Flow Through Porous Media
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Lecture - 32
Miscible Displacement (Fractured Porous Media)

I welcome you to this lecture of Flow Through Porous Media. We were discussing about Miscible Displacement and more particularly dispersion that happens.

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Dispersion of solute or tracer in porous media and in the last lecture we discussed about these dispersion in when a fracture is embedded in a porous media.

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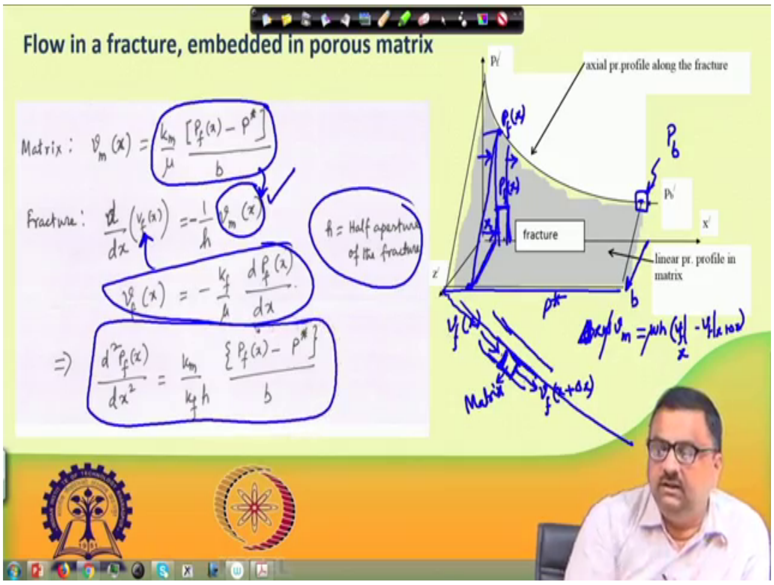

So, what we had at that time was that this is the flow that we talked about one is the linear flow along the fracture, another is the linear flow away from the fracture and into the matrix and we say this would be the situation when the flow rates are little high on fracture.

So, the pressure in the fracture is much higher where has this would not be the case this flow into the matrix would be insignificant when we are flowing some contaminant is going into the sub surface or when there is a very low flow rate, fluid flow that is very low flow rate. So, there in that case such type of flow from fracture into the matrix would be insignificant. So now, when there is a flow from fracture into the matrix, the way this one thing is handled I mean at least we try to put together case here.

We said that let us say the outlet of the fracture there is a pressure, there is some pressure and some distance away from the fracture which is say at a distance b some pressure is maintained here. So, some pressure is maintained along these and that pressure is let us say P star and some pressure is maintained at the fracture outlet and we call these pressure as P b. So, what we have here is the velocity say for example, I pick up one differential element and at that point. So, this is x this distance is x and let us say for this differential element on the fracture the pressure here is P f as a function of x.

So, that is the P f existing. So, this is basically this is P f at x. So, that P f and here it is P star. So, any fluid that is flowing from fracture into the matrix so, that will have a pressure gradient which is P f at that particular x minus P star corresponding value here. So, P f minus P star. So, P f x minus P star and over this depth is b. So, let us say this depth is b k m is the permeability of the matrix and mu is the viscosity. So, this could be the superficial flow as far as the flow from fracture into the matrix is concerned.

On the other hand if we look at this differential element; if we look at this differential element here and if we try to see how much of flow takes place through this differential element inside the fracture; that means, we have the fracture and we have the matrix. So, we picked up a differential length dx ok. So, we these differential length dx at x; so, here the distance is x let us say at x the velocity would be vf at x and here would be v f at x plus delta x. So, here it is v f x plus delta x here it is v f x. So, the difference between these 2 velocities I mean where it is going. So, v f into the cross sectional area will give me the flow rate.

Here it is v f again into cross sectional area will give me the flow rate that is going out. So, this difference is arising because some fluid has gone into the matrix side ok. So, now, if we work with 2 h as the aperture of the fracture; that means, this fracture aperture is 2 h and we are working with the symmetric half of the problem we are working only with this half. So, this symmetric half of the problem; for this symmetric half of the problem these width is h which is half aperture of the fracture. So, over these h so one can find out how much fluid is going in.

