Flow Through Porous Media

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Lecture - 24
Miscible Displacement (Uniform Velocity Over Capillary Cross – Section)

I welcome you this to this lecture of Flow Through Porous Media; we have already discussed about Darcy's law, we have already discussed about Ergun equation, we have already discussed about how to characterize a flow through porous media at least the basics. And also we have discussed about various transport mechanisms already, the other than viscous flow other than the Poiseuille's flow what other ways there could be transport and if there is a mixed mode; that means, both Poiseuille's flow and Knudsen diffusion happening together. So, what would be the effect on it? And can you then come up with an effective permeability to take into account various effects.

So, we have discussed about all these aspects. Now we are going to talk about in this new lecture module we are going to talk about something called miscible displacement. So, what we have here the topic is miscible displacement; that means, a fluid that is being injected into the porous medium; a fluid that is injected into the porous medium now that gets mixed with the fluid that is already present in the void space. So, if there is a mixing for example, I mean injecting a dye there is a porous medium already sitting there porous medium in the void space there is some water already present there and now I am injecting some water into the pore.

So, we said that it will follow Darcy's law that will give you the pressure drop and so there will be superficial velocity, there will be interstitial velocity and all this. Now suppose I put a dye at the inlet. So, then the dye is miscible; that means, what by miscible what I mean is if I put it in a beaker full of water drop of ink that ink travels all over the beaker. So, that is a mixing we are talking about, this mixing is because of Brownian motion of molecules. So, they will go from higher concentration to lower concentration and travel into those other places. So, this dye now as it travels through this porous medium this dye will not remain static this dye will also travel upstream.

So, these dye will also travel downstream into ahead of its front, it is not like just if I had injected a water and so water would be displacing water from the porous medium, then we would have expected that this water we continue to flow through the porous medium. But since it is the dye this dye will get mixed with the downstream fluid who the fluid that it is pushing it will get into that fluid by virtue of diffusion.

So, what kind of; what kind of profile one gets at the outlet? First of all what is the use of this kind of I mean why do we want to know that how much a dye will penetrate into the porous medium. You will very soon see that there would be; there will be various reasons for which we have to; we have to perform miscible displacement in porous medium. It comes to let us say if you are working in hydrocarbon area there would be lot of fluids it is injected to produce more oil and water and the fluid that is injected that is that gets mixed with the fluid that is already in place.

In some cases this one can characterize the porous medium and for that they inject tracer and how these tracer travels through this porous medium that is also extremely important. Because from the tracer signature and the outlet of the porous medium one can get an idea what kind of mixing is taking place inside the porous medium and from there how the conduits are located, I mean we cannot go into the porous medium to see where what is happening there, but by looking at the signature of this tracer at the outlet we can get lot of important information from them.

And generally this miscible dis because porous medium means meant for I mean though we only talked; only talked about Darcy's law and all, but all the time we would be doing, we would be injecting something which is different from what is already in place.

So, moment I put something which is in being injected, how it would; how it would travel through the porous medium ok? It need not have to travel as a single front; that means, at the outlet in one go you will find suddenly all the new injected fluid will start flowing in. The inside the porous medium there would be some amount of mixing taking place. And so one needs to characterize this mixing because for various different applications of porous medium this is this kind of miscible displacement is very common. So, now, let us talk about this.

