Flow through Porous Media

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Lecture – 34
Immiscible Flow

I welcome you to this lecture of Flow through Porous Media, what we are going to discuss now is Immiscible Flow. Immiscible flow means flow of two phases which are not miscible an example of immiscible system is mixture of water and oil.

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So, what we will discuss in this this section is primarily relative permeability and two phase flow and what are the complications arising from the arising when two phases are present together in porous medium.

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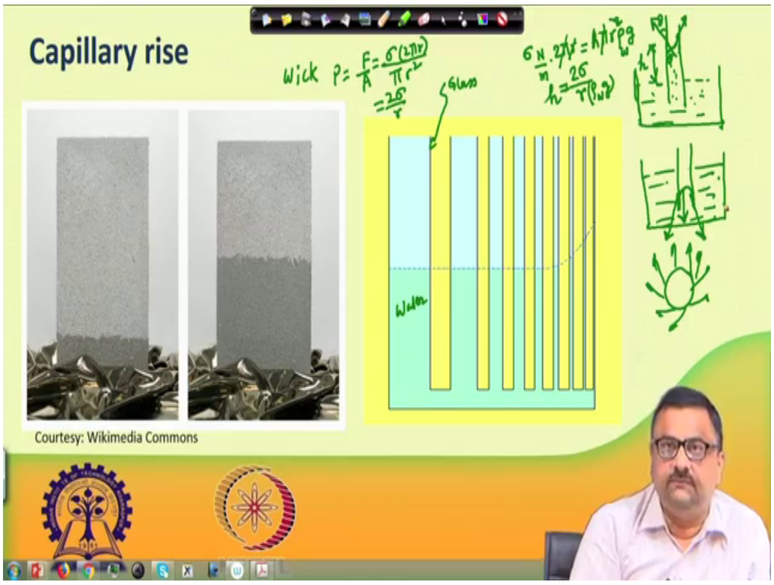

Before we get to this discussion, I must point out a couple of things about these surface tension. Because moment you have two phases present water and oil they are together, one can have this interfacial tension and this interfacial tension, that will govern; that will be instrumental in deciding the outcome here. So, before we get to these the discussion on this two phase flow, I think we will just very briefly touch upon what is surface tension, what is the definition. Here in these pictures one can see some solid structure and then it is one can see these water is rising slowly with time ok.

So, it is against a gravity. So, what is the force that is pulling this water up we can say wick action w i c k wick. So, this is something which we have seen before and this is another picture where capillaries of different diameters they are placed side by side by the rule of hydrostatics, we expect that everywhere the height would be same. But what is observed here is that as the capillary gets thinner the level is higher. So, what is contributing to this, what is instrumental in pulling the liquid up here? So, let us say these are made of glass and the liquid that we have here is water.

So, water has some affinity towards glass so, that is why water is rising in these smaller capillaries. So, what is this force that is causing the capillary rise? We have already studied before what we have studied here is that, if I introduce a capillary in water, we expect that the water rises in this capillary and not only that it forms a meniscus like this. So, there is a rise additional height by which this water level goes up. The same thing when we do it with mercury, we see that if this is the capillary and if this is the mercury level, we see that the mercury is not entering into this capillary mercury is up to this point and that to the meniscus is the other way. So, this is what is the situation with mercury.

So, it is not only entering and it is in fact, depressed here it is the rise and here it is the depression in liquid level. So, what is causing this rise? So, we said that what we have studied earlier is that there is a surface tension acting and the 2 pi r is the area over which this surface tension acts. So, that time we said that surface tension has a unit of Newton per meter and then we are multiplying it with 2 pi r the perimeter of this circle over which the surface tension is acting; that means, if this is the perimeter of this circle, then surface tension is acting all along like this.

So, it is trying to pull this. So, that is why it 2 pi r is the area; 2 pi r is the perimeter ao, this has meter Newton per meter into meter. So, this is Newton. So, this is the force by which this is being this liquid is being pulled upward and then I mean at equilibrium, the weight of this liquid it has to support. So, this has to be equal to if this height is h, then h into cross sectional area which is pi r square. So, h is the height, pi r square is the area. So, h into pi r square gives a volume then we multiply this by rho water, we said this is water and this is glass. So, volume multiplied by kg per meter cube. So, meter cube into kg per meter cube we have now kg into g, that is the downward force due to gravity and that has to be supported by surface tension.

So, we equate these two and lot of times we said that there is an angle involved which is this angle theta, this theta and this theta is very small. So, that is why this side if this angle is theta then this angle also has to be theta and we have to take the vertical component of these; that means, we have to multiply this by cos theta and since theta is very small; theta is very small so, that is why cos 0 is 1. So, one can ignore and just simply consider sigma or if theta is not that small one can consider sigma cos theta also here. So, from these one can find out what is the h, how much the fluid will rise from these expression.

