x minus m x dot for the ith phase. So, ith phase mass flux from the left ith phase mass flux mass flux that is living from the right that difference multiplied by a delta t exactly the same thing we are doing. So, on the right hand side it is only we are talking about any accumulation that might have taken place as far as the ith phase is concerned; and that would be in this case you can see that rho phi rho phi delta v and delta v is a delta x so those are here. Instead of rho we are talking about rho I phi is there a delta x is there multiplied by we have a saturation term is i here, because we are talking about the ith phase and these ith phase is not see when when it is a single phase the entire pore volume was occupied by that single phase. But now since we have two different phases or maybe multiple it is it is it is actually when we write i i is one to two if you write it as two phase. So, when we have multiple phases or more than one phases then this entire void space is not occupied by the ith phase only the part which is represented by a si the saturation of the ith phase.

That is any accumulation that can take place is you have to multiply this with si, because this entire void space is now not occupied here the we have done it del t of rho phi delta v. Here we have del t of ith phase density phi delta v and then we have to multiply this by the corresponding saturation term to make sure that the you are talking only the one part which is 5 by 45, if you are looking at green phase I is the green phase only we are talking about 5 by 45 part of the void space so that has to be accounted here. And the fall out of these is that even if the density is constant, there could always be a readjustment of the saturations.

That means I am having a two phase flow and at every grid let us say I have I have defined these I have defined these I have split into hundreds of such grids hundreds of such differential volumes. Now at every volume there could be this readjustment someplace the saturation of blue phases increasing someplace saturation of green phases increasing that can very well happen that for that you do not need to have an compressor you do not need to have a compressible system it could very well be incompressible and still there could be readjustment of this some places. The green part will be more and blue part is less in some void space down the downstream down the line somewhere it could be the rivers.

So, that is why even if this rho is constant. In fact, what we will do in these cases we will take this entire part out and, but there could be del s del t I mean that is that saturation can change with time as the flow takes place. In fact, in fact suppose I have this entire pore space is filled with water and then somebody or entire pore space is filled with wetting phase and now we are injecting non wetting phase. So, naturally I would expect gradually the non wetting phase which is now being injected non wetting phase will be more and more it would be crossing more and more it would be occupying the void space.

So, there is always the saturation is changing, so so I would in that case I will say saturation of wetting phase is decreasing and saturation of non wetting phase is increasing grid by grid. So, then this del s del s high del t term makes perfect sense ok. So, that is why now this would be a steady state in a steady state process what we will do is because of the saturation now we will have this accumulation term. So, any steady state process we have to take care we have to look at this and only this term will not this term will not be important only if you can if one can ensure that this entire porous medium.

This particular ratio of 5 by 45 and 40 by 45 is maintained and the fluid that you are injecting that has the that is a mixture of two phases exactly in this ratio, if one can do that then that would be considered a steady state process for this two phase system. And in that case I would agree that saturation will not change with time so this is something which we are heading to and so there. So, this definition of saturation definition of capillary pressure and flow of flow or displacement of oil and water these are very important and I discussed this in this module that is all I have for this lecture module.

Thank you very much.