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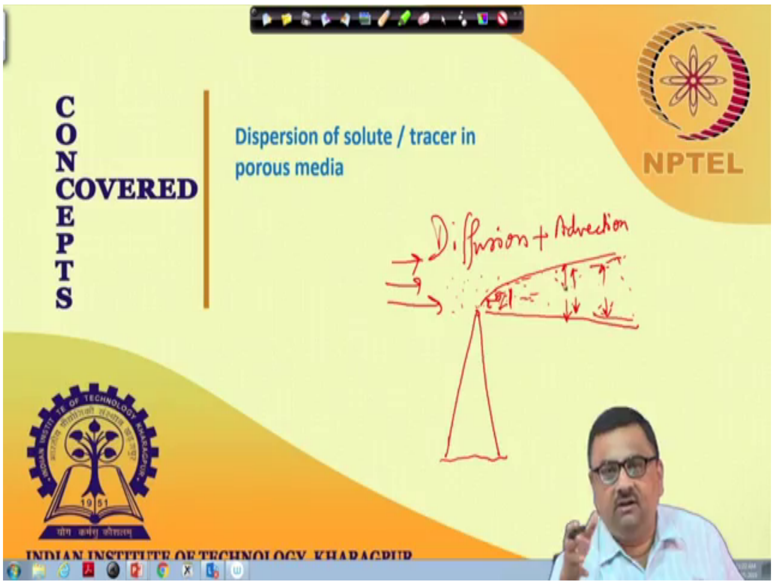

So, what we have here is a process known as dispersion of a solute in porous media. So, dispersion, what is the dispersion? First of all dispersion there are I mean this is a new term suddenly we coined; we had earlier talked about something called diffusion. So, what is the difference between a dispersion and it diffusion? That would be the first question. Diffusion is I mean I can think of let us say a very in a very crude way this could be the; this could be an example you have seen the smoke coming out from a chimney. So, if this smoke comes out here and you will find that this smoke also diffuses. So, smoke is coming out and then diffusing everywhere.

So, this is diffusion; that means, because of Brownian motion the molecules are traveling and from higher concentration of smoke to lower concentration of smoke these molecules are going around. Think of another situation where the wind is blowing in this direction. So, what will happen to the smoke? You will find that the smoke would be traveling in this direction. And generally if you have seen this, if you have tracked if you have seen a smoke or coming out of a chimney you will find that it would need to take a shape like this. It would take a shape like this and you will find that this is gradually increasing in size and it is the here it looks more concentrated.

But here it is; here it is more fuzzy, here it is more appears to be more dilute, but this kind of you will get this kind of a situation there. So, here the diffusion is also taking place, the smoke is also traveling the both ways and at the same time smoke is being carried because of flow of here. So, when you have a diffusion plus and let us say advection, one can think of this as the dispersion.

So, this is a dispersion; when I put a drop of ink in a static pool of liquid that is not dispersion that is simply diffusion. But if I have a flow of water going through a pipe and I introduce a drop of fluid drop of ink then that is dispersion, because the ink is being flowed through the pipe because of water that is carrying the ink and at the same time the ink is diffusing.

So, that is known as dispersion. So, we are interested here in dispersion not diffusion, because we are talking about the miscible displacement; that means, a fluid is present there and that fluid is being flushed out by another fluid which is injected into the porous medium and this new fluid how it mixes inside the porous medium as it flows through the porous medium. So, that is something which is of interest to us.

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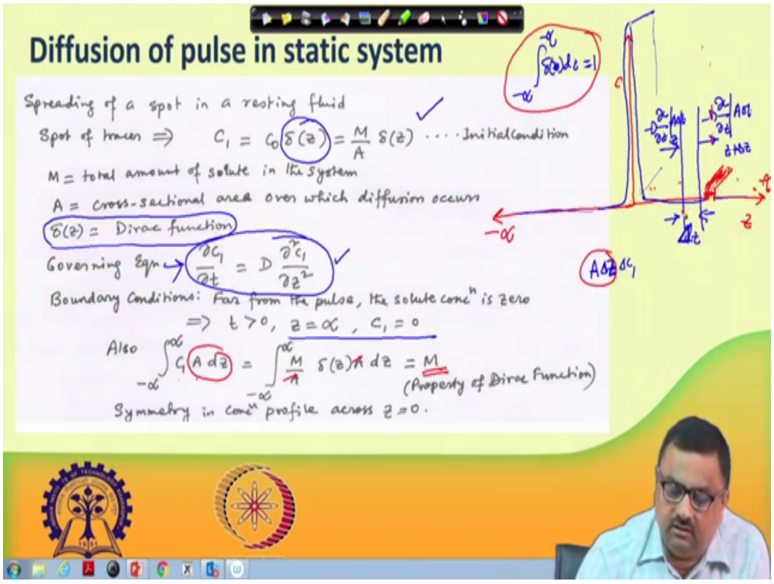

