Flow through Porous Media
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Lecture – 31
Miscible Displacement (Step Change in Concentration) Contd.

I welcome you to this lecture of Flow through Porous Media. What we were discussing was Miscible Displacement. In particular we are talking about mixing that takes place as flow occurs inside a porous medium.

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So, what we discussed before is basically we are talking about dispersion of solute or tracer in porous media and we have first looked into how it occurs in a capillary and then came up with concepts called dispersion coefficient, which involves both diffusive component as well as the effect of velocity because mixing is contributed both by the tendency of molecules to break free and move into other territories, so, that is that is one thing.

Second thing is this is also enhanced by the flow that takes place because mixing is also a function of that. So, we came up with a concept called dispersion coefficient for a capillary and then we try to extend that to porous medium. So, now, what we were discussing at the end of last lecture was the dispersion coefficient in a in a porous medium.

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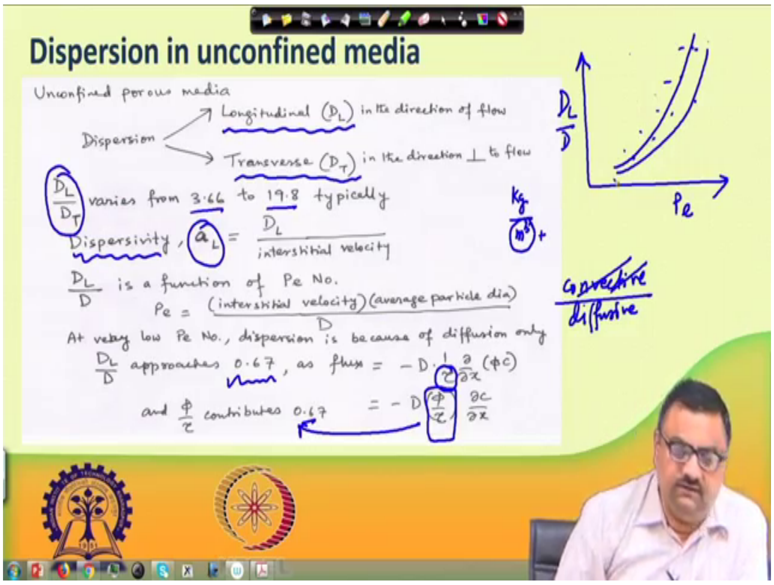

So, here now we are going to discuss about two different types of dispersion coefficients because this would be important. So, far we have been talking about only unidirectional flow, but once you have an infinite porous medium on this. So, then there would be a longitudinal dispersion and there would be something called a transverse dispersion.

Longitudinal means that the flow is taking place in this direction. So, in this direction there would be a dispersion happening we have talked about Taylor dispersion, these are all longitudinal dispersion. But at the same time there could be another spreading or occurs that occurs in the transverse direction as well. So, what research as they have done what (Refer Time: 02:29) they have done was that they have calculated these dL by dT and they found that longitudinal dispersion is dominant; obviously, it varies from 3.66 to 19.8. So, the dispersion in this direction in the direction of the flow these dominates, but there would be some amount of spreading also in the transverse direction.

Now, in most in many cases this transverse direction dispersion is not considered because what the application may require only considering the longitudinal dispersion. In this context there is a term which is defined which is called dispersivity dispersivity is I mean in porous media literature porous media related software and generally they ask for what is the dispersivity we should consider. So, dispersivity essentially this is defined as a L which is this longitudinal dispersion divided by interstitial velocity. So, this there is a lot of times in softwares you may be asked to feed in this data what is the dispersivity.

So, dispersivity is nothing, but longitudinal dispersion divided by the interstitial velocity and dL by D this is this itself is a function of peclet number and peclet number we all know is the interstitial velocity average particle diameter divided by D.

We said that this is basically the convective divided by diffusive contribution convective contribution to mixing divided by diffusive contribution. So, that is why in the numerator we have this interstitial velocity and in the denominator we have the diffusivity term. We have already studied we have seen that peclet number if it is small we we go for pure diffusion whereas, if peclet number is very high will go for pure conviction. So, this is we are already familiar with this peclet number now this dL by D I mean there is there is a there is a I mean this is this is a common trend among you know among these researchers here is that, there is people tend to plot dL by D as a function of this peclet number and they found at ds I mean they have picked up different kinds of porous medium there different types of particles they the constructed porous medium and they had to run it.

