Flow Through Porous Media
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Lecture 23
Flow Equation (Fluidized Bed)

I welcome you once again to this lecture on Flow Through Porous Media. What we were discussing was the Ergun equation as an alternative to Darcy's law, particularly when you have full information of the particles that are constituting the bed, the porous bed. So, now, we said in this in the lecture that I am going to in this next round of video lecture what I am going to discuss is a special case, where a porous medium may not behave exactly the way we thought it would. So, what is; what is; what is that? What kind of exception we can have so let us talk about it in this lecture.

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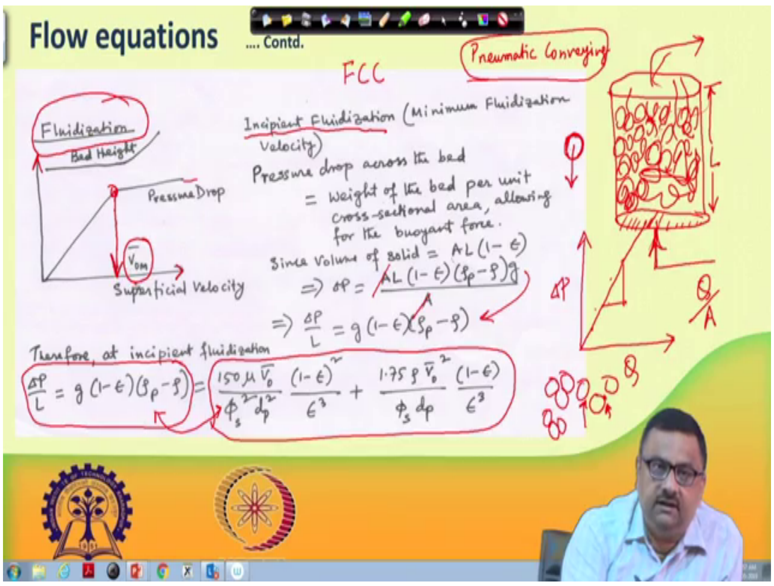

So, what we have here is a process which is known as fluidization; a process which is known as fluidization. What is exactly fluidization? Let us say I have a packed bed and here we have these. So, I have these particles and these are constituting so these are all filled with particles and we are having a flow let us say, we have a flow rate of Q over that overall cross sectional area is let us say E. So, superficial velocity is Q divided by A that is the superficial velocity.

So, that there is a flow from the bottom and the flow is taking place and flow is going out and then if you have this as length L and if you see there is a pressure drop delta P across this bed so one can find out one can apply Darcy's law, one can apply Ergun equation to find out what would be, the how pressure drop is related to the flow rate?

So, now, suppose we continue to increase this velocity or increase this flow rate. So, if we increase this flow rate and if we do not have a restriction that this porous medium is to be held as it is, in that case what we expect is what is shown here, in this plot. So, here you can see in the x axis it is the superficial velocity and a y axis for this particular line we are talking about pressure drop.

So, let us say we are plotting pressure drop versus superficial velocity. We have already seen we have; we have; we have walked with Darcy's law where we said that we talk about pressure drop versus flow rate and we will get lines and we pass aligned regress the line to the center and to find out this the slope and from that we find out what is the permeability of the bed. So, in this case also we can expect when we are plotting pressure. So, pressure will increase with superficial velocity, it will increase, but how long generally we are operating here.

So, we can see that it is linear, we are happy with that and we can we find out what is the slope and what is the permeability accordingly, but if you go beyond some threshold value of the superficial velocity, one will find that the pressure drop would be will not change any further and if simultaneously, if somebody plots bed height, this is bed height. So, if y axis is a bed height as a function of superficial velocity, one finds that the bed height was all along constant, we always said it is L and L is constant, I mean when we define Darcy's law.

So, this is; this is this was L and L was constant all along, but beyond this threshold value, I mean these here, there are some in windows, it will not be a you know very smooth transition. There would be some curves, some humps, etcetera, I do not want to get into that, but overall trend is that the bed height that we talk about here these bed height was constant all along, but beyond this point one sees that the bed height starts increasing with superficial velocity. What does that mean that? Now, the bed height is changing, but pressure drop is not changing pressure drop remains constant.