So, if we look at the spreading of a spot in a resting fluid. So, we can think of spreading of a spot in a resting fluid. So, we have discussed this once very briefly let us now articulate this little better, it is something like this. Suppose, I have this is the concentration versus distance let us say we treat this as an one dimensional problem; that means, this is z tending to plus infinity and this is z in the minus direction, this is the origin it is tending to minus infinity and this is tending to plus infinity and here we are plotting let us say concentration. So, here initially there is a spot of ink put in a resting fluid.

So, we are trying to understand the diffusion of a pulse in a static system so; that means, suppose we give a pulse here, pulse means it has a very high value at z equal to 0 and everywhere it is 0 at time t equal to 0. So, this is this we have provided, this is a similar simulated here as a Dirac function; Dirac delta function if this is a Dirac function delta z. So, this is the Dirac function ok.

So, you a spot of ink was placed at z equal to 0, we are treating this as a one dimensional problem; that means, there would be a area perpendicular to this screen and that area could be area A let us say; let us say A is the area, but we are treating this as a one dimensional system; that means, this we have produced this all over the area perpendicular to the if this there is a third dimension here.

So, we have produced this Dirac delta function there. So, or one can consider this these to be per unit depth perpendicular to the screen. So, anyway the problem is one dimensional. So, what we did already once is that we can pick up a differential length dz and we said that the amount of flux in this direction which is minus D del C del z at z and amount of flux that is leaving this right hand phase is minus D del C del z at z plus, actually let us call this delta z not dz.

So, this at z plus delta z and then we said that difference in this flux. So, this is flux if we multiply this by area A that gives me the flow rate. So, here also I multiply this by area A this gives me the flow rate; flux is volumetric flow rate divided by unit area divided by area. So, if you multiply by area this area will cancel out.

So, this becomes volumetric flow rate and if we still multiply this by delta t, if we still multiply this by delta t; that means, overtime duration delta t how much of; how much of volume has gone into this system, how much of sorry how much of solute has gone into this system and here this gives me if you multiply by delta t, how much of solid left this system. The difference between in minus out is accumulation and accumulation will take place over A delta z that is the volume; A delta z is the volume and let us say the change in concentration earlier at t it was concentration was C and now or C 1 and then after at t plus delta t the concentration is C 1 plus delta C 1.

So, then delta C 1 is the extra concentration that you have and so that is the accumulation if we look at the unit this is area is meter square delta z is in meter. So, this gives me meter cube and delta C 1 would be kg per meter cube. So, meter cube and meter cube will cancel out. So, you have kg; if you do not want to with kg you can work with moles. So, moles per meter cube. So, this meter cube and meter cube will cancel out. So, this would be the number of moles here that is accumulated.

And so in that case here also the number of moles that have gone in over duration delta t, this is the number of moles that has left the system at over this duration delta t. So, this minus this has to be the accumulation and once we have equated these in minus out is accumulation and we had taken delta z to the denominator of the left-hand side and delta t from the left-hand side from both these terms it has gone to the right-hand side.

So, this delta t tending to 0 this delta C 1 by delta t became del C 1 del t and this term here became, when we take divided it by delta z this term became the d del square C 1 del z square. So, that is exactly what is the governing equation in this case. So, this we have written it here as governing equation, this is also referred as Fick's second law this can be yes.

So, this is Fick's second law and this this type of treatment of doing it with the flux here one assumption here is that the concentration of the solute is small. So, now, if this is the governing equation; now what is the; what is the boundary condition or what are the initial conditions here? So, spot of tracer one can see here that C 1 is equal to trick, this is treated as C 1 is equal to C naught delta z where delta z is that Dirac function ok.