They had run and after they did all these they plotted they made a log plot and they generated a curve something like this ok. So, this dL by D as a function of peclet number generally at low peclet number or peclet number tending to 0; this dL by D it has some magic number everybody has converged to which is 0.67 and generally they attribute this whole thing to the fact that when peclet number is truly small or insignificant, this convective part is not at all present; that means, it is flowing at such a low rate that conviction does not have any effect. So, it is completely diffusion this transport is diffusion control.

So, if there is a diffusive transport. So, any flux any longitudinal in longitudinal direction any movement is basically arising from the diffusion and so, it will follow minus D del C del x type form and then they what they had done is if somebody divide these D bulk diffusivity by a factor known as tortuosity to account for the fact that here the diffusion is taking place inside porous medium not in a beaker not in a not it is not a free diffusion. So, I mean it has to go through all the tortuous pathways the diffusion pathways are now increased.

So, that is why they have put in one tau here and there is some factor there some tortuosity factor there and then some factor which is the arising far from these arising from these this particular situation, that inside porous medium diffusion is not molecule is not free to move molecule will be restricted to move in tortuous path. And then further the concentration the definition of concentration here I mean if you see you can write it a concentration we write as kg per meter cube. So, when we write kg per meter cube that is kg per meter cube of that solution. So, so many say I am writing this is a so, many kg per meter cube is the concentration or so, many moles per meter cube is a concentration. So, moles per meter cube or the solution.

But here this meter cube ideally I mean if this meter cube is not meter cube is solution which is in only in the wide part plus there is a solid volume. So, if you are expressing this concentration as so, many moles divided by the porous medium volume. So, one has to account for that fact that that effect. So, one has to bring in that porosity term. So, that is why one has these ratio. So, these ratio basically contributes to 0.67. So, otherwise it is it is. So, this is this is something which is which people have already seen. So, there are good amount of literature available where in porous medium there were studies made and reports made on what is the dL by D for a particular porous medium as a function of peclet number ok.

And these they have done for different samples and they have they have created sort of a domain I mean range within which this say for example, within this window they will be a if the grains are fine grains or coarse grains depending on that these lines will change and how they will change. So, that kind of studies have been done in this regard.

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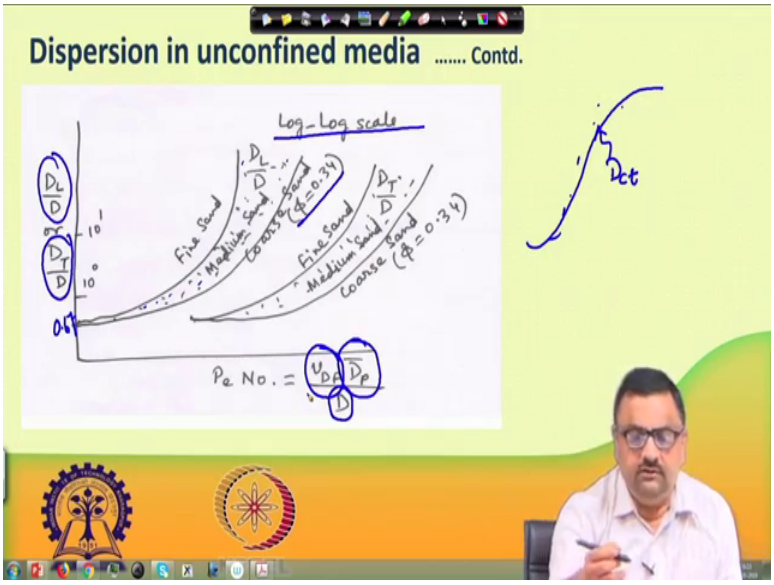

So,. So, this is what I was I was referring to that these dL by D or longitudinal dispersion or transverse dispersion and then they put these on a log scale. So, log scale when they put you can see this value is converging to 0.67 and then these are for these data points all this point these are these are for longitudinal dispersion and they found that this is the line for fine sand and this is the line for coarse sand with porosity 0.34 and all are the all data points they are within this band ok.

Similarly, for transverse there a day dispersion in transverse direction. So, they are also they had created a band. So, this kind of studies have been done and generally if an unknown sample has to be studied. So, then one has to run the dispersion test; that means, one has to conduct a dispersion experiment and then one has to find out what is the dispersion coefficient I mean to say that I give a step change I get I get these are the points and then I have to fit a line through that and what should be the value of what we call that time was Dct.