So, what that means is this bed is expanding these bed is expanding. So, this bed once it was some length now it is at a higher length, if you do not have any constraint on the top the bed will simply start expanding. In fact, this is one way these filter medium filters are cleaned I mean if one runs say for example, somebody is working on a gravity filter for example, if you have a sand bed and continuously pouring water and water contains lot of suspended solids you want to get rid of those. So, merge that the all mud and all those.

So, you expect those merged to be those suspended solids to accumulate at the look and cranny of these you know pathways here and what we get is filtered water. So, that time the flow is from top to bottom. Why top to bottom? Because one uses the gravity so by gravity the flow. So, you do not need any external pump or anything simply pour water and I mean that is how you know filters work a lot of times. You simply pour water and because of gravity water simply drains and whatever suspended solid is there that is held up in this bed. After some time this bed would be this, this all those pores most of them pathways would be all restricted ok.

So, at that time one needs some cleaning. So, at that time it a practice is that there would be a blow from the bottom. So, once they blow from the bottom, then you can explain you can expect this bed to this bed to expand, this bed to expand; that means, all the mud or all the suspended solids that has accumulated in between so, those suspended solids now, during this reverse flow you expect that since the bed is expanded. So, earlier one particle another particle are sitting here another particle are sitting here now, in the expanded form this is here this is here this is here.

So, naturally whatever was accumulated there now, if you give a high reverse flow these accumulations they will be carried down, carried to the top and will be separated. So, now, once again the bed would be cleaned ok. So, these are this is something which is which can be done, this is called you know back flushing. So, now, this makes this creates a very important situation here. This process of the bed getting suspended so; that means, flow. So, we are coming back to fluidization. So, when we have this fluidization; that means, we have put this flow and then it has crossed this threshold value.

So, at that time these particles in the bed they get suspended ok. So, these condition this is known as incipient fluidization; that means, the fluidization has set in. So, if you continue to increase the flow rate we will see these bed would be expanding and there would be movement of these solid particles and then what is the; what is the end of it? I mean if you continue to increase the flow rate further, then these particles will be simply carried downstream, these particles will be will go out of the system.

And in fact, this is one situation this is one technique which is used in a process which known which is known as pneumatic conveying. Grains instead of putting it in putting them in the sack and carrying separately, the grains can be stacked in a chamber and there could be a high flow of air and these would be this would be carried, then this entire bed will be then fluidized and then these grains will be carried downstream to wherever you want to take it, just in this by establishing a airflow. So, this process is known as pneumatic conveying. So, that is the end of it.

So, if you continue this process. So, at the end you will find these would be simply conveyed through the pipe, these particles will be conveyed through the pipe and it would be just a two phase flow in that case solid grains and air they are flowing together through a pipe to a destination where you want to take it, that is known as pneumatic conveying. So, now, this fluidized bed they have certain advantage and advantage is something like this that lot of times these packed beds, they are used for used in used to conduct catalyzed reactions.

So, suppose a and b they are fed so, they are put in a packed bed and this packed bed, these are basically, in that case catalyst pellets. Catalyst pellets means these are you can think of tablets, inside that catalyst material is loaded and these the fluid that is flowing through these packed bed, they will come in contact with that catalyst and they will be there this a plus b forming c plus d and this k this reaction is catalyzed. So, this kind of packed bed catalyst, packed bed of catalyst is very common. Now, one found that instead of working on a packed bed, where the catalyst pellets will be all together.

So, there would be; there would be some places, you have you may not have access fully or have full control over the flow, instead of that if one works on a fluidized bed setup; that means, where the catalyst pellets are suspended then one can access these catalyst pellets even better. So, that is how there is this; there is this famous FCC unit which is Fluidized Catalytic Cracking unit, which is used in many petrochemical complexes, you must have heard FCC unit.

