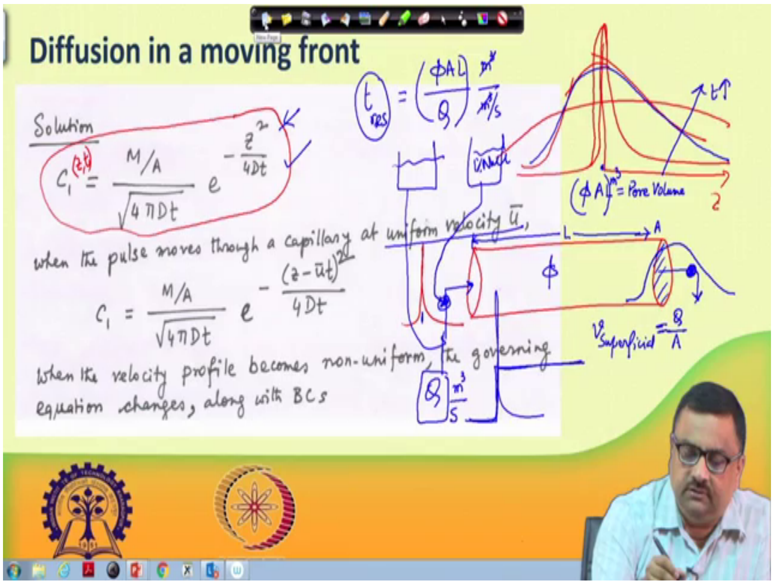
Flow through Porous Media
Prof. Somenath Ganguly
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 25
Miscible Displacement (Laminar Flow in Capillary)

I welcome you to this lecture of Flow through Porous Media. We were discussing about Miscible Displacement that is we if we have a mixing that is taking place inside a porous medium, because already there is a resident fluid sitting inside the porous medium and then we are injecting another fluid which can mix with the resident fluid. So, how this mixing takes place? How we can characterize this mixing or by looking at the signatures at the outlet, what understanding what inference we can draw about the pore transport inside the transported pore level inside this porous medium? So, these are some of the things which we are trying to address.

(Refer Slide Time: 01:14)



So, we started with the definition first that we started with the case where you have a static pull of liquid in which a drop of ink is placed. So, it has nothing to do with porous medium and we ended up with an equation we said we have not given you the we have not given you the derivation as such, but we showed that what are the governing equations we used to arrive at this equation and we said that this C 1 is a function of position and time. So, these gives me suppose I have this as my concentration as a function of z a one dimensional problem and if we have Dirac function and if we place these inside a beaker pull of water.

So, this Dirac function will gradually we will see that gradually we will see that this is this is going to be this is this is going to take a shape like this so; that means, this pulse that I provided here that is gradually spreading out if we with time. So, now, if we place this whole thing inside a porous medium; that means, suppose I have a porous this is the porous medium and here we introduced a pulse like this so; that means, how you are introducing a pulse what we are doing here is we are having a flow through this porous medium continuous flow is going on injection of fluid is going on say let us say we have water inside this porous medium and we are also injecting water through the porous medium.

And then what we did is, I have a small toggle here and or small port here through which using an injection syringe, we introduced some color in it or introduced some salt. So, high concentration of salt in this for a very short time we injected a pulse. So, this is this will simulate something simulate to a Dirac function. If we if we there are two ways we can do this introduction one is like a pulse like this or what we can do is, we can have two different containers one container was the simple water that was being injected and another container is containing the salt. So, this here we have a valve.

So, suppose this is the salt and this is the water. So, water was continuously being injected and there this is this is let us say one percent NaCl solution one percent NaCl solution a slight amount of the small amount of salt put in there. So, this let us say we here we have some way to measure this concentration at the outlet. So, we could have introduced it as a pulse; that means, we could have just for a fraction of a second we open this so, that is one percent NaCl solution goes in there and then again we go back to the to our original water flow or we can have it as a step change; that means, the water was flowing in, but then at some point we pull the toggle and then it is continuously the one percent NaCl will be flowing not water anymore.

That means, we could have given a pulse which is which is something like this or we could have given a step change something like this. So, step change we have given in concentration. So, we could see we expect that this will come out at the outlet at some point when it will come out at the outlet? Because it has a certain amount of residence time inside the fluid. So, it will take some time for the these pulse that we have given not for the step change that we had given to show that will show up at the outlet after some time what is that time? Let us say I have a flow rate Q, I am talking about a flow rate Q.