So, what should be the value of Dct such that these line fits the experimental data points perfectly. So, this kind of studies have been done systematically for different sets of porous medium and these are the or this is how the observations are summarized. So, these V DF in this context V DF is the interstitial velocity Dp bar is the particle diameter. So, because porous medium is consisting of particles. So, these particle diameter what is the average particle diameter? That is Dp bar and this D is the bulk diffusion coefficient. So, this type of exercise has been already done.

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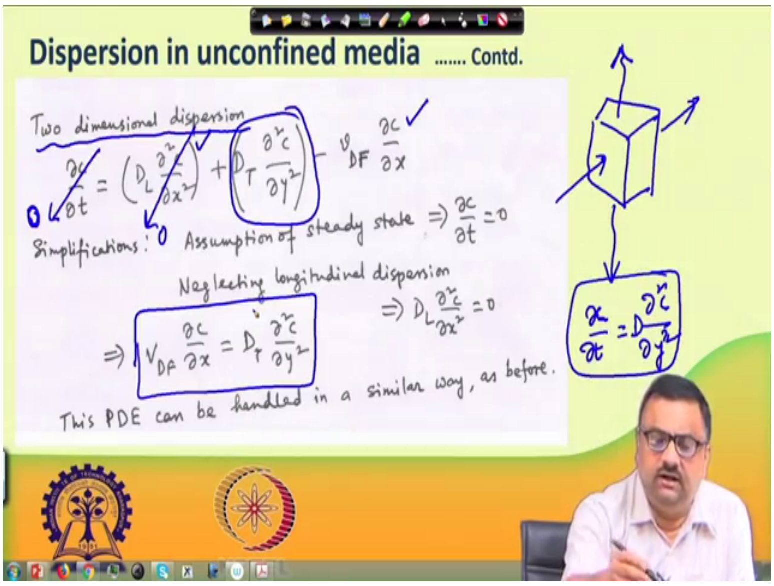

Next thing what could be of interest in this context is two dimensional dispersion. So, in two dimensional dispersion you can see we have been we have been only talking about what? We had picked up a differential element we picked up a differential element and we tried to find out how much is going in by diffusion, how much is living from the other side by diffusion, how much is going with the flow from this side, how much is living with the flow from other side. So, that difference. So, diffusion in minus out convection the solute is entering through convection that in minus out. So, those terms and difference is accumulation. So, that is how we had these this is the diffuse contribution from the diffusion term, this is the contribution from the convection term V DF is the interstitial velocity, this we have studied in previous lecture.

Now, if someone has both longitudinal dispersion and transverse dispersion; that means, they have another dispersion component in this direction here. So, that has to be accounted. So, one can one can put these D T, one can one can put these this contribution of transverse dispersion and they have included this term as well and calling it a two dimensional dispersion. So, now, once again there are some simplifications possible to this equation first of all if one assumes steady state, this term will go to 0 and then this if one neglects longitudinal dispersion I mean if this needs to be simplified in the event this no longitudinal dispersion can be neglected when can it be neglected convection dominates diffusion is not dominating.

So, in that case one can only focus on convection for the longitudinal part because that is what is dominating and then one will left to it this form of an equation. So, so this part is considered 0 these longitudinal part also goes to 0 and so, it is play between this and this. So, that is what we have written here is that the convective term this goes to the other side is equal to D Tdel square c del y square. And by now this kind of partial differential equation you have already I mean I have I have discussed this in one of the earlier lectures is, I mean one or two lectures back we had discussed about how to solve this using Laplace transform, this is this is very similar to what we had there because that time we were solving del c del t is equal to D del square c del y square. So, instead of. So, this is this is now the equation.

So, how to solve this kind of equation generally the solution would be in error function term and that that approach. So, with the right boundary conditions that approach we have already discussed. So, this is also another way to handle things analytically without complicating because if you bring in this longitudinal dispersion, there may be analytical solution available, but it is most likely one has to resort to the numerical solution. So, instead of that still one has the analytical solution for this kind of equation. So, one look for opportunities that if one has if this kind of assumption be made that longitudinal dispersion is neglected then the treatment would be much simpler and easier.