And this C naught is M divided by A. What is M? M is the total mass of solute that you have that is present in the system. So, the drop of wing that we placed, what is the total mass? If it could be in kg if all these C is in kg here, it would be in moles if all these C's are in moles. So, this M is the let us say we are working with kg. So, M is the kg of solute is that ink that you placed. So, kg of solute that is placed at this point at t equal to 0, so, that M amount is sitting there and then gradually it is diffusing that is gradually diffusing out.

So, at t is equal to 0 C; so, spot of tracer means C 1 is equal to C 0 delta z; delta z it is the Dirac function. So, the way delta z works is that delta z is will have a value of infinity at z equal to 0 and delta z will be 0 everywhere ok. And on top of that delta z it has another property which is if you integrate from minus infinity to plus infinity delta z dz; delta z dz that would be equal to 1. So, this is also another property you have for Dirac function. So, this is what a timing we have this Dirac function in present. So, Dirac function means I have a spot of dye placed here and that is Dirac function says that the concentration is infinity at z equal to 0 and concentration is 0 everywhere.

And now this process starts, as per this governing equation. So, this is the governing equation, this is the initial condition and we need some boundary conditions as well here. The boundary conditions are one thing is far from the pulse, the solute concentration is zero; that means, far away from the pulse; that means, at infinity now definition of infinity is in your hand. If whether you call 1 centimeter to be infinity or 100 kilometer to be in infinity it depends on what kind of what are the; what are the length scales of this system. So, you for; so far for far from the pulse the solute concentration is 0; that means, for t greater than 0 and z equal to infinity C 1 is equal to 0.

So, we are focusing only on the right part of the you know; you know of this problem. So, here because the left part would be simply the mirror image, right part of the. So, if we solve only the right part; that means, from z equal to 0 to z equal to infinity that would if we get a profile of conservation profile for the right part that can be simply we can take the mirror image of it and construct the left part. So, that is z is equal to plus infinity we are looking at where C 1 is equal to 0. The other condition here is that minus infinity to plus infinity C 1 A dz.

See at any location; at any location let us say we are looking at this location, we are looking at this location. So, at this location let us say the concentration is, let us say the let us say; let us say at this location the concentration is, at this location at this location the concentration is; at this location the concentration is C, say C 1. So, C 1 multiplied by the area, so area is perpendicular to the screen. So C 1 multiplied by A dz. A d z is the or A delta z is the volume of these differential element. So, at volume of this differential element.

So at every point A dz will give you the, if C 1 is the concentration and corresponding volume is A dz, so if you multiply C 1 with A dz, this gives me what? C 1 is in kg per meter cube, A d z is meter cube the volume. So, kg per meter cube meter cube. So meter cube meter cube will get cancel out it is kg. Kg of what? Kg of solute. Solute where?

This (Refer Time: 19:00) kg of solute is contained within this differential volume whose dimension is dz and A is the area perpendicular to the screen.

So, A dz is the volume of that differential element and this has a mass of C 1 A dz. So, if you have to sum all such masses everywhere. So, this; so, if you do that from minus infinity to plus infinity these gives you the total M, that is the amount of kg of solute that you have placed at z equal to 0, at time t equal to 0 and you see that same thing is happening here because minus infinity to plus in C 1 A dz is instead of C 1 you are writing it as M by A we have already seen this. And then this and I said is A will cancel out here and delta z dz M will come out of integration because M is constant and these delta z dz this this is the property of Dirac delta function. So, essentially this is equal to M.

So; that means, at any time the sum of all the concentrations everywhere, what we mean by this? Let us say if I erase everything here, I am looking at the concentration profile.

