Flow Through Porous Media
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Lecture - 28
Miscible Displacement (Step Change in Concentration) Contd

I welcome you to this module of Flow Through Porous Media what we were discussing was Miscible Displacement; that is if one gives a pulse or one gives a step change in concentration at the inlet and simultaneously the flow is taking place at constant flow rate. So, what would be the mixing and what we expect at the outlet? So, this is something which we are discussing and we have already established in the static system when we give a step change how it behaves.

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CONCEPTS COVERED

Dispersion of solute / tracer in porous media

So, now we are going to convert that to the flow process.

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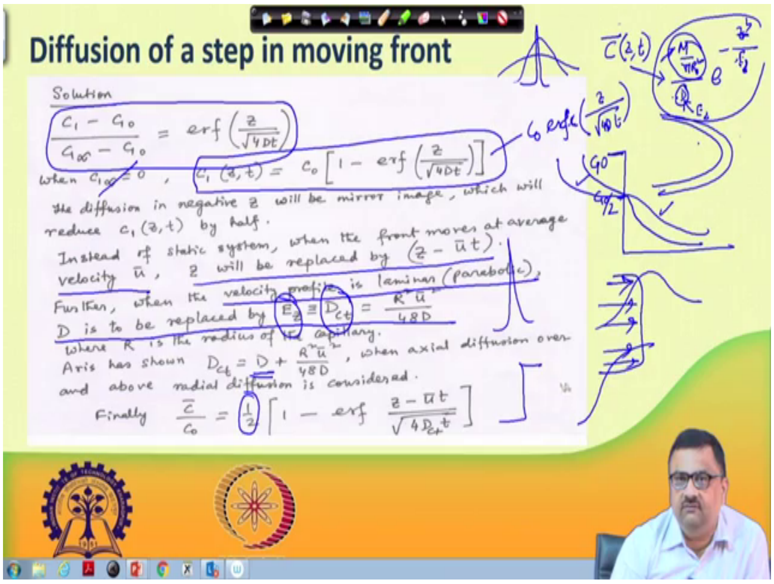

Diffusion of a step in moving front

Solution

$$\frac{C_1 - C_0}{C_\infty - C_0} = \mathrm{erf}\left(\frac{z}{\sqrt{4Dt}}\right)$$

when $C_{1\infty} = 0$, $C_1(z,t) = C_0\left[1 - \mathrm{erf}\left(\frac{z}{\sqrt{4Dt}}\right)\right]$

$C_0\,\mathrm{erfc}\left(\frac{z}{\sqrt{4Dt}}\right)$

the diffusion in negative z will be mirror image, which will reduce $C_1(z,t)$ by half.

Instead of static system, when the front moves at average velocity $\bar{u}$, z will be replaced by $(z - \bar{u}t)$.

Further, when the velocity profile is laminar (parabolic), D is to be replaced by $E_z \equiv D_{ct} = \frac{R^2\bar{u}^2}{48D}$ where R is the radius of the capillary.

Aris has shown $D_{ct} = D + \frac{R^2\bar{u}^2}{48D}$, when axial diffusion over and above radial diffusion is considered.

Finally $\frac{\bar{C}}{C_0} = \frac{1}{2}\left[1 - \mathrm{erf}\,\frac{z - \bar{u}t}{\sqrt{4D_{ct}t}}\right]$

So, we are discussing about dispersion of solute or tracer in the porous media. This is something which we ended up with, this was the equation for a static system and we said that when this C 1 infinity can be set to 0 then this would be the governing equation C 1 as a function of z and time, it can be given as this into one minus error function. And what is this error function? Generally, it is customary to write this as c 0 into complimentary error function since it is 1 minus something. So, it is customary to write complimentary error function that is erf c z divided by square root of 4 dt.

So, now we said that this is based on that C 1 0 being held there as constant, C 1 0 for all time t greater than 0 this is held at C 1 0 and this is held constant. But in when there is a step change and this step change is flowing through a capillary, we expect this concentration to be like here there is a buildup. So, similarly there would be a depletion. So, we can assume this instead of this C 1 0, it is operating as C 1 0 to C 1 0 by 2 so; that means, instead of this as the profile, we are looking at this as the profile and this as the mirror image of this as the profile. So, this is something which we can work with. So, that is the right away what they said is that when there is a flow we are not maintaining C 1 0 rather anything that is diffusing to the downstream has to come from the upstream.