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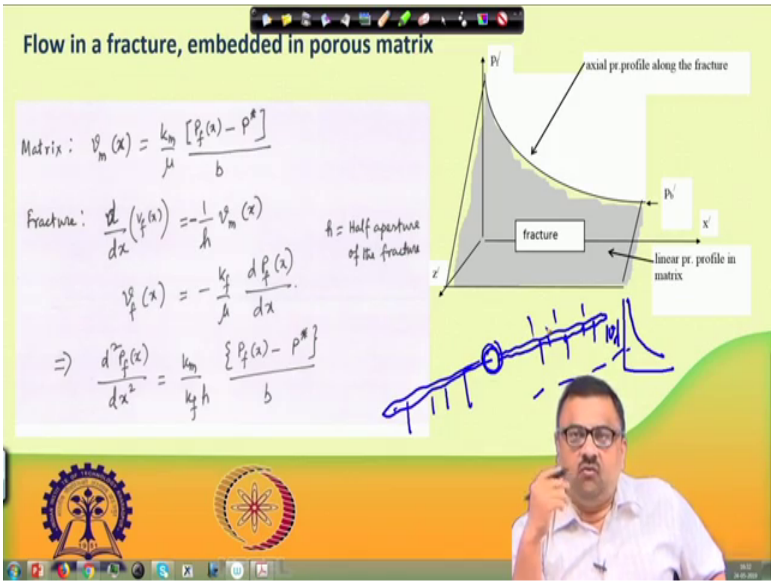

So, this is these are the in windows one has with regard to this dispersion, what I have next is something little different let me let me talk about this. First of all when it comes to flow through a fracture, then the in a porous medium there can be a fracture. If flow through a fracture it would be it would be something like this that if this is the well if this is the well and from the well there could be a fracture running; that means, we already talked about it this there could be a crack running. Either this or it could be either these or it could be that there are there are multiple small fractures and they got somehow connected there fracture can occur in the fracture can occur due to different reasons, it could be that one has by mistake over pressurized the porous medium, I mean put additional pressure and so, it could not bear and so, there is a crack developing inside.

It could be that or it could be due to thermal stresses, you heated it and the heating was not uniform or there is a sudden heating or cooling and because of that thermal stress is developed and so, because of that it has cracked. So, it could be it could be in a porous medium that one has I mean one was cross, it could be just a manufactured or processed porous medium there also the same thing can happen or it could be even somebody is looking at the subsurface processes, they are also the same thing can happen that in a in a underground reservoir because of thermal stresses thermal stresses means one injects water and this reservoir itself was at a higher temperature one is injecting cold water. So, these thermal stress can cause fractures I mean because the in certain locations it can happen, oh and there could be other geological reasons tectonic stresses and other reasons because of which there could be fractures.

And at some point when it comes to reservoirs, there are there is a possibility that one can resort to something called hydraulic fracturing I mean in intentionally inject some liquid and create a pressure so, that a fracture generates around a well the to take to leverage fittings, I mean which you can you can. In fact, very well see that we have already seen when we look at a well when we look at a cylindrical system, we have talked about this v r the radial velocity in earlier lectures.

So, there we have seen that the vr is vr is increasing I mean if we if we plot mod of vr that is increasing very rapidly as r decreases so; that means, near the well the velocity has to be radial velocity has to be very high whether you are injecting or producing. So, that is why I am writing it as mod of v r. So, because of this high velocity so, it is it is better if one can have some more area for collection of this fluid because you are collecting the fluid from all over the reservoir and you are collecting it through a small place. But instead of that if you could expand if you could create some kind of manifold through which these fluid can be collected or through which the fluid can be injected then this type of sudden rise of v r at smaller r can be avoided and so, this can help tremendously with regard to pressure drop.

And pressure drop means pressure that is that is one has to generate this pressure somehow the another. So, that is. So, that that would that would simplify the matter very much if one can create a manifold. So, one way of doing it is to create a fracture around the well so, that the fluid can be delivered or fluid can be injected through this. So, it is basically acting as an extended source we had a point source earlier and vr was drastically increasing. Now instead of a point source only now we have a line source attached to it. So, this is this is so, now, this is either line source or line sink. So, these are these are some of the reasons for which one does fracturing and when one does a fracture. So, when the flow takes place through a fracture what kind of dispersion we can expect?

I have already mentioned before that if one has the I have already discussed before that if one conducts the dispersion experiment in a core that is already fractured, we would be seeing an immediate breakthrough and then gradually this is that there is a there is a long tailing of the response.