So, let us say we have some depth in this direction fracture must be having some kind of depth here. So, if that is w for example, then we will have w into h that would be the area over which v f is acting. And, similarly w into h into v of x plus delta x that is the flow rate that is living this differential element and these difference in flow rates have to be equal to the amount that is flowing into the matrix ok. And the amount that is flowing into the matrix is basically what is the area over which this is flowing? This side is dx and perpendicular to the screen we have that w. So, dx into w would be the area.

So, dx also if we put this all in perspective dx into w into v m, that would be equal to w into h into v f at x minus v f at x plus delta x. So, we will go for in minus out and that has to be equal to the fluid that is going into the matrix. So, then if we work this out we will find that d dx of v f x equal to minus 1 by h vmx if we solve this further ok. Because, you have to set limit delta x limit this is already I have written it as dx actually I should be writing it as delta x. So, limit delta x tending to 0. So, when go to the right hand side it will be minus d f x minus dv f dx. So, minus dv f dx would be equal to 1 by h into vm w and w will cancel out.

So, that is exactly what we have done here and vf x any at any point in the fracture the velocity vf x is given by minus kf by mu dP f dx. It is I mean we are just simply applying Darcy's law for the fracture. This is possible we said that fracture I mean if somebody wants to look at flow between 2 parallel plates, then it if there is a famous cubic law which is b square by 12 b is the fracture aperture. So, it is a so, one can toggle between the aperture of the fracture and permeability of the fracture so, through this relation kf equal to b square by 12.

And this b square by 12 where arises from Navier-Stokes equation one can show this we can take this up as an assignment at some point. So, this is v f x equal to k f by mu so, this quantity. So, v f is just like any other flow through a porous medium, it is the way to follow Darcy's law and so, this is the equation. So, now, if we club these now if we put these expression for v m. So, these v m if we instead of these if we if we place this one here in v m and this expression for v f if these goes there. So, then it becomes d square P f x dx square this is going to km by k f h P f x minus P star divided by b if we simplify this equation.

So, this equation can be solved easily this equation can be solved easily and that is what we are going to talk about here in a moment.

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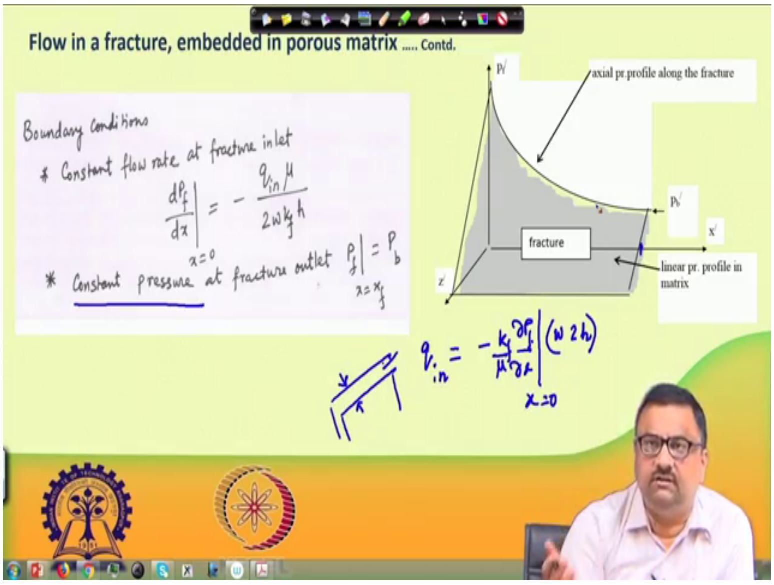

So, now, let us look at quickly the boundary conditions at this point boundary conditions is the constant flow rate at fracture inlet. So, at fracture inlet flow rate is constant. So, look at the way we have written it here q in is the flow rate; q in is the flow rate and that has to be equal to minus kf del P f del x at x equal to 0 and then further minus k by mu del P f del x that is that is what is superficial velocity, but q in is the flow rate.