So, any gain in the downstream is equivalent to loss in the upstream. So, concentration if these and these are mirror images instead of C 1 0, you should consider this to be C 1 0 y 2 straight away. So, that is why this half is appearing here. So, and we put there here one

C bar I mean it is just to remain consistent with what we had done earlier for Taylor dispersion. In fact, that we will go there in a moment let us I I mean just make note that we are going to put a factor half for these reason; considering this to be an exact mirror image. An instant of a static system when the front moves at an average velocity u bar then z has to be replaced by z minus u bar into t this we had done earlier for moving coordinate system.

So, and further when the velocity profile is laminar, now you make note here I mean if I had this front to be perfectly uniform; that means, this is the velocity front, every layer the entire cross section is travelling with the same velocity then we could have solved these in a different way; that means, we could have considered these velocity u bar as the exact velocity and the diffusion that is taking place that is the diffusion that is just the diffusivity right. What we did last time if you recall in case of a Dirac function when it is traveling, we said it can travel under two conditions: one is uniform velocity profile; that means, velocity over the entire cross section is everywhere it is same.

So, what; that means, is simply as if it is in a static system only this system is traveling over some time. So, it is as good as giving this Dirac function some residence time in the static system wherever it would have gone, the same thing would have happened because there is no stretching of this no parabolic velocity profile, no stretching of this pulse. So, it would it is as if this is remaining a static system only it is given that residence time inside the capillary. So, when it reaches the outlet over that residence time whatever diffusion would have happened that diffusion had happened there. So, this is one way of looking at it second.

So, in that case when we did these that time, we had written it as M divided by pi r 0 square which is M by a then there was some term here and then e to the power of minus z square divided by some term here. And, this z minus we have considered the velocity multiplied by t whole square and here wherever this D term is appearing here or square root of D term is appearing here. So, this D term this is just simple diffusivity, because it is under just simple diffusion only it is instead of sitting in a back system, it is given the residence time. So now, we have hypothesized it as if I have given just the residence time in a static system same as whether it is traveling through this capillary.

Now, on the other hand when the when these Dirac function is placed in a capillary where non-uniform velocity profile was set in by virtue of laminar velocity profile where so, where these pulse is stretched in the form of a parabolic distribution. In that case also you remember if you look at the final result of Taylor dispersion we stuck to that same form of equation M by a divided by some term involving pi Dt and then e to the power minus z square that some term is there divided by some term involving d dt etcetera.

So, these same form we maintained only instead of D we had here E z. E z is the dispersion coefficient instead of D here we had Ez which is the dispersion coefficient. So, now, D is gone instead of D we are writing R naught square v naught square divided by 48 D that is the dispersion coefficient. So, they said that the outlet if we want to do this the only change we have to do is we have to change the diffusion coefficient to the dispersion coefficient now and obviously, the velocity this for the moving coordinate I have to use the average velocity. Here in this case for uniform velocity it is same as you know velocity at every point is same as the average velocity.

In case of parabolic velocity profile velocity is not same everywhere, but one is to consider to average velocity. So, then they know that the center of this pulse how far it has it is moving. So, we what we essentially see here is that, when velocity profile is laminar or parabolic then D also has been replaced by this dispersion coefficient. Last time we call it E z now in case of a step change the general literature says this is as dispersion coefficient for capillary ok. So, Dct which is same as R basically radius of the capillary square average velocity square divided by 48D.

So, and and further there is one point I must note here that the Taylor beyond Taylor Aris who was the next scientist who worked on it, he showed that if somebody wants to consider axial diffusion you remember when Taylor did these analysis that time we considered axial convection and radial diffusion. But, Aris showed that on top of this if one considers axial diffusion also then simply that is a term of the dispersion coefficient term that term will where there you have to add simply 1 D to these R square u r square by 48D so, that this now this would be capable of including the effect of axial diffusion. So, this is something which Aris has shown. So, that is I mean that is up to somebody who if they want to consider axial diffusion or ignored axial diffusion that choice is there.

So, now finally, what one ends up with is these C bar now we are calling it C bar I mean I do not I am assuming there would be Taylor dispersion inside the capillary. So, Taylor dispersion inside the capillary only thing is I have an instead of Dirac function I have given a step change ok. So, now, there in that case we had a concentration profile by Taylor dispersion for Dirac function for a pulse, for a step change we are arriving there also in case of a Taylor dispersion, we were we talked about C bar. And C bar was a function of z and t. Why C bar because we had averaged it over the R right we said that radial concentration profile in radial direction is of no use. So, over the entire cross section we will average it. So, that is why instead of C 1 as function of r z t we moved to C bar z t.