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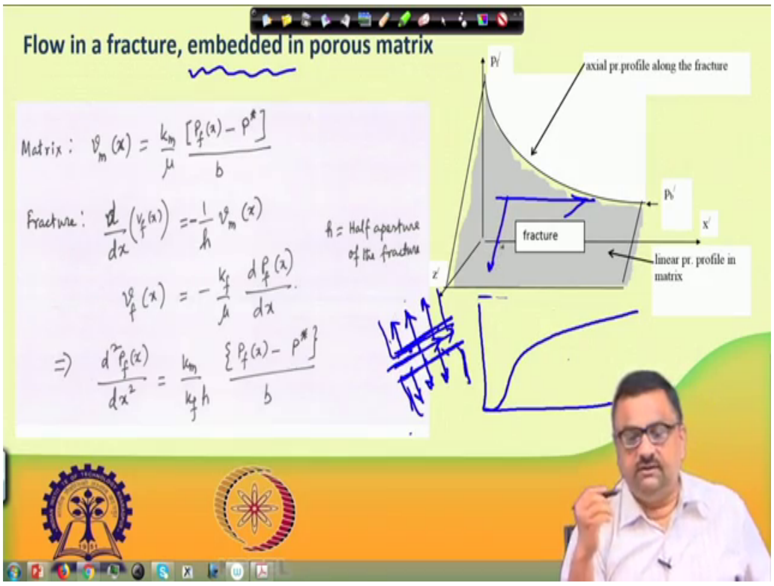

So, it was immediate breakthrough and then that is a long tailing to reach this beyond 0.6 or so, then there is a long tailing is to reach that 1.0. So, this type of profile we have already seen when there is a fracture present. Now when it comes to flow through a fracture, there are two ways we can think of this flow to happen one is I mean when you have a fracture embedded inside a porous matrix mind it embedded in porous matrix.

There are two kinds of flow that takes place; one is when I am injecting at a very low rate at a very feeble rate this could be for example, I am injecting a contaminant into the fractured porous medium or I am injecting some tracer. So, these are injected at a very low rate and the fracture is it is a thin fracture we are so, so. So, in such situations it is it is probably it is it is better to consider the flow through the fracture as a unidirectional flow; that means, this is the fracture and this is the adjoining matrix. So, we can expect the flow is happening through the fracture and then there is some diffusion molecular movement of cause solute into the adjoining medium. So, this is something which we could think of. So, it is more of an unidirectional flow through the fracture and then there is some amount of diffusion.

On the other hand, so, this is this is this is for all practical purposes this is true if somebody is looking at a transport of some contaminant, transport of radio nucleotide radio nucleotide some transport of transport of the transport of tracer which are done at a very low rate, the pressure here is not much. Of course, a pressure this side is higher than this side. So, there is a flow, but the overall pressure at which this whole scheme is working is at a low pressure. On the other hand when you when someone is someone is having a flow through a fracture which is at a very high rate; for example, one is trying to fracture. So, that time they will do it at a high pressure or when they are producing they are injecting I am injecting the fluid which is of value to which is which is a value.

So, there this here we here the objective is to have the highest amount of production. So, there the pressure at which this fracture is held is a much higher pressure. So, in that case one can one can think of the fluid flowing from fracture into the matrix. In fact, this is the this is the case where we call the there this is some kind of a regime the earlier regime that I mentioned it is a linear flow regime; that means, flow is taking place linearly in one direction along the fracture. But now we are talking about a regime which is more of a bilinear regime; that means a flow through the fracture and perpendicular to this there would be a flow into the matrix.

Earlier we considered this to be earlier we considered this to be diffusion that molecules they will diffuse into the adjoining porous matrix because of their Brownian motion and everything. But now we are talking about a Darcy flow in this direction because the pressure in the fracture has been jacked up. So, pressure in the fracture is much higher, pressure in the fracture is much higher compared to that joining matrix. So, we would expect that there would be a Darcy flow. So, these they at that time these fracture is acting just as a line source. So, there are two ways to look at these. So, I just wanted to do first of all I wanted to differentiate these two types of flows. So, that is why I have I have put this plot here and these shows and this kind of two linear two perpendicular to two linear flows that are mutually perpendicular to each other.

One flow is along the fracture and the other flow is from the fracture into the matrix.