So, it has to be multiplied by w into 2 h. So, that is the total flow rate that is injected into the fracture 2 h is the fracture aperture. So, w is the depth was that is perpendicular to the screen ok. So, fracture means fracture has a dimension which is 2 h in this side we were working with the symmetric half so, that is why you considered h. So, 2 h and perpendicular to these basically fracture has a third dimension. So, the third dimension is w. So, that is why we have this expression here. So, this is one in one condition, that the flow is going into the flow is entering.

So, whatever flow rate is entering into the fracture so, that has to satisfy this condition and the other condition is one can impose a constant pressure at fracture outlet. So, then in that case P f at x equal to x f that is the x equal to x f here. So, at x equal to x f the P f is equal to Pb that is what with the pressure we are holding it here. Pb prime we have written P f prime we have written we made this dimension less this is z prime and x prime we will discuss this in a moment how to make this dimension less, but basically this is how it is. If somebody says that the or that is fine I mean there could be a where you will be possibility that entire fluid that is flowing will all going out through the matrix and there is no flow left at the outlet. If that is the situation then if one wants to find out that is a limiting condition, then in that case one is to set instead of these one is to set del P f del x or dPf dx at x equal to xf equal to 0. So, no flow out from the fracture outlet so; that means, all the fluid that is going into the matrix that is so, that way only it is everything is lost. So, that is a very limiting situation its hypothetical situation. Anyways so, this is these are the boundary conditions one has.

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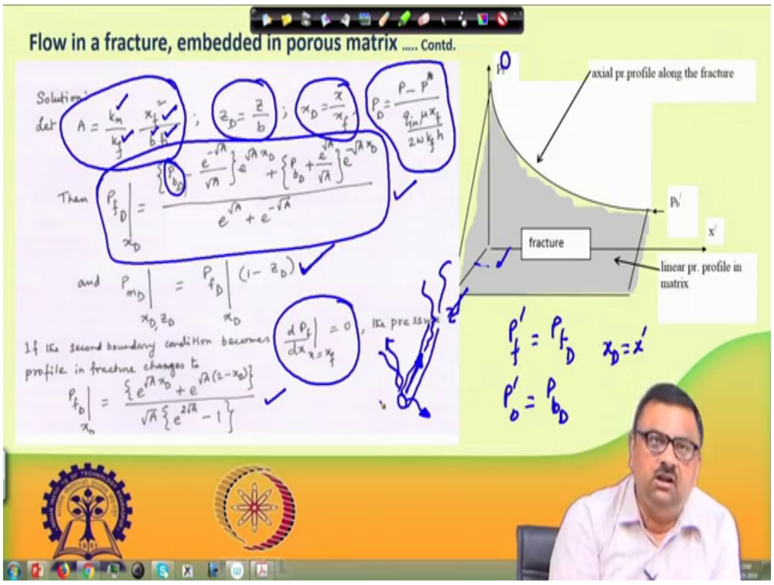

So, now we can make this dimension less very quickly, one dimension less term is this e dimension less term e and then dimension less z direction length dimension less x direction length. So, now, we can see what is x prime what is z prime and all these. So, and dimension less pressure is written as this. So, once this is done the solution of that equation that we talked about just now that d square P f dx square is equal to something on the right hand side the solution of that equation will take a form something like this. So, pressure at any dimension less location would be function of A definitely.

That means A is the function of what is the matrix permeability, what is the fracture permeability and what is the length of the fracture, what is the half aperture of the fracture and what is the width of the matrix over which linear flow we are considering. So, based on these so, at any location xD and depending on the value of PbD dimension less pressure at the dimension less pressure at the outlet. So, this is. So, here I am I am actually there is a slight mismatch, we are writing about we are talking about this as prime this prime. So, basically P f prime is same as P f D.

Similarly, P v prime is same as P v D this is basically done pressure at point b dimension less pressure at fracture dimension less it is like this ok. So, x D is same as x prime it is the dimension less distance along the fracture similarly Pm prime which is Pm D here at x D and z D. Now here at any location x and you have travelled up to let us say some distance z. So, here the pressure would be here the pressure is simply linear, I mean we know this pressure has to be linear we have already seen this right, that if any if there is a Darcy flow through a porous media the pressure drop should be linear.