So, this entire flow rate is not changed flow rate is Q all the time. So, if flow rate is Q and let us say we have this cross sectional area which is A. So, what is the superficial velocity in that case? Superficial velocity is v superficial in this case is Q divided by A that is the superficial velocity right. So, if Q divided by A is the superficial velocity what is the residence time that the fluid will be in this in this porous media? How long it will take for the porous medium to reach the outlet? So, we can we can think of let us say this fluid this porous medium has a porosity of phi, the cross sectional area is A and let us say this length of the porous medium is L. So, what; that means, is the volume of this porous medium is A into L.

So, volume of this porous this entire medium is A into L so, if we multiply this with phi this is the pore volume. So, this is equal to pore volume. So, essentially I expect that if I am injecting at a flow rate of Q. So, let us say Q meter cube per second is the unit, Q meter cube per second is a flow rate which we are injecting and phi into A into L A is in meter square and L is in meter, phi is dimensionless since its porosity.

So, let us say this is in this is in phi into A into L meter cube. So, then how long it takes for the fluid that is injected here to reach the outlet that would be if we if we do this phi A L is the total void volume, this Q has to travel through this void volume right this divided by Q in that case this numerator is meter cube and the denominator is in meter cube per second.

So, this meter cube and meter cube they will cancel out so, this second goes to the numerator right. So, this is this many second would be required for the fluid to reach the outlet. So, phi into A into L divided by Q. So, this I can call the residence time. So, this is the residence time that one must provide. So, after this residence time I can expect this pulse to show up at the outlet, I mean in a very you know moral sense. Similarly if I give a step change, I would see another step change coming in out and come in at the outlet. So, there must be some kind of detector to measure the concentration. So, after this much of residence time, I expect these pulse or these step change to arrive at the outlet.

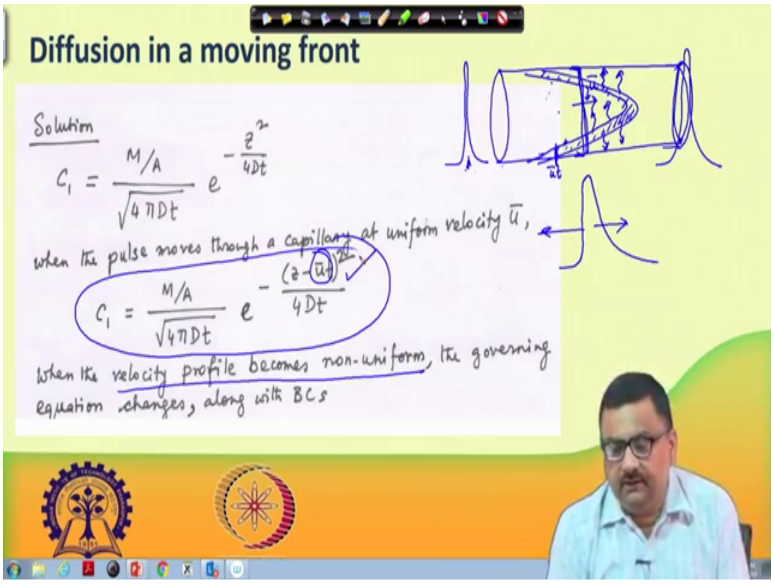
So, this we must know at the very onset. Now if this is so, then suppose I have this pulse this pulse is in static system and this is the equation that was followed at that time. So, now, suppose these pulse I am introducing this pulse here, I have introduced this pulse here at the inlet and this pulse is traveling to the downstream. So, then I expect. So, essentially I am giving this pulse a residence time of t raise the residence time of t raise after which this pulse arrives at the outlet ok.

So, now if we try to find out what would be the concentration at this? So, here let us say this is the this pulse and already it was it was a Dirac function here, but it has already diffused like this, this is just like just like this. So, let us say it has gone up to this. So, basically you have given this pulse this much of time so, this is we said that as time increases these would be the shape of the pulse. So, after residents after you have given t raise of residence time this much of residence time whatever the pulse shape is had the flow being all in plug flow; that means, as if this is this is just a static system I am simply travelling it all the way there. So, I am providing as if this is sitting in a beaker and I am providing a residence time.