So, one can find out h is equal to pi and pi will cancel out. So, h would be equal to 2 sigma divided by 2 sigma r and this r square there would be 1 r coming here and then there would be this rho wg; so, h is equal to 2 sigma divided by r into rho wg. Some people even converted this to a pressure what is the pressure acting on this fluid because of this surface tension? So, what they have done is they have taken this force which is equal to sigma into 2 pi r and then divided it by area over which this is acting and then they find out what is the pressure. So, they have done it with sigma into 2 pi r divided by pi r square which give 2 sigma by r. So, that is the pressure by with this fluid is being pulled upward.

So, these are these are some other things which we have seen and we said that in case of a mercury, in case of mercury there is a reversal. So, these angle first of all is downward. So, angle is downward. So, there is a vertical when net is net there is a vertical force these angle it was theta is very small, but here the theta is much larger. So, that is why so, you have a you have pulling in the downward direction and so, that is why the one can do similar force balance and find out how much would be the depression. So, this is something which we are familiar with already and the use of surface tension in this context.

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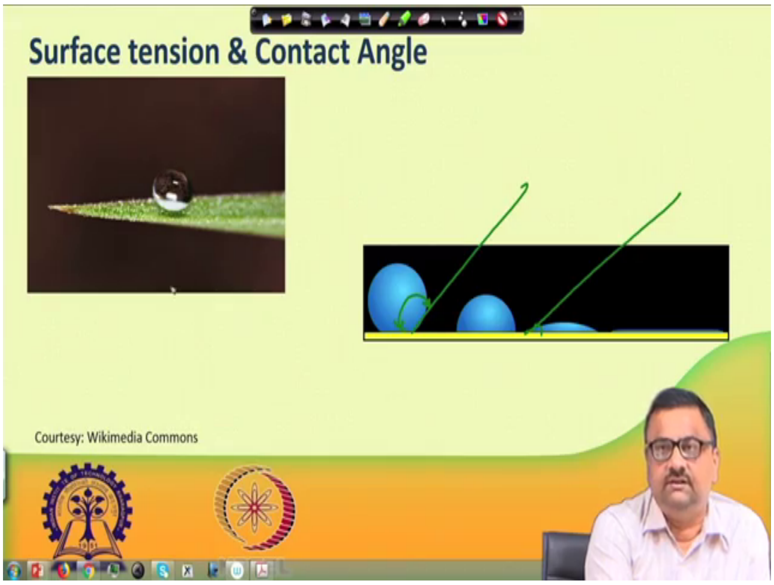

So, now we have also looked at these contact angle; that means, we have various ways to look at these contact angle and there people have observed, what would be the contact angle by placing a droplet and then taking a picture of it. So, one can see here that this one for example, has a very low contact angle whereas, these has much higher contact angle. So, this has a much higher contact angle; so, when the contact angle is more than

90 degree. So, you would assume that this is not going to wet the surface whereas, here the contact angle is minimal, here this contact angle is truly close to 0. So, this is weighting the surface, here it is not weighting it is just sitting like a droplet and we have seen similar droplet on leaves because these are highly hydrophobic surface. So, water forms a droplet like this.

So, these are some of the issues with surface tension and contact angle, I just like to brush up these ideas before we get to these two phase flow.

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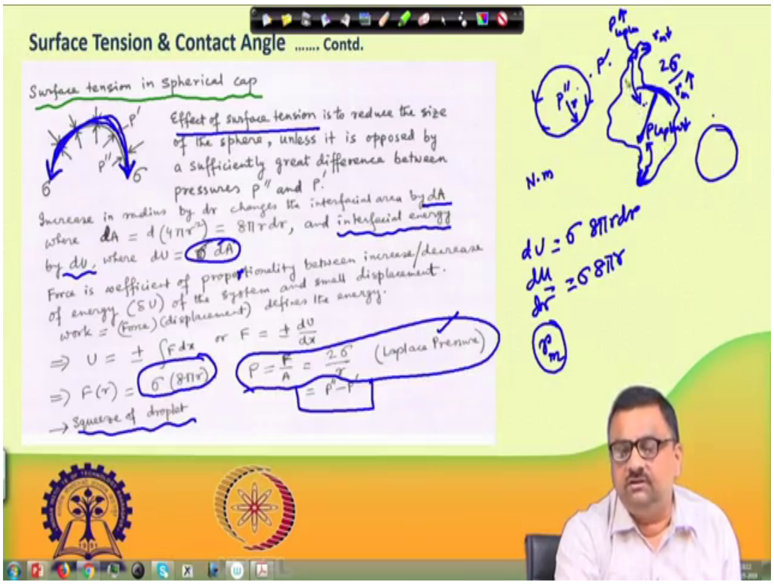

So, now if we try to get to the definition of these surface tension as it applies to the our flow situation, we look at a classical definition of surface tension in a spherical cap. So, what we have here is we see that for a spherical cap one has let us say we have these contact line here. So, on one side we have a liquid and we have other side we have, where the I mean there are two different phases ok. So, this is the interface.