So, it would simply drop from corresponding P f value at that location of x and P star at the end. So, anywhere in between it is simply a linear interpolation. So, this is the pressure profile in the matrix. So, this at this is the way one can look at what would be the pressure profile and from there one can look at how much would be the flow into the matrix what is v m and what is v f. So, from these one can calculate. And the hypothetical situation that we are talking about, where all the flow ends into move going into the matrix from the fracture and here nothing is left here.

So, that type of condition can be one can have this as the boundary condition and in that case they will end up with these as the pressure profile. So, these are some of the ways one can handle these. So, this is once again I must point out that this is something which we have this is something which we have when the pressure in the fracture is very high. And, that can happen under situations where one is producing from the fracture at a very high rate or one is injecting something into the fracture at a high rate or someone is intentionally wants to fracture the fracture it further.

So, there in increasing the pressure so, that a fracturing occurs so, these when these are the situations then one can expect the fluid to flow from fracture into the matrix because the pressure in the fracture is very high. On the other hand when it comes to the tracer transport contaminant transport; that means, some contaminant is generated and it is deposited in the sub surface. So, we expect that flow to be very feasible flow. So, there the fracture will not be at a very high pressure compare to the matrix. So, one would not expect I mean one can I mean I can tell you the difference here.

Suppose I have sent the fluid packet now fluid packet has a choice of flowing through the fracture or fluid packet has a choice of going to the going into the matrix. Now you look at the fracture permeability here and matrix permeability there they are orders I mean about not only orders I mean I would say at least four or five orders of magnitude different.

So, this will because of this high permeability flow will tend to go to the fracture only. Only if this pressure is very high then only one can think of flow this fracture acting as a line source; however, if there will be a very feasible flow and then continue like this. And, the fracture would be acting as a short of short circuit as far as rge flow through porous media is concerned. So, this is this is one aspect of flow now let us look at the other side of it.

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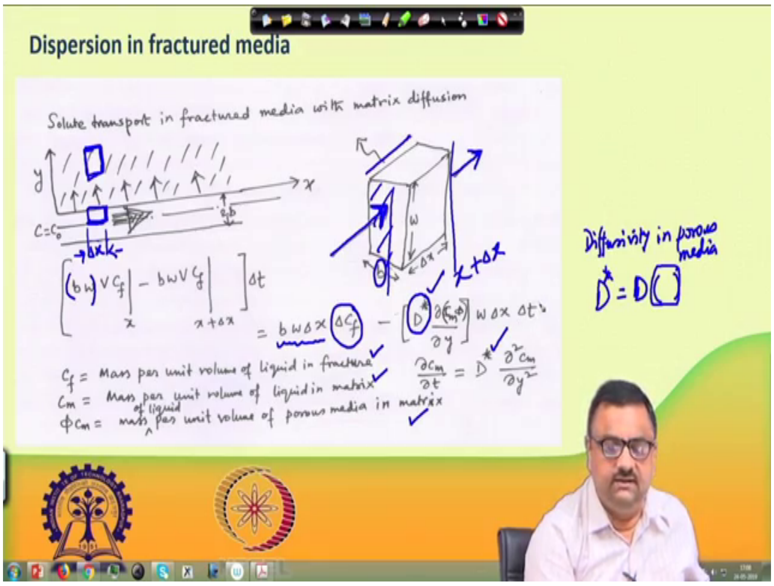

The other side of it is the solute transport in a fracture media when there is unidirectional flow through the fracture. So, when one has unidirectional flow through the fracture and they are they have. So, there is a flow taking place through the fracture and there is a flow from there is a diffusion now this is mind it this is different the solute is diffusing into the porous medium.