So, there instead of having catalyst as packed bed, one prefers to work with fluidized bed concept. So, where the particles are suspended, but they are not leaving the chamber. They are simply suspended. So, how do you theorize these, this type of a process? Earlier, we said it was a packed bed and we had Darcy's law we had Ergun equation and all this. So, now, when you have this fluidized bed what are the conditions? How would you; how would you explain it?

So, what they have done here is first of all they said that the pressure so, the point incipient fluidization where this is the minimum fluidization velocity, this is where fluidization starts and fluidization continues for some time until this entire bed is simply carried downstream, that is pneumatic conveying. So, that is down the line somewhere pneumatic conveying will happen.

So, in between this entire superficial velocity here is basically, here it is if the bed is in fluidized state, but the fluidization process started here. So, this is referred as V OM bar. So, this is the superficial velocity, at which the fluidization has started. So, this is called minimum fluidization velocity or this is same as I mean this is the concept is same as incipient fluidization velocity. So, that is the point where fluidization started and if somebody wants look at or find out what is that velocity one must make note of the fact that on one hand you have so, this point belongs to this line right.

So, one hand this point has to satisfy Ergun equation. So, that is what we have tried to we have written here. So, on the right hand side I can see the Ergun equation for delta P by L. So, if you; if you; if you; if you leave out this part, if you leave out this part, on one hand we are equating delta P by L with this quantity, which is the Ergun equation. On the other hand now, we have a new definition of delta P by L, what is that is what here given here.

Pressure drop across the bed has to be weight of the bed per unit cross sectional area allowing for the bow end force; that means, at the time of when the when this; when this is fluidized think of a particle, if this is the particle, these particle has a weight, this is this particle that is constituting the porous media one of these particles it has the weight. It is already suspended in a fluid medium, because flow is taking place from the bottom to top, so, there is already a, buoyancy acting on this ok.

So, weight of the bed per unit cross sectional area, allowing for the bow and force. So, there would be a buoyancy which is acting upward, weight acting downward. So, what would be the corresponding force and that force per unit area that would give the pressure drop. So, from there if you can come up with a delta P by L so, one has to make sure that the delta P by L that you are putting here by came here you are obtaining by Ergun equation when that delta P by L becomes equal to that pressure gradient becomes equal to the pressure gradient that could suspend a particle; that means, this pressure gradient is sufficient for this particle to support.

Basically, weight minus buoyancy whatever force is acting downward the delta P by L, because of the flow should cancel it then only this particle will get suspended. So, the pressure drop across the bed from that concept would be weight on the bed per unit cross sectional area allowing for the bow and force. The volume of the solid is AL into 1 minus epsilon volume total volume is AL, out of that solid fraction is 1 minus epsilon.

So, AL into 1 minus epsilon where epsilon is the porosity of the bed. So, delta P would be in that case AL 1 minus epsilon, that is the volume of the solid; that is the volume of the solid multiplied by rho P which is the particle density; that means, solid density. So, multiplied by rho P into g that is weight downward minus there would be a buoyancy acting upward and balancing is the weight of the fluid of same volume as that of the particle so; that means, minus AL 1 minus epsilon that is the volume of the particle into rho this time not rho P that is the rho is the density of the fluid into g.

So, this is after accounting buoyancy this becomes the total force acting on the particle weight downward bouncy upward. So, that divided by area A area is the cross sectional area A, so, that gives you. So, this is not just for one particle this is for sum of all particles. So, this is the volume of all the particles and multiplied by the buoyancy corrected weight and divided by this. So, this is the buoyancy corrected weight of all particles divided by this cross sectional area. So, this gives you the delta P.

So, once this delta P is supported by the delta P that you get by Ergun equation, then at that point one can say that this incipient fluidization will start. So, then this delta P by L is one can cancel out this A with A this AL will come in l will be here and the denominator. So, delta P by L would be simply g into 1 minus epsilon into so, this is arising from here, g into 1 minus epsilon into rho P minus rho. So, that is exactly what is this delta P by L.