So, and that was equal to this form we had discussed in the Taylor dispersion. So, now, in this case we have this same C bar, but for a step change in concentration is equal to half into C 0 these this C C bar they there also we had a C 0. So, here also it is C 0. So, C bar divided by 0 is C equal to half 1 minus error function of z minus u bar t the moving coordinate system has to be set in divided by square root of 4 Dct this time which is the dispersion coefficient for a capillary multiplied by t. So, this is the final expression; this is the final expression for concentration profile for a step change in concentration in a capillary.

So, just like we have a final expression C bar final expression C bar actually they are the C naught does not make sense. So, that is why we had this M by pi R naught square because there M matters in case of in case of a pulse what how much kg you have placed that is important. So, M matters. So, that is why it is written as M by pi R naught square. So, you have a form in case of a Taylor dispersion which relates C bar z t in the form of, but there we had an exponential term. Here instead of exponential term you have error function, but same C bar we are relating instead of M by pi R naught square you are having C naught half C naught 1 minus error function of this quantity.

So, now this is something which we will carry forward when we when we go to porous medium, this is something because we have a or other we can take that also, we can continue with that exercise as well of, but if you are working with a step change in factor this is truly speaking you can use either of these one is just put a pulse. So, for that what would be the stretcher spreading of that pulse and otherwise you put a step change and continue and see how much. So, I have given a step change here and after the residence time I see the step change is showing like this. So, what how much it has how much is the spreading how much the front is made.

So, this is something we can see. So, now, in we have now seen how to do these two things one is Dirac function to spreading or step change to spreading. So, how we can analyze these, how we can analyze these for a capillary? So, these are the two expression this is one expression or the other expression we have discussed in a previous lecture.

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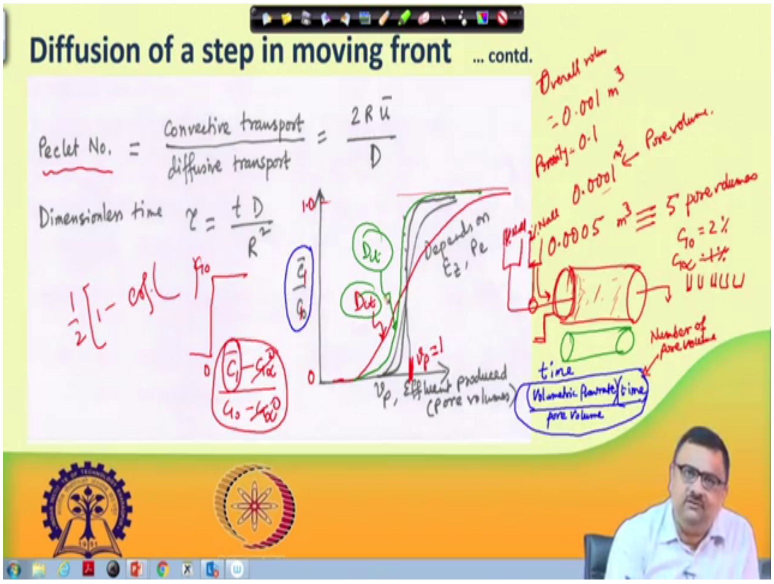

Diffusion of a step in moving front ... contd.

Peclet No. $= \frac{\text{Convective transport}}{\text{diffusive transport}} = \frac{2R\bar{u}}{D}$

Dimensionless time $\tau = \frac{tD}{R^2}$

So, now we can we can look at let us say we said C bar by C 0. So, we have we can plot this C bar by C 0 as a function of we said we have choice for the x axis two choices we have, one could be simply time or we it could be q it could be the volumetric flow rate volumetric flow rate multiplied by time volumetric flow rate multiplied by time.

So, this could be the way x axis or this could be the x axis and table have taken it further they said why volumetric flow rate multiplied by time will give me the cumulative volume ok. So, instead of that you divide it by the pore volume or void volume in that for that for that particular media. So, what these gives me now number of pore volumes number of pore volumes if you divide it by the pore volume. So, what is the idea here that.