There is no pressure this is not a pressure driven flow this is simply diffusion. So, when one has this type of a situation then one will go for then one will go for let us say I pick up a differential element here. So, I pick up a differential element here these differential element I have seen or rather I would say on I will take a symmetric half I will not consider let us say this one I will only work with a symmetric half so; that means, this side it is only b though the fracture aperture is 2 b. So, half aperture is only b. So, that is why we have b here.

So, this side it is b and perpendicular to the screen we have w; that means, that means perpendicular to the screen we have the w width of the fracture there. So, that is why we wrote it has w and let us say we picked up a length delta x. So, delta x is the length. So, if we if we have such type of a differential element. So, if we look at how much is going in, how much is coming out from that side and how much is diffusing, but this time it is diffusing only from one side because we have only working with the symmetric half. This particular phase is all because of symmetric it is that basically del c del z would be equal to 0.

That means no concentration it that is no flow possible it is because of the symmetric this wall is you can consider this to be impermeable. So, so this we have only diffusion from one side ok. So, we consider the symmetric half now this for this symmetric half we have if I look at how much of flow that is taken how much of how much solute that is going in because of convection. See b w is the area through which the flow takes place bw. So, this area is bw. So, b w into v that gives me the volumetric flow rate, that goes into this system and that v into Cf at x concentration at x. So, this is at x and this is at x plus delta x; so, the flow that is going in flow into concentration.

The solute that is going in at x minus b w VCf the solute that is reliving at x plus delta x multiplied by delta t that is a duration. So, over these duration this is the volumetric flow rate let us say meter cube per second and C f let us say in kg per meter cube or or alternatively you can go for porous per meter cube. So, if it is kg per meter cube. So, kg of solute in minus out over duration delta t, that has to be equal to the accumulation first of all. So, the accumulation is accumulation is here the change in concentration in the fracture it is Cf. So, let us say delta C f is the change in concentration over these duration delta t.

So, this is the volume of this fracture no porosity is attached generally fracture has a permeability, but fracture porosity do not able do not do unless this fracture contains sand or some other thing. If it is a through fracture nothing existing inside; so, it does not have any porosity. So, b w delta x is the volume of this volume of this differential element and this is the change in concentration. So, that is the change in concentration we tell you with the kg per meter cubes. So, this much so, this much of kg of solute got accumulated in these and there would be some loss from this phase.

And that phase is basically the area is w into delta x, this length is delta x and w is this height. So, over this area over this particular area there is diffusion out. So, that is why it is D star into del c del y multiplied by the area. So, this is the flux, this is the superficial the sorry this is the flux mass flux from fix first law multiplied by the area flux means the kg that is transported divided by divided by area divided by time. Now you are multiplying it by area w into delta x and the time. So, the all those denominator area and time will cancel out and so, this remain only what is remained is the kg.

The only catch here is that now we have to introduce this porosity here because once again here it is the concentration means the concentration of volume of the element that remain same, but once it enters into the matrix, the concentration is if the question comes in the concentration is concentration per unit volume of porous medium or concentration is concentration per unit volume of liquid and these two are different for fracture these two are not different. So, do you one need not have to consider, but here one has to consider this porosity term.

So, this is as far as the mass balance equation for the fracture is concerned and when it comes to this mass balance into the matrix. So, one can pick up a differential element here in in the matrix and one can draw these simple this is the diffusion happening no pressure driven flow. So, it would be simply del c del t equal to D star mind it this we are it as D star and not D. So, D star is the diffusivity or diffusion coefficient you call it is diffusivity in porous media diffusivity in porous media so; that means, this has to be equal to bulk diffusivity multiplied by diffusivity means which is reported in literature for so, say salt it is some one point something into 10 to the power 10 one point something in to 10 to the power minus 5 Cm square per second.

So, this is that bulk diffusivity inside a beaker whatever you observe, but then multiplied by some factor because this diffusion is happening in a tortuous path ways. So, that is why we are calling it D star. So, D star is equal to D multiplied by some factor which accounts for the tortuosity of the path way and all these restrictions in the pathways inside porous media. So, this is one thing this D star is appearing here. And this D star is appearing there as well and this phi term that is appearing on both these both the sides and so, it got cancelled out the for the porosity term see it is appearing.