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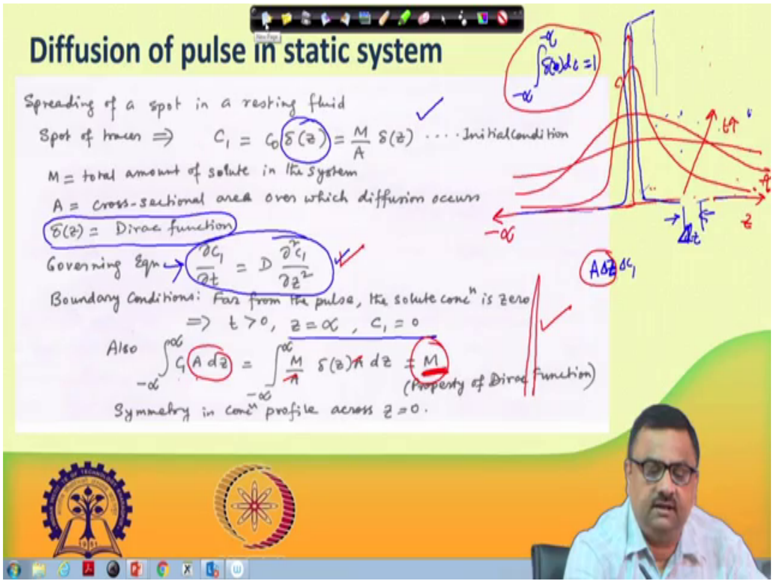

So, we would be seeing this this was at time t equal to 0 as time progresses, we would be getting concentration profiles like this we would be getting concentration profiles like this. So, as time progresses the concentration is building up here and concentration is decreasing here.

So, at one point it would be all flat, same concentration everywhere. So, that is what we are reading too. So, then what we said is that if we sum all the comes, all the solutes that are present in various differential elements everywhere, if we sum them up we will be ending up with this total M who what is the M is basically that amount of solute that was placed at z equal to 0 at time t; at time t equal to 0.

So, this same M. So, basically that same kg of solute you are not importing anything from outside that drop of ink that you placed in the beaker that same ink is, same ink is diffusing. So, that M if we take a stock at any time that how much of solute is present in the beaker. So, at any time wherever it is diffused, wherever it has gone and in I mean anywhere between minus infinity to plus infinity sum of all the solute if you sum them up that has to be equal to M that you have placed at t equal to 0 at z equal to 0, but because there is no other input at any time no other output from the system at any time.

So, this is; this is; this is equal to M and that is; that is what we see and also you can see here another condition could be applicable that there is a symmetry at z equal to 0, what because by symmetry only we are only working with z equal to 0 to infinity the right part of the problem and we said that a left part will be the mirror image.

So; that means, there is a symmetry existing in the concentration profile at z equal to 0. So, with these conditions being known and if this is the governing equation and this is the initial condition this equation can be solved. You can one can the common way to solve this type of equation is Laplace transform or you can look at similarity transformation as well.

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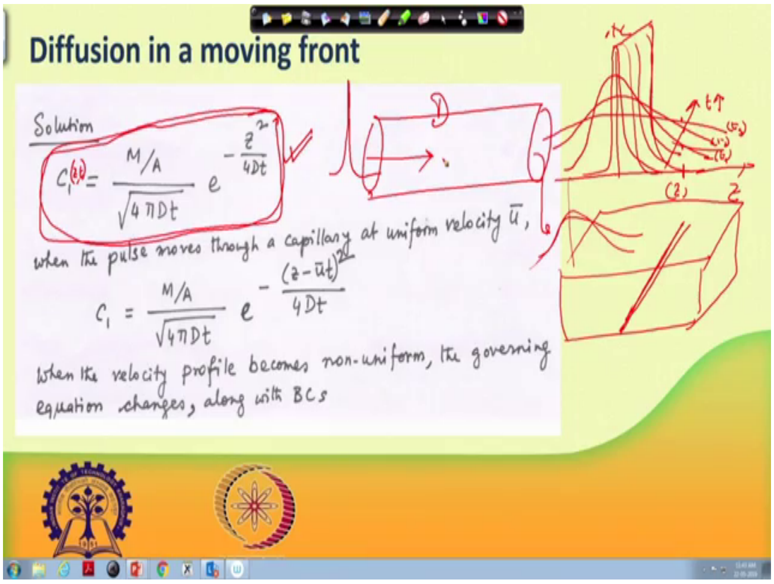