So, they are for at in incipient fluidization on one hand these delta P by L has to be this much; that means, the pressure drop that the system has that pressure drop should be sufficient to suspend the particles that pressure drop should be sufficient to overcome the gravity with the buoyancy correction. So, that is on one hand that is to be true and on the second this pressure drop is where this pressure drop is arising from? This pressure drop is arising from Ergun equation and these two are to be equated, if one has to find out what is this V O bar.

So, if we equate these two the V O 1 that we solve it. One has to solve for V O bar and if V O bar is solved then the V O bar that one finds out that V O bar would be equal to this V OM bar, which is the superficial velocity for incipient fluidization.

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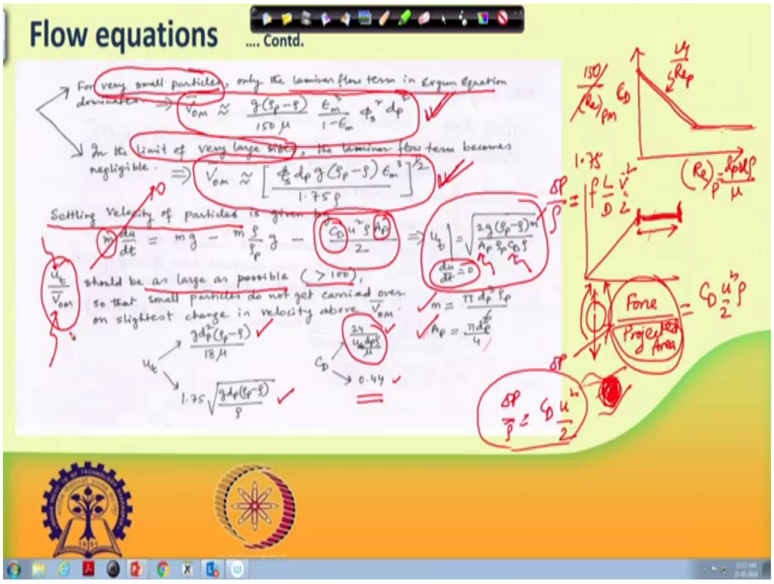

Let us now look at so, what they have done here is, for very small particles they have; they have broken it down ok. We had these, which Ergun equation we it has on the right hand side two terms right, because they did not want to work with V O bar square and V O bar and all those.

So, instead of that so, only in one case they have only worked with 150 divided by R e porous media that term. So, only the laminar flow term in Ergun equation was used. So, when this was used in equating these two; these two term that we mentioned just now. So, one found out that this should be the V OM bar 150, I can see is appearing. Whereas, if you do not work with this laminate part, if you want to work with the second term which is 1.75, friction factor is 1.75. I can see 1.75 is appearing here. So, one finds out that these is the tar.

So, you have these are the; these are the these is the in superficial velocity at incipient fluidization, if one is working with very small particles so, there would be this, because after all R e P has this dp term. So, dp is if the diameter of the particle is very small whereas, if the limit of very large size is, if this domain the diameter of particle would be much larger so, this is contributing very small particle is taking you to the Reynolds number of porous media small and in this case Reynolds number of porous media large.

So, these are the two cases and for that you have the corresponding incipient fluidization values. So by so based on so, by looking at these parameters, one can very well say that well, if you want to expect this bed to be fluidized, at least you have to take the superficial velocity to this level that is given by this expression. See on the right hand side what do you have? Basically, particle density, fluid density and particle diameter and then (Refer Time: 20:34) factor if any and of course, the porosity of the bed. So, that is something which is; now, porosity of the bed will remain same all along until the incipient fluidization.

So, this is the initial porosity of the bed epsilon. So, at that incipient fluidization these should be the superficial velocity. So, until you cross this superficial velocity, there is no scope for fluidization, that is for certain. Now, in this context one important matter is that we said that this pressure drop would be going like this and then it then the fluidization starts and then at one point there would be pneumatic conveying.