So, effect of surface tension we are calling these as surface tension; this is the sigma the surface tension effect of surface tension is to reduce the size of the sphere unless it is opposed by a sufficiently great difference between pressures P double prime and P prime. So, what that means, is that I have a water droplet water droplet will tend to that there is a surface tension if I cut it I will see that there is a that the cut the surface I will see that the surface is being pulled in such a way so, that these size of the sphere gets reduced and what is opposing it is a pressure difference; that means, inside there is a pressure outside there is a pressure and these two pressure differ there is. So, this inside it must be a higher pressure and outside there must be a lower pressure and these higher pressure is forcing it not to collapse forcing it not to not the surface tension not to reduce the size of this sphere any further. So, that is what a surface tension means for a spherical cap.

So, classically we had this increase in radius by d r changes the interfacial area by dA. Suppose I have a sphere of radius r and increase in radius by dr amount will change the interfacial area by dA amount where dA would be simply d of 4 pi r square because the surface area of a sphere is 4 pi r square. So, d of 4 pi r square so, if we do that we get these 8 pi r dr ok. So, these 8 pi rdr would be is equal to dA. So, if one increase these spheres, this sphere by from r to r plus dr if the radius is changed, the corresponding change in surfaces; this interfacial area dA would be equal to 8 pi rdr.

And then correspondingly the interfacial energy will change and interfacial energy is energy means it is basically same as the work done right. So, it is basically Newton in to meter force into displacement. So, when we multiply this sigma to dA, sigma has a unit of Newton per meter and dA has a unit of meter square the area. So, Newton per meter into meter square using Newton meter. So, these becomes the energy the interfacial energy du. So, if one changes the radius from r to r plus d r corresponding change in area would be 8 pi rdr and corresponding change in interfacial energy would be sigma into 8 pi rd r that is what we are saying.

So, now force is essentially coefficient of proportionality; coefficient of proportionality between increase or decrease of energy which is delta u of the system and small displacement. So, that work is equal to force into displacement defines the energy. So, now, one can see this internal; one can see this interfacial energy in this case as equal to plus minus is whether it is increase or decrease so, you have this integration of Fdx. So, that is what is the definition of force.

So, then on other words you can write force as equal to d u dx and here in this case the d u is already we know is sigma dA so, and dx is basically here we are talk about dr. So, we get sigma dA is equal to. So, dudr it would be sigma into 8 pi r right. d u is equal to sigma into 8 pi r dr. So, d u dr is equal to sigma into 8 pi r. So, that is exactly what we have seen here. So, F is equal to sigma into 8 pi r.

So, if somebody looks at the pressure, the pressure is equal to F by A and the area would be once again 4 pi r square. So, if we 8 pi r divide by 4 pi r square would be 2 by r. So, you get P is equal to 2 sigma by r. So, this is classically what is defined as Laplace pressure and for mechanical equilibrium that has to be equal to P double prime minus P prime. So, that means, the pressure inside this phase it has to be higher than the outside and these difference in pressure would be equal to 2 sigma by r, where r is the radius of the sphere and sigma is the surface tension.

So, that is 2 sigma by r and that is called Laplace pressure, the pressure difference or the additional pressure that one must have to consider in such in this sphere. Now, what will happen if someone squeezes this sphere; suppose the sphere is being squeezed to a shape like this, one takes the pics of the hand and then squeeze it like this. So, then it would be this Laplace pressure, Laplace pressure would be then in various locations the Laplace pressure will change inside this squeezed droplet. For example, here there is a sharp turn here. So, here you will find that the radius this would be the r here, we will very soon show that 2 sigma by r this r is radius for a sphere, we will very soon will show that these are is actually we will change it to mean radius of curvature rm. Particularly in a case where you have multiple curvatures; that means, if you have an arbitrary surface not exactly sphere.