So, over residence time whatever diffusion can take place it has taken place and now it has arrived at the outlet with this much this is the front. So, we can we can think in that line ok. So, if that is so, if we want to utilize the same equation one thing is for certain that this is for a static z right z is for a static system so; that means, the z it is always the wherever this the z is starting from the origin and that dye was placed at the origin, but in this case the z we have to see one thing is that the center of this center of these pulse that is moving right.

So, at any time how much the center of the pulse has moved that must be. So, one has to look at these with a moving coordinate system rather than a stationary coordinate system. So, that is the only difference here you can see the only change when the pulse moves through a capillary at uniform velocity u bar, pulse moves through the capillary. So, not in a not a porous medium as such the we have gone up to a capillary. So, let us.

(Refer Slide Time: 11:21)



So, with this understanding suppose now we at are looking at a situation, where we have a pulse going through a capillary not a porous medium. So, if it is flowing through a capillary of cross sectional area a.

So, here it is just open capillary no not a porous medium. So, if we if we give a pulse. So, the center of the pulse is here now the center of pulse is shifting and the center of pulse after residence time is given the center of pulse will reach at the outlet. So, if this is the so then one has to work with a moving coordinate system because z has to be calculated see mind it, here the concentration is changing with time. So, the changed with time with reference to this z, but if the center is continuously moving instead of a static system. So, all the z that you calculate now when the center is here the z has to be calculated with reference to that center.

So, accordingly here is a correction made that when it is moving z minus u bar t you see it it makes perfect sense, because it is at after time t this center will shift from this point to toward location which is equal to this center will go up to a distance u bar into t after time t. So, your center is here. So, all the z value that you are looking at, that z value has to be with reference to z say let us say I am looking at this is my new z. So, first thing you have to do is subtract u bar t from that z.

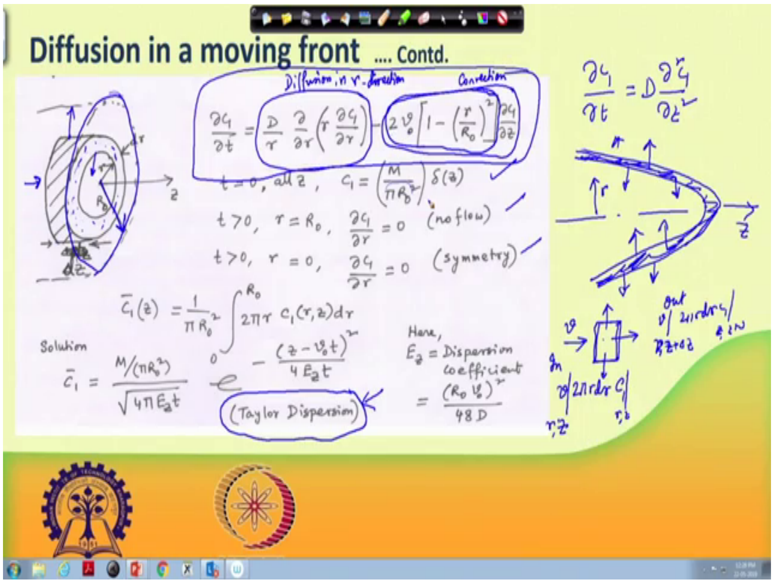
So, then you are talking about z with reference to this. So, this correction has to be done with z. So, we are talking about a moving coordinate system. So, this when a pulse moves through a capillary, these must be taken into account particularly this part. So, when the velocity profile becomes non uniform the governing equation changes along with the boundary condition. So, what does this mean? Here we are assuming that as if these front that is traveling, suppose I have pulse that is traveling we have as if we have this entire fluid is flowing at a velocity u bar ok. So, then it is alright, but in many situations it is not exactly moving at a velocity u bar rather one can see a parabolic velocity profile like this.

So, in that case if there is a parabolic velocity profile, then the pulses that we have given the pulse will also be stretched in a parabolic velocity profile like this. A parabolic velocity profile is common, we have already seen for a flow through a pipe for laminar flow through a pipe one ends up with that functional form we have seen 1 minus r by capital R whole square that is a parabolic form right. So, now, this is. So, this pulse will be stretched like this. So, this is where we have higher concentration and so, now, earlier what were what we were thinking that, if there is a single front like these if there is a straight front like this. So, as if this was the center of the pulse and this is what is this is how it is diffusing.