So, it should it is just like any other diffusion equation. So, Cf is the mass per unit volume of liquid in fracture, Cm is mass per unit volume of liquid in matrix and when we when we converted this phi into Cm is mass of liquid per unit volume of porous media in matrix. So, these are the two equations we have one is this this is. So, we have two equations now if we look at these we have one equation which is valid for the fracture here and another equation for the matrix. So, these are the two equations we have here. So, these are the two equations that needs to be solved simultaneously.

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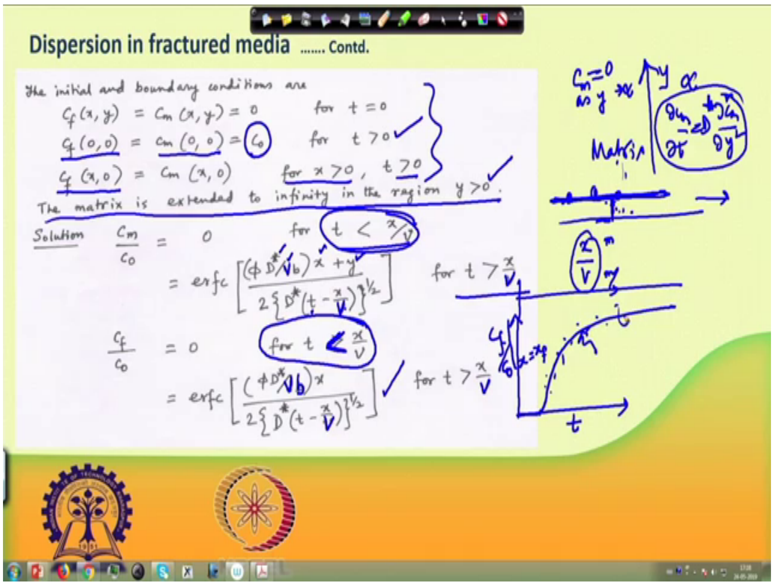

So, what we have here is so, we have certain boundary conditions here the boundary conditions are at Cf x y; that means, at the velocity sorry at x C f x y is equal to Cm x y is equal to 0 for t is equal to 0. So, at time t equal to 0 everywhere it is concentration is 0 and then once time starts; that means, once t is greater than 0 Cf 0 0 x is 0 y is 0.

That means at the inlet of the fracture the whatever is the concentration that is equal to Cm 0 0 that is the concentration see I must point out here is that look at the axis here the x axis is at the edge x axis is at the edge of the fracture it is not at the central. So, this is the central line I agree, but the x axis and the y axis starts from the edge of the fracture. So, we assume that is all the well mixed and everything ok. So, it is as if I have an you might have noticed already that we are working only with the V. So, it is like you have a uniform velocity profile V.

So, that is the velocity with which the flow is taking place and the x axis is defined here. So, 0 0 is basically this corner point not inside the fracture it is at the interface of fracture and matrix and y is equal to 0 means this interface line not the central line of the fracture. So, this you one must make note here. So, if we see in this case. So, it is Cf 0 0; that means, at the inlet point and then Cm 0 0 it is C 0. So, that is what you have introduced for t greater than 0 and that is what you are maintaining and another condition one has to satisfy which is Cf at x comma 0 x comma 0 means Cf at any location but y is equal to 0; y is equal to 0 means at the fracture matrix interface.

So, the concentration in the fracture at the fracture matrix interface is equal to concentration of the matrix at the fracture matrix interface x 0. So, basically these so, we are talking about if this is the fracture and this is the matrix. So, we are saying that at this location we are solving something; so, Cf that we calculate here and the Cm that we calculate there. So, these two are same value these; two are same value these 2 are same value for every x; so, for x greater than 0 and t greater than 0. So, whatever happens from fracture whatever value we calculate and from matrix whatever value we calculate they have to be equal.

So, now, these are the boundary conditions one holds. So, these and further one other condition is the matrix is extended to infinity in the region y greater than 0. So, this is the y direction, this is the x direction in the fracture and this is the y direction which is into the matrix. So, one assumption is that it is extended up to infinity so; that means, when you when you solve this del Cm del t is equal to D star del square Cm del y square.