Sphere has only one radius of curvature, but if you have an arbitrary surface, then you can have several radius of curvature at a particular point depending on how the it is think of a cloth and you can twist you can turn it in many different ways at a particular point. So, one can have a slope in one direction some other slope and the other direction as far as that particular point is concerned. So, similarly here we have; so, in that case this is called mean radius of curvature. So, one would look at the mean radius of curvature at this point so, that would be something like this would be the mean radius of curvature let us say.

Similarly, mean radius of curvature here is this would be the mean radius of curvature; whereas, what would be the mean radius of curvature here I by looking at it I can see if this is the curvature, the mean radius of curvature would be at least this much. So, here the mean radius of curvature because the curvature that I see of this surface is this would be the curvature. So, the corresponding radius of curvature would be this much. So, naturally if we try to find out what is the Laplace pressure here inside, these Laplace pressure here inside these Laplace pressure we have to calculate these Laplace pressure as 2 sigma divided by these r m which is much larger.

So, naturally if rm is much larger, then the Laplace pressure would be much smaller here; whereas, in this case the rm the mean radius of curvature is much smaller in this case mean radius of curvature is much smaller, I can look at the curvature of this surface I can see mean radius is less. So, naturally the here the Laplace pressure would be 2 sigma by rm. So, here the rm is low; that means, 2 sigma by rm is higher. So, here the Laplace pressure is P Laplace pressure would be much higher and here the Laplace pressure would be much lower, much lower.

Now, Laplace pressure is the pressure difference P double prime minus P prime here it is P double prime minus P prime. See P prime is same around same P prime is existing so, naturally I can say that here the pressure, the actual pressure would be much higher compared to this place. So, if the pressure is much higher at this place. So, automatically fluid will flow from here to there fluid will flow from here to there.

So, automatically you will find that this will again take the shape of a sphere just to because it cannot hold a high Laplace pressure. So, there will be a flow from high pressure to low pressure. So, these are some of the implications. So, if you squeeze a droplet, one can see that this droplet again will come back to its original shape because of this movement of fluid from higher pressure to lower pressure.

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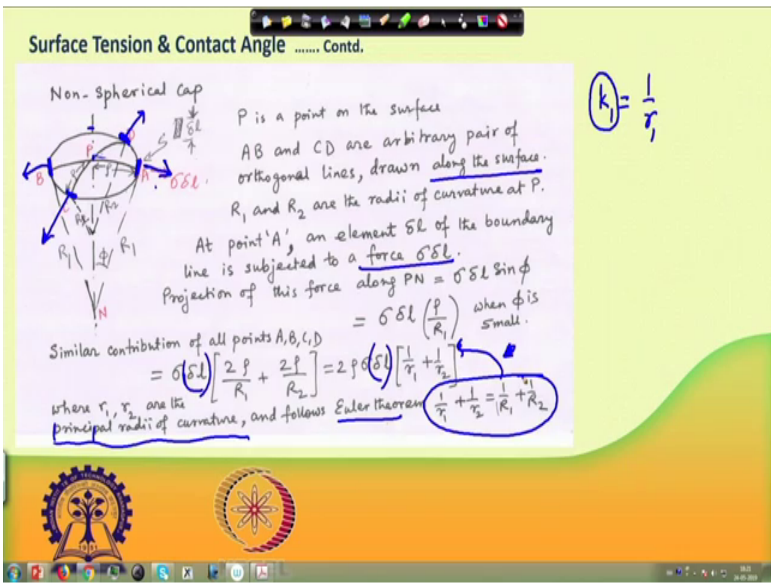

So, now we look at the situation where you have a non-spherical cap. Non spherical cap means the last time we had a sphere, now we have a surface which is having multiple curvatures. Curvature is basically, curvature is also one parameter curvature is inverse of the radius of curvature, curvatures are given by k. So, k 1 is equal to 1 by r 1 like this. So, if the r 1 is the radius of curvature; so, corresponding curvature would be defined as k 1 which is 1 by r 1. So, anyway this particular surface it has different curvature in a sense that, this particular point which is given here as P, this particular point I can see one curvature if I follow this line C D and that corresponding radius of curvature is r 2 see I have drawn in normal I would say at P I have drawn a normal which is N P.

So, if I look at the radius of curvature; if I look at the curvature along C D it has a radius of curvature which is r 2. For the same point if I look at this A B if I draw another line so, this A B will have radius as radius of curvature is r 1. So, basically this is a surface which has multiple curvatures, it is not like sphere that it does have single curvature. So, now, what we write it here this is called a non-spherical cap. So, here we write P is a point on the surface A B and C D; A B and C D these are arbitrary pair of orthogonal lines drawn along the surface.