So, it can diffuse to this direction and this direction and then we had the governing equation by called to taking a differential dz and then volume we calculated a d z and in minus out accumulation and Fick's second law and all those things we had done already. But here in this case if these pulse itself is stretched like this because of the parabolic velocity profile, then we will see that diffusion is taking place in this direction, here in this direction, here in this direction, here in this direction. So, it is I mean it is creating a slag and you know it is a completely different volume.

So, this is something. So, what we what you said is when the velocity profile becomes non uniform; that means, you do not have a single velocity existing everywhere all along the cross section, then the governing equation changes along with the boundary conditions. So, if had it been a standard flat front this equation could have been just fine.

(Refer Slide Time: 16:06)



So, with this understanding if we try to look at what we mean by a parabolic velocity profile, we like to do you would like to look at something which is the first of all let me tell you that for a capillary, this particular phenomena is known as Taylor dispersion that dispersion; we are talking about see first of all we said diffusion in a static system then we are trying to extend it for a flow system.

So, moment we turned we consider diffusion plus flow that will be called dispersion ok. So, we have already given an equation we have just extended the static case only with a correction on z, we said instead of a fixed coordinate we are going to a moving coordinate. So, z minus u bar t instead of z square z square term we now have z minus u bar t whole square.

So, dispersion has already started there, but now we have said here that, it could very well be that the this it could very well be that the front may not be uniform and non uniform in the sense the front could be parabolic, we have already seen that it could be the velocity profile could be it could take a parabolic form. In fact, that is what we have seen in the flow equations where when we define friction factor, at that time we looked at the velocity. If we look at the velocity profile it is basically 2 average into average velocity into 1 minus r small r by capital R whole square.

So, that itself means that it is parabolic. So, if we have if one has this kind of parabolic velocity profile and if one wants to know what is the concentration profile. So, this is this is this is provided by Taylor. So, it is referred a Taylor dispersion. In this case first of all here we let us say we pick up a washer shepherd object, let us say this is the pipe, this is the capillary, this is the capillary, this is the capillary with radius R 0 out of that we pick up an annular area, annular area is given by this blue dots this annular area this annular area we are talking about and this annular area is between inner radius is r and outer radius is r plus dr this width is dr and it has in the direction of flow this is the direction z in which the flow is taking place, let us say this annular shepherd area we have taken this washer shepherd area it has this thickness of dz.

So, if we try to do these. So, z is in this direction in which the flow is taking place. So, if with certain assumptions we will get into the assumptions in a moment, if somebody does the mass balance equation in the same way as we have done Ficks second law; we have already seen that this del C 1 del t is equal to d del square C 1 del z square if for a one dimensional system. If one extends this to a cylindrical system R z system ok. So, we have r in this direction for a capillary r is the radial distance and z is the along the center this is along the central line the z is put here

So, then in that case this right hand side will have instead of this is this is for a Cartesian system one dimensional Cartesian system, but when you go for R z system. So, you are considering the diffusion in r direction so; that means, D by r del r of r del C del r this is diffusion in r direction in r direction and further you have this is arising from the convection; that means, the there is some material that is being delivered because of the velocity itself. So, that is so, that is by convection.

So, what are what are what are these terms let us let us look at quickly. We are considering we said that we did that the Taylor dispersion was supposed to simulate this case of parabolic velocity profile and a pulse is getting pulse is getting stretched the pulse is getting stretched like this ok. So, this is the pulse. So, we are interested in we are saying that the diffusion can take place in these directions. So, this direction is basically r direction because basically this is r and this is z. So, you have assumed that as if the diffusion. So, if we if we pick up this when we work with this differential element, we have a diffusion that is taking place in this direction and this direction; that means, in r direction, but in z direction there is no diffusion that is one thing.

So, you are not you are assuming first of all this when we worked with the earlier case there the diffusion was only taking place in z direction, there is no scope for diffusion r direction because the velocity profile is all flat. So, there that time the slag was all straight slag was not stretched, but in this case to simplify what Taylor did is Taylor said in this case diffusion in z direction for the time being you ignore later on Aris has shown that the actual diffusion can also be included in these and then the solution what would be the solution that has been found out.