So, then if we if we put as number of pore volumes or people generally shorten it in and say that pore volumes ok. Pore volumes means what how many number of pore volumes for example, if this material has a volume of let us say 0.001 meter cube and it has a porosity of 0.1. So, total volume let us it overall volume is this, overall volume is this and porosity is equal to 0.1. So, then one can say that 0.0001 would be the pore volume. So, instead of plotting it with point this way what they do is they when say let us say I have injected 0.0005 meter cube. So, instead of calling it 0.0005 meter cube I call this as equivalent to 5 pore volumes; that means, 5 number of pore volumes 5 pore volumes.

So, now if you plot these as effluent produced in terms of pore volumes and C bar by C 0. Moment we do it C bar by C 0 C 0 is the change in you remember I mean we have talked about C 1 bar and C 1 0 right. So, if we this if we do this then we expect that this will be scaled in that case it is scaled between 0 to 1.0 moment you. So, this become dimensionless right because your C 1 0 is the concentration that that can be the maximum concentration it can reach. So, because the step change was from 0 to we are assuming 0 to C 1 0.

So, then it is C 1 bar by C 1 0 if we plot C 1 bar. So, so this any concentration at any point, this is C 1 this is the average concentration. So, when you divide it by C 1 0 then automatically this becomes this is normalized within this 0 to 1 scale. So, now, essentially if we had given a steep change we expect that at vp is equal to 1; that means, at one pore volume you have that one poor volume means you have just that is equivalent to the residence time that is equivalent to the if this was the overall volume this is the porosity one pore volume is equivalent to 0.0001 meter cube this is this point.

So, I expect if there is no smearing nothing, it is just a plug flow and no mixing nothing i would have expected it was all 0 and suddenly boom it is 1.0 and continuing, but because of this mixing which we have already accounted through this Fick’s second law and all I we would expect that these lines will be smeared and how much smeared? It depends on of course, the dispersion coefficient we that is what we are interested in. So, they depends on dispersion coefficient and in this context there is a dimensionless number which is important which is called Peclet number.

Peclet number is equal to convective transport divided by diffusive transport what are the contribution of convective transport to that of diffusive transport? So, it can be written as 2R u bar by D. So, u bar is the average velocity which is the flow is taking place and diffusive transport D is the D is the diffusion coefficient ok. Similarly, one can think of a dimensionless time tau which is equal to tD by R square.

So, this is also a dimensionless, this term is dimensionless this term in is dimensionless this term is referred as Peclet number. So, this these lines they will vary I mean depending on the value of E z or depend on the value of Peclet number, I mean either way you can think of it that these lines will change and so, basically if one has a porous medium is given and then somebody runs such type of a test suppose there is a core given and they were asked to run a test. So, one can very well put two reservoirs, one reservoir has let us say if this is a sandstone core generally its customary to injectors not just direct water, but some amount of salt to avoid any clay swelling inside this.

So, let us say you are you have 1 percent NaCl which was it was inside it is one percent NaCl and then same 1 percent NaCl was flowing. And then you have another line here for 2 percent NaCl. So, let us say this is 2 percent NaCl and this 2 percent NaCl also can be injected and there is a valve and then suddenly at one point you simply turn the valve. So, that instead of 1 percent NaCl now it is 2 percent NaCl which is being injected. So, instead of 1 percent now, it is 2 percent anything getting injected. So, essentially you have come you have provided a step change in concentration.

And, at the outlet you are collecting fractions, that is possible or there are some instruments where this flow can be directly fed into that instrument and instrument will continuously give the record with time. So, either way see you one can get these concentration values at the outlet. So, if somebody plot these concentrations, the one can get this kind of a plot if they if they let us say this change from 1 percent to 2 percent. So, if you can make this dimensionless; that means, in this case what do you have? In this case you have C 1 C 1 0 is equal to 2 percent and C 1 infinity is equal to 1 percent.

So, you can find out what is the; so, you have you have here we are writing it as C 1 bar minus C 1 0 instead of that you can you can write it as C 1 C 1 bar minus C 1 infinity divided by C 1 0 minus C 1 infinity. So, you can you can make on this is also dimensionless if you have because, you have C 1 infinity is 1 percent. If you are not working with sandstone if you are working simply a pack of glass beads; this kind of glass beads these are used for electro osmotic pumping that for micro when one needs a pumping action at a at a very low flow rate by applying a voltage. So, those are so, in case suppose one is going for electro osmotic pumping and where these are filled with glass beads instead of sand stone and water is just fine. Then in that case C 1 infinity would be 0. So, C 1 infinity would be 0 it is 0.