So, see you can you need to introduce a similarity variable these are very standard the equations one can get the solution of these equations. So, let us look at what is the final form of this solution, this is what we have as the final form of the solution; what does it have? It has C 1 is equal to M by A divided by square root of 4 pi Dt e to the power minus z square by 4 Dt. First of all what is C 1? C 1 is a function of z and t; that means, we said that this was the concentration profile time t equal to 0 was this and then gradually this, then gradually this, then gradually this like this. So, this is as time progresses the concentration changes. So, let us say this axis is z, this axis is concentration.

So, this let us say I look at this point at z equal to something. So, at this particular z the concentration C is changing, it was here at time sometime at some other time is this some other time is this. Again, if we hold on to one particular time let us say we hold on to this this is the concentration at a particular time ok, at some time t 1, this is at some time t 2, sometime t 3 like this. So, at time t 1 the concentration changes with z I can see here. So, concentration is a function of the C 1 is a function of z and t and this C 1 is now I can see on the outside M divided by A; M was that kg of solute, kg of that ink that I have placed in the beaker and A is the area perpendicular to the screen that we have.

So, beaker is not beaker as a cylindrical coordinate system and in this case, we are looking at a one-dimensional system. So, in this case since we are working with a one dimensional system we should be this is more of this is something which is not exactly I mean we are essentially saying that perpendicular to the screen we have an area A. So, the Dirac delta function that we have is something like this. So, I do not know one has to ideally want us to solve in cylindrical system if we are interested in doing it in a beaker as such. But this is; this is a; this is like you have a rectangular trough and then a rectangular let us say a box and then you have these colors that are placed along a line.

So; that means, you have a rectangular box and then full of water and then you have place the color along certain line ok and then you are studying how this color is changing. So, anyway this is; so this is divided by 4 pi Dt; D is the diffusivity and D is the D is z that D is featuring in Fick's second law the governing equation. Del C del C 1 del t is to D del square C 1 del z square we said D is the diffusivity right we have already defined what is diffusivity and what is its unit etcetera.

So, this is at D and the t is the time at which we are doing it, e to the power minus z square divided by 4 Dt, z is the distance from the origin and 4 D t. So, this is the governing equation that can define the concentration profiles that we discussed here. These concentration profiles can be given by this equation, this is the concentration profile which we get for to do these to do these equations.

Now, coming to coming to think of it what is the use of this in porous medium? What we are going to do next is if we have to; we have to this is for a static system. Now, if we place it in a flow system, but you remember this equation very well because we will be working around this equation many times. We will simply change this equation and we will simply change this equation and particularly if in our standard, our planned here is that if this is the porous medium and if we introduce a Dirac function here and these Dirac function is traveling through the porous medium and by the time it comes out these Dirac function must have flattened like this, just like it is happening here.

So, with time it is changing. So, here also when it travels through porous medium, we are giving a residence time inside the porous medium. So, as it comes out these Dirac function would be flattened like this. How much it got flattened why it got flattened how much it should have gotten flattened if it is a homogeneous medium. So, these are some of the questions we are going to ask now ok. But do you remember very well this form of the equation? Because this is extremely important that C 1 is M by A divided by this quantity e to the power minus something z square by 4 Dt.

So, remember this equation very well because these equation we would be working on very often because we need to find out now if we introduce this Dirac function in concentration as the flow takes place through this porous medium at the outlet we are interested to see how much this pulse is spreading. And from that we want to extract some information on how the transport takes place in porous media. So, this is something which we are heading to and so this is where I want to end this this particular module of the lecture.

Thank you very much.