But for the time being when Taylor has arrived at the first equation, Taylor assumed that axial diffusion; that means this in the z direction that diffusion is ignored only the diffusion in r direction is included. So, this is the diffusion in r direction Taylor has taken into account and second thing is the fluid is being delivered here in this case by convection what is convection? Suppose I have a differential element and I pick up a differential element there could always be diffusion in the r direction ok. So, minus d del c del z 1 some something will come go out this way that is a diffusive process, but since the flow is taking place in this direction. So, naturally this velocity v will carry some amount of solute with it and some amount of solute will leave the differential element with the flow.

So, if these are not same. So, in that case there would be an accumulation arising from these and that will contribute to this del C 1 del t. So, that is exactly this term because v is a velocity term, v is in meter per second. So, if we write v into area cross sectional area of the differential element. So, that would be then. So, what is the cross sectional area anyway we can very well calculate. The cross sectional area of this washer shepherd element is 2 pi r dr right. So, 2 pi rdr v at a particular z ok. So, this gives me this is the velocity in meter per second 2 pi rdr is the cross sectional area of this annulus. So, this product gives me meter per second into meter square; that means, meter cube per second. Meter cube per second and if you multiply this with C 1, this gives me meter cube per second into kg per meter cube. So, this gives me kg per second mind it.

So, there is always some kg per second going in, if you multiplied by delta t so, many kgs going in. And what is coming out from this, the one that is coming out this is in this is in and what is out? Out would be v at z plus this C 1 is also at we are we are looking at C 1 as a function of r and z. So, similarly here also v at v as a function of r and z. So, this is v as a function of r and z, here also v as a function of r and z plus delta z that is going out similarly you have 2 pi rdr C 1 at z r and z plus delta z. So, this is out.

So, this in minus out this also one has to consider alongside these diffusion and accumulation. So, this is this has been considered and the velocity is written as 2 into average velocity into 1 minus r by capital R 0 whole square. So, this is the this is the this whole thing is the velocity term basically this is the v this is the v and here we are having a minus sign because here in this case the del C del r del r itself had a minus sign mind it the flux, but here we do not have that minus sign to start with. So, that is why you are ending up with a minus sign here.

So, this is v del C 1 del z this term will come here I mean it has to happen v del C del z has to come because these in minus out has to be accounted and this is the diffusion in r direction. So, one has to solve this equation. So, to what Taylor has done is, Taylor has solved this equation and Taylor has used these boundary conditions that at t is equal to 0 for all z at t equal to 0; that means, time when you start this is a Dirac delta function M by this is a Dirac function M by pi on this M by a instead of a you can write pi R naught square that is the area ok.

That is the cross sectional area and for t greater than 0 there are two conditions one has to satisfy, one is r is equal to R 0; that means, add up wall of this capillary at the wall of this capillary r is equal to R 0, there is no flow possible no flux nothing. So, d del C 1 del r minus d del C 1 del r equal to 0 no flux. So, that is why del simplified it is del C 1 del r equal to 0 no flow and for t greater than 0 and r equal to 0 intuitively we can see there has to be some symmetry existing. Because everything that is happening there, it is there has to be some kind of symmetry at r equal to 0 for example, if you I mean to start with the velocity profile I can see that if we work with the upper part we can ignore the lower part. So, there has to exist a symmetry at r equal to 0. So, if there is a symmetry then del C 1 del r has to be equal to 0.

So, these are the conditions initial condition and these two boundary condition and this is the governing equation Taylor had to work with. So, to work with this kind of a stretched pulse. So, Taylor said that if you if there is a flow through a capillary, one has to consider this stretching and the one has to consider these dynamics will be completely different and in porous medium basically these are capillaries of size 1 micrometer. So, naturally this type of dispersion will be extremely important.

So, that is so. So, now, let us. So, I am going to close this lecture here, in the next lecture I would be continuing. So, you have to you have to you have to remember this equation and the contribution of individual terms and we will continue from here and we will see because Taylor take the aim of the Taylor was to arrive at the same form of equation that we had worked with earlier e to the power minus z square that same form of equation Taylor with Taylor can Taylor wanted to arrive at so, that the situations can be compared with reference to a static system.

So, let us see what Taylor has done as a solution. So, this is the governing equation that I mentioned, the boundary conditions that I mentioned just now and then Taylor had worked with these scheme of things to find out how the that dispersion will get modified, how the dispersion will change if instead of a plug flow instead of uniform velocity front if you bring in the parabolic velocity profile, which has to happen if there is a laminar flow in a capillary. So, I am closing this lecture here, I will continue this exercise in the next lecture.

Thank you very much.