So, if one interest is to run a fluidized bed, then one needs to know what is this window right, because if the if it incipient fluidization starts and immediately, it starts becoming the you know pneumatic conveying comes in so, then or immediately at this point pneumatic conveying is in pneumatic conveying develops. So, then one gets a very small window. So, what; that means, is if there is a slightest fluctuation in superficial velocity and that can happen in due to many reasons.

So, if there is a slightest fluctuations in superficial velocity, the entire bed will be simply running out, you know move moving away from the system. So, that is not wanted one wants a good window over reach this window for the fluidization in velocity. So, even if there is some perturbations to this fluidization velocity still, the fluidized condition will remain, it will not go to pneumatic conveying condition.

So, to ensure that one must look at what is the settling velocity of a particle. So, a particle if you leave it in leave it as it is, if you leave it in here, this particle will have a settling velocity; that means, first of all this particle will be accelerated, because of gravity particle will be coming downward. So and on the other hand when the particle is coming downward, there would be buoyancy of course, it is acting upward plus since, the particle is moving downward there would be this drag force, acting upward.

So, once so, particle if you leave it from the rest, it will accelerate and beyond some point you will find that these three forces that we are talking about one was gravity, one is buoyancy and the other is drag. So, these three forces gravity is downward buoyancy upward and drag is upward these three forces they will balance each other how they will balance each other? Because as the velocity increases drag force increases. So, as the drag force increases, this will counter the gravity.

So, at one point, some velocity continuously increases drag force, also increases at one point, they balance each other. So, at that point the forces are all balanced so that means the net force acting on this particle is 0. So, in that case the particle will not change. It will not accelerate anymore that is called terminal velocity of the particle. So, particle will continue to move downward at the terminal velocity.

So, if one needs to work here and one needs to work with a good window, so one must compare, what is the terminal velocity of these particles as against what is the superficial velocity that you are imposing and how the superficial, where the superficial velocity stands with respect to the terminal velocity. If supervision velocity and terminal velocity they are very close one can think of, I mean that possibility that particles may get carried away and will not remain fluidized.

So, one has to ensure that there is a good gap between the terminal velocity and superficial velocity so that the fluidized condition can be retained. So, here we have done the same thing. Here, settling velocity of particles, this is given by see, first of all mass into acceleration for any particle with mass m and u is the velocity. So, du dt is the acceleration. So, mass into acceleration that gives me the force on the particle, that is essentially mg, which is the gravity force acting downward minus m rho by rho P into g. This is m by rho P is the volume of the particle, rho P is the particle density, m is the mass of the particle. So, m by rho P is the volume of the particle multiplied by rho, which is the density of the fluid.

So, this is volume of the particle multiplied by density of the fluid multiplied by g. So, this is essentially buoyancy, so, this is buoyancy. So, weight of the particle minus buoyancy minus drag force. So, drag force is given by, there is something called drag coefficient, there is something called drag coefficient which is known as C D. So, drag coefficient into so drag coefficient I mean it is covered in fluid mechanics if this is; this is; this is something like these, that this drag coefficient is a very similar term as friction factor.

Generally, the drag force is drag force; drag force divided by the projected area, projected area means for a sphere the projected area is a circle for a rectangle, for a cube this projected area is a square something like this. So, if you if there is a fluid flowing and if you have a; if you have a particle here, so, the drag force would be the projected area would be the area that the fluid sees ok.

So, this is for a sphere it would be a circle. So, force divided by projected area this projected area is defined as A P in this expression, this A P force divided by projected area. So, this is equal to drag force drag sorry the drag coefficient CD into u square by 2 the velocity kinetic head basically and into rho. So, it is basically, you can think of this C D which is called drag coefficient, this is very similar to friction factor.

So, this also can be plotted exactly in the same way as a function of Reynolds number of the particle this time, not a porous media. Reynolds number of a particle and particle Reynolds number is given by particle diameter, particle velocity, rho of fluid, viscosity of the fluid. So, that is how the particle Reynolds number is and typically this also is inversely proportional to particle Reynolds number this is 24 by R e particle number particle Reynolds number.