So, along the surface I am following this is not a straight line joining A B, this is the line A B which I have drawn following the surface only. Similarly C D is joined following the surface only. So, that is why it looks curved because we are following the surface, surface itself is curved. So, A B and C D are arbitrary period of orthogonal lines drawn along the surface and r 1 and r 2 are the radi i of curvatures plural of radius is radi i. So, radii of curvature at P; so, at point A, A is this point.

So, first of all I think one thing you must note here is that, there is these you are drawing a circle here, this ABCD these are this is basically a circle because these if you follow this point A point B point C point D they are all row distance apart from point P. So, if you look at here follow the line, follow the surface I am following a surface and going from P to A that distance is rho. Similarly I am going the surface and going from P to B this distance is rho, I am following the surface I am going to P to C this distance is rho.

So, basically this ABCD this is basically a circle, but this circle is drawn in such a way so, that the all distance of all these are points from point P, they are same as long as you follow the surface go to that point; from point P. So, now, we have these r 1 and r 2 are the radi i of curvature at point P we have said. Now, at point P we pick up an element delta l of the boundary line. So, we pick up an element delta l of the boundary line. So, we pick up this boundary line I extended it the length of this boundary line is delta l. So, these delta l will be pulled; this delta l will be pulled in this direction and in this direction because of this surface tension. So, this is subjected to a pores force sigma delta l.

And projection of this force along P n if we look at the projection of this force along P n, this is P n. So, projection of this force I am sorry I did this direction wrong; this is being pulled in this direction, this is sigma dl; in fact, that is what is written here that is drawn here sigma delta l. So, this is being drawn here already. So, sigma delta l if this length is delta l; if this length is delta l so, there will be a pull because of surface tension and that would be sigma into delta l in this direction. So, now the projection of this force in P N direction so, this would be pulling from point P only so, the same force is being pulled. So, this its projection along P N would be sigma delta l sin phi, if this angle is phi right.

So, then this is sin phi, when phi is small one can write sine phi as rho divided by r 1 rho is this distance, P A following the surface and r 1 is A N. So, this is written a sigma delta l rho divided by R 1. So, similar contribution will come from point B so; that means, you can pick up another such differential length delta l. So, there would be a point B similar such delta l would be their point D, similar such delta l will be there and point C. So, these are being pulled in all directions that is what surface tension is all about.

So, similar contribution from all points ADCD so, one will end up with sigma delta l into one contribution from here rho by R 1 plus rho by R 1 rho by R 2 plus rho by R 2. So, 2 rho by R 1 to rho by R 2, now it would be 2 rho sigma delta l; sigma delta l and then 1 by capital R 1 1 by capital R 2 that is written as 1 by small r 1 plus 1 by small r 2 what is the difference? r 1 and r 2 see these ABCD or these are arbitrary orthogonal pairs but so, there could be multiple curvatures possible.

So, at any point there would be something called a principal radi i of curvature, what are this principal radii curvature? At any point there could be multiple number of curvatures possible out of that the maximum radius of curvature and the minimum radius of curvature these two are called principal radi i of curvature. And there is this Euler theorem that says that, if you draw an arbitrary orthogonal line and if you come up with these 1 by R 1 and 1 by R 2 just the way we have found out.

The some of these 1 by R 1 plus 1 by R 2 would be equal to 1 by small r 1 plus 1 by small r 2 where r 1 and r 2 they are those two principal radi i of curvature; that means, that a maximum and minimum. So, that is this; that is already established that is why we placed it and we call 1 by R 1 plus 1 by R 2 is 1 by small r 1 plus 1 by small r 2. So, now, this will give me; so, now, we instead of these we would be writing here we are; so, now, when it comes to these; we have to somehow come up with something called a mean radius of curvature. And this; so, the exactly the way we had done it earlier P double prime minus P prime, that same exercise we will repeat with these non-spherical cap to arrive at the Laplace pressure in this case.

But in this case we have two radi i of curvature R 1 and R 2; capital R 1 and capital R 2. So, accordingly I have to find out mean radius of curvature that is important. Earlier case we have found Laplace pressure as 2 sigma by rm, where rm is r is simply 2 sigma by r where r is simply the radius of that droplets radius of that sphere, but here since it is a non-spherical one, now at least we came to two principal radi i of curvature the maximum and minimum radius that are existing at that point.

And now we have to define something called mean radius of curvature and expressed Laplace pressure in terms of mean radius of curvature. Because you will see very soon we will get to capillary pressure where we have to work with mean radius of curvature instead of these; because it will not be sphere always; that we will be talking about the interface which has different type of; which is different from sphere. So, the general radius of curvature has to be defined, that is all I have for this lecture.

Thank you very much.