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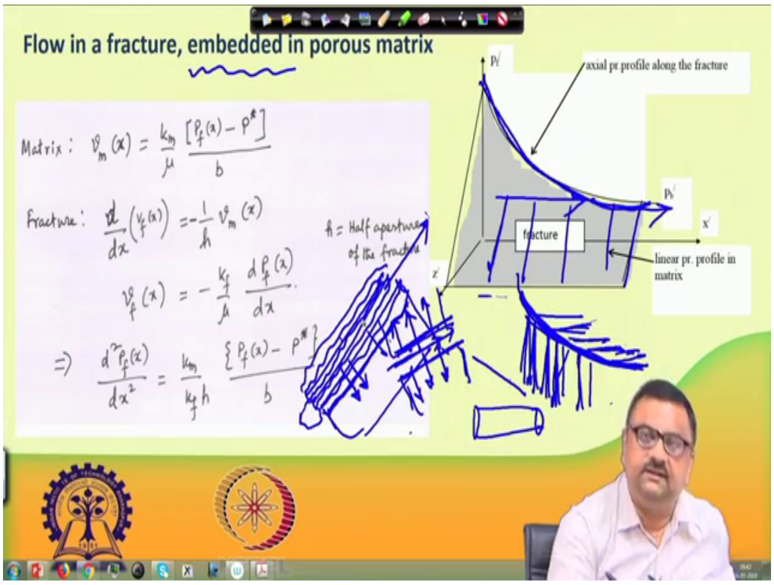

So, basically if this is the we expect that as if. So, the pressure in the fracture is following this line. So, these line we have already shown here by this line. So, pressure in the fracture has to go down because we have flow through the fracture. So, there is this a pressure has to be go down flow will take place from higher pressure to lower pressure. But at the same time this pressure in the fracture is higher than the pressure in the adjoining matrix. So, if I follow these lines here, this is the pressure profile in the matrix so; that means, these are, so, along the in these. So, there is pressure is higher pressure is lower. So, there would be flow from fracture into the matrix and here also flow from fracture into the matrix ok.

So, this is what we are referring as so, called by linear flow; one linear flow along the fracture, another linear flow perpendicular to the fracture and into the matrix and we expect this line not to be straight line rather this line is carved, this line would have been straight line if there is no loss into the matrix no fluid going into the matrix, but since any element as and any flow that goes here some part is going in this direction, but some is lost into the matrix. So, naturally I mean you remember if I had a one dimensional flow through a core, I would have expected the pressure drop to be linear, but here in this case the pressure would not be linear pressure would be carved because there is continuous loss of fluid into the into the matrix.

So,. So, this is this is something which is the pressure profile that fracture will follow and this is the pressure profile which matrix will follow. So, this is something which I have tried to draw here. So, this is the pressure along the fracture and this is the pressure towards the towards the matrix. So, now, we need to. So, so what I will briefly do in the in the next lecture is I will try to quickly show how this type of two linear flows at least can be can be conceptualized and then next we would we would focus on one I mean only single linear flow which is done for a tracer test and we will try to find out how dispersion takes place in this in that situation. Because in the in the when there is fluid flowing from fracture into the matrix the flow is somehow somewhat different.

You can think of here that the flow is flow is going from here to this direction and also flow is moving in this direction. So, we if there is a very little chance for this fluid getting diluted when you are collecting at the outlet because it is the same fluid which is flowing. So, one can one can think of suppose I have this is the fracture. So, moment we introduce a packet of this solute the packet of solution which has a higher concentration of solute, these packet will be immediately stretched to a straight line I mean that is that is what these that is what these two linear flows all about.

This will be stretched in the form of a straight line and then these straight line will start growing these straight line will start growing, this straight line will start growing and go into the matrix flow will continuously take flow will take place in this direction, but this will continuously this will also grow. So, when it comes to this fluid that we get from the outlet once it gets. So, I am pushing all the fluid out from fracture into the matrix. So, there is very little chance that these gets diluted these particular fluid gets diluted compared to the situation, when there is not this flow is absent we are only having one linear flow.

So, that is why in this case probably the chance of dispersion is somewhat less if at all dispersion happens dispersion will happen here inside this. So, one front is coming and getting into the porous medium. So, one has to now look at this dispersion which is just a classical dispersion of porous medium it has nothing to do with fracture. From a line source fluid is entering and this is undergoing dispersion one has all those longitudinal dispersion dct etcetera. So, those will be appearing here, but when it comes to the fracture, we do not have that possibility. On the other hand when we have a linear flow through the fracture only and no fluid going into the matrix in that case there is a truly a dispersion happening at the outlet end in the fracture. So, these are some. So, this issue I would like to delve into this further in my next lecture and that is all I have for this module.

Thank you very much.