So, this is called drag coefficient. So, just like we had that time delta P is equal to delta P by rho in case of pressure drop flow fluid delta P by rho was f L by D V square by 2. So, here also we have these projected area, so, these gives me the delta P. So, delta P here also we have this delta P, where delta P by rho would be equal to C D into u square by 2. So, this is; this is; this is so, it is a very similar expression. So, this is what is utilized here. So, this is essentially the drag force.

So, now if we equate this and if we set this term to be equal to 0; that means, when these and it has reached the terminal velocity. So, then ut at du dt equal to 0, this can be simplified as this expression 2 into g into rho P minus rho into m divided by this quantity A P rho P C D and rho. So, now, this ut divided by V OM bar that I have been saying here, that we need a window; we need a window. This ut should be much higher compared to V OM so that we can, we get a longer window. So, u p by VOM bar, this should be as large as possible and preferably greater than 100.

So, that if there is any fluctuation in the superficial velocity at, the time of when it is in fluidized state the it should not reach the terminal velocity that it leaves the system. So, that small particles do not get carried over. So, on slightest change in velocity above V OM bar, this will happen if these ratio is not high enough and high means at least to the tune of 100. Now, here if you can see these ut, what is this terminal?

So, if V OM bar we have already found out, what is V OM bar. We have already gotten this expression of V OM bar here. We have another expression depending on the size of the particle. So, we have already gotten this expression for V OM bar, how to get this ut? We have already arrived at an expression for ut, but these expression requires information of m, information of A P and information of C D. The other terms are known rho P rho it is a rho P is the particle density; that means, the solid density, the porous medium is consisting of solid particles.

So, that solid density is rho P rho. The density of the fluid g is the acceleration due to gravity. So, these are all known to us, m here is the mass of the particle. Mass of the particle is volume of the particle multiplied by the density of the particle. Volume of the particle is four third pi r cube or pi by 6 dp cube. So, if you write it in terms of diameter instead of radius. So, pi into dp cube by 6 multiplied by the particle density.

So, this is the m and A P is the projected area, projected area in this case for a sphere, it would be pi r square and pi r square is given as pi dp square by 4, where dp is the particle diameter right. So, projected area means this. As I said when there is a fluid flow the sphere, the fluid will see a circle. So, that is the projected area.

So, mp is known A P is known. So, now C D, now C D has one hand. we said that if you are in laminar region it comes to 24 by R e. So, C D can take this value or C D can take 0.44, which is the, it is that constant value that in case of porous medium we have 1.75 right f versus R e p m, there we have this 1.75.

So, here it is 0.44. So, you know depends on whether you work with this regime or this regime, the corresponding terminal velocity will take this equation gd p square rho p minus rho divided by 18 mu or terminal velocity will take this, if it is in the, if it is in this region of particle Reynolds number. So, it either of these will happen. So, it depends on which region of particle Reynolds number one is in. So, these are the two expressions for ut that are available.

So, the job here would be first calculate this V OM bar. If somebody wants to design a fluidized bed system first calculate V OM bar, find out the V OM bar and then simultaneously one needs to look at what is the terminal velocity, what is the size of the window over which this fluidization will occur. So, this is extremely important and so, you have to parallely find out what is the terminal velocity and take this ratio and see whether this ratio is sufficient at least to the order of 100 or not.

So, these this I think this gives you one give you this, this entire exercise this gives you and feel how this fluidization will occur, where the fluidization will take place at under what conditions and how you can predict where the fluidization is going to take place and where the fluidization would be stable enough. So, that it does not get carried downstream. So, this is all I had here as far as this particular lecture module is concerned.

So, we had talked about packed bed, we have talked about Darcy’s law and then we said that if we have good information the particle size etcetera, the Ergun equation, its subcomponents and everything and then we said that the one offshoot of this is this fluidized bed and instead of a packed bed and then if one needs to find out at what superficial velocity this fluidization sets in and how, under what condition this fluidization would be stable.

So, we have some handle over it, I mean how do how to you know predict these cases, these situations we have talked about that in this lecture. So, here I am closing this lecture on flow equations and I will start a new concept altogether from the next lecture.

Thank you very much.