So, you are assuming when you are solving this equation you are assuming I mean simultaneously with the fracture equation you are assuming that, Cm you will tend to be Cm is equal to 0 as y tends to infinity. So, that is something which you probably used as a boundary condition so, that is why they are saying this. So, now, you have a solution here I you can see here Cm by C 0 there is a solution Cf by C 0 there is a solution and these are the expressions that one has the solution. Now it is given for 2 conditions one is for t less that x by v another is t greater than x by v what does x by v signify? You can see very well x is in this direction this is the x direction.

So, x divided by v v is in meter per second v is the velocity at which the flow is taking place. In fact, since we are putting at this is the same we that we have been continuing all along v is the velocity in the fracture. So, it is I mean there at that is that is whether it is some place it looks like it is a capital V or small v, but it is the same v. It is the same v we are talking about which we are discussed earlier also. So now, this v is the velocity through the fracture now this v is on the other hand.

So, if you have x divided by V that gives me what? That gives me the time taken to reach time taken for the front to reach the distance x right. So, x is in meter V is in metre per second. So, when you take x by V meter divided by meter per second. So, if a fluid front moves at a velocity v the time required to reach a distance x would be given by x by V. So, what is means is when t is less than x by V means at that x location I am say let us say I am referring to this x location and I am asking what is the value of Cm as a function of y and what is the value of Cf as a function what is the value of Cf at that point and what is the value of Cm what is the value of Cm as a function of y from that location.

Now, if the front has not reached there, then naturally this question does not arise. So, this is 0 this is everywhere it is 0. Once the front reaches then only we start considering these diffusions. So, that is why the at the out at the outside they said t less than x by V Cm by C 0 is 0 because the matrix was all concentration was 0 and the same thing Cf by C 0 is 0 for t this is this this should be less t should be less than x by V Cf by C 0 is 0 it should be less than v so, less than x by V.

And when t is greater than x by V then only there would the solution is required and then they have done this they have obtained the solution, this is the analytical solution available for I mean when the salt the fracture equation and the matrix equation simultaneously with these boundary conditions. So, these are the, this is the equation.

So, you can see here Cm by C 0 is a function of the matrix concentration is a function of the first of all the position what which x which y and then the porosity the matrix diffusivity the velocity at which the flow is occurring b is the half fracture aperture x is the location V and what time we are talking about. So, with time also this is changing. So, similarly here also we have the Cf by C 0 the concentration of the fracture and that is given by this quantity which is D star divided by V into b. So, this is again t minus x. So, this this are this is the way this Cm by C 0 and Cf by C 0 these are handled. And now if somebody plots Cf by C 0 Cf by C 0 as with as a function of time at x equal to xf let us say the end of the fracture. So, now, they should be getting for some distance it is all 0 until t is greater than x by a when t is less than x by v this is all 0.

And when t is greater than x by V this equation will be followed and if you follow this equation, it will go like this and then start telling if somebody plots these exactly then they will find that now you can plot the tailing of the response, what happens in the fracture by using this equation ok. So, this will give the tailing of the response. And, now this tailing of this response can be fitted to find out what is suppose some unknowns are there I do not know what is suppose if it is a sub surface or in case where we do not know or we have no characteristic parameters for the fracture.

So, now we can find out V is in our control D star we have a priori knowledge, porosity we have some a priori knowledge, but probably the aperture b we do not know. So, one can find out what is the value of b which satisfies the experimental data point that you have gathered at the outlet and now you want to fit a line using this equation. So, what should be the correct value of b for example? Correct value of b so, that one can get this kind of profile.

So, one can go back now and find out. So, by looking at this looking at this tracer response, this is one can perform a tracer test and by looking at this tracer response now they can go back and characterize, what is the characteristic parameters of these fracture or what other some other parameters if one wants to know. So, they can give these kind of data points using these equations and they can find out those parameters. So, you can see these are some of the some of the advantages of working with this kind of analytical models ok. This is all I have as far as this lecture module is concerned.

Thank you very much.