So, it becomes C 1 bar by C 1 0. So, this is this is what you are working with. So, now, if somebody quickly take measures this value and find out. So, they are registering they are recognizing this, they are measuring this C 1 bar with time. So, that is what against fractions they are collecting fractions and everywhere they are measuring C 1 bar.

So, they are finding this ratio and plotting it. So, we expect this ratio to turn to from 0 to 1 near about this location vp is equal to 1. So, the one generate let us say someone with an unknown core plug they have, they have generated this plot ok. So now, one can go back and use that equation that we have already shown we have already shown an equation right which was half of 1 minus error function of; this we just discussed before half into one minus error function of z minus u bar t divided by square root of 4 D ct t.

So, this is. So, all everything is known I have a profile only unknown here is this Dct; so, by looking at this concept. So, these are experimental measurements at this time now ok. So, these are experimental measurements and we know what velocity this fluid is being injected and at the outlet at a given time what are the concentrations. So, only unknown is this Dct. And so, if this only unknown is this Dct in that case one can find out what is the Dct that corresponds to this particular profile what is the Dct that corresponds to this particular profile ok. So, now, I have this is one plot and then then for another one another core another similar core you have, there you saw that this was the profile.

So, for this new profile a new one you can have a different Dct. So, instead of looking at the profile you can directly look at the dispersion coefficient and then compare which one has more dispersion inside that is more mixing inside the porous medium and which one has less mixing.

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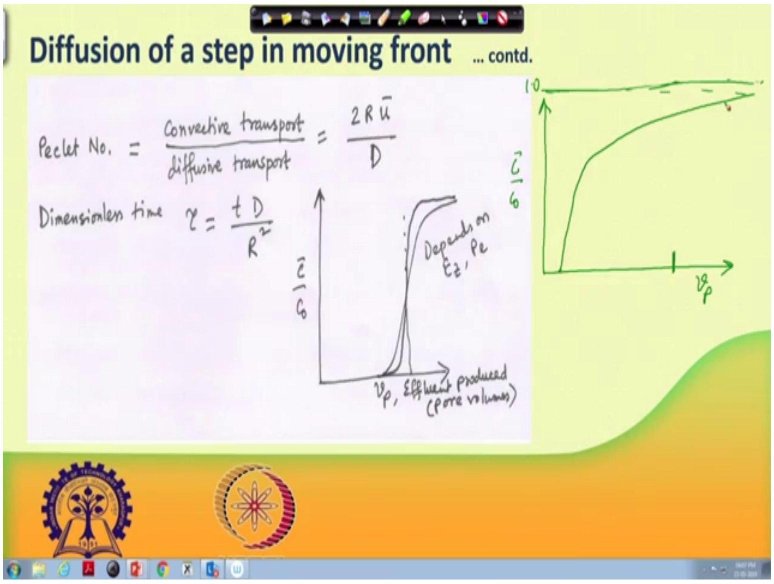

Diffusion of a step in moving front ... contd.

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$$\text{Dimensionless time} \quad \tau = \frac{tD}{R^2}$$

And, coming to coming to think about this mixing if you will I must point out suppose I see a profile of the C bar by C 0 versus vp the number of pore volumes and it came like this. Let us say this is let us say 1.0.

So, this is 1.0 and it came like this and I know from the measurement of porosity that one pore volume comes here and I see a profile something like this in for an unknown porous medium. So, what in that case my immediate conclusion would be that, there are some high permeability streaks or some channels existing some high permeability zones existing inside the core. And so, flow is not uniform or flow is not as we have thought in case of a homogeneous porous medium with bundle of capillaries, there are some zones through which the flow is taking place faster ok.

So, by looking at these plots I can I can make lot of you know this make lot of judgments about the quality of this porous medium whether it is homogeneous whether it tends to whether it will lead to more mixing or so, by looking at those numbers by looking at that Dct value. So, these are these are some of the elements that we need to think about when we do these miscible displacement. I will continue this exercise now; now we have already covered most of the fundamentals of dispersion process now we focus only on dispersion in porous medium exactly what are the trends ok. So, that is something which we are going to address in the next lecture. So, this is all I have as far as the present lecture is concerned.

Thank you very much.