Flow through Porous Media

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Lecture – 20
Flow Equation (Contd.)

I welcome you to this lecture of Flow through Porous Media. What we are discussing in the last class is how what are the alternatives of Darcy's law we have for flow equations constituting viscous flow. And if we if we try to bring in the concepts of friction factor, Reynolds number as we have studied in flow through a open flow through a pipe, if we bring in those concepts how we can relate pressure drop with the characteristic parameters of the porous media and the flow rate?

So, the topic that we are discussing was flow equations and particularly flow equations I mean we are making a general you know the analysis what is the origin of Darcy's law and what other alternatives we can think off.

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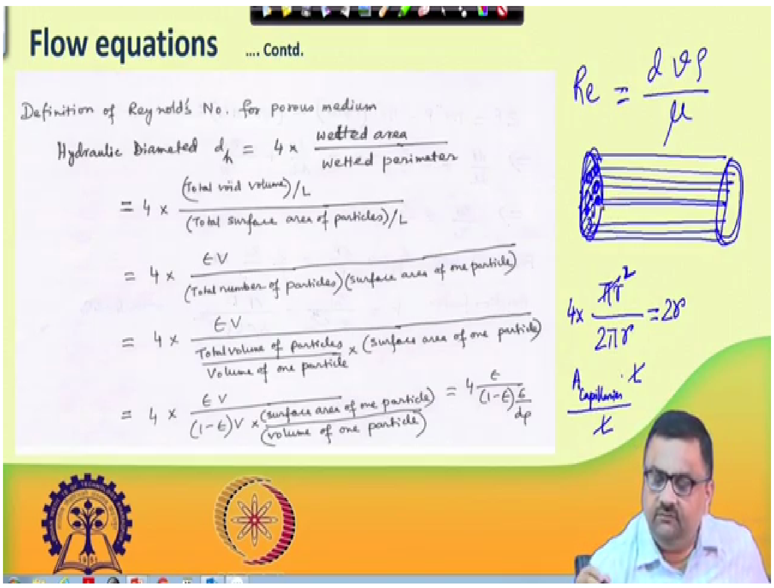

What we were discussing at the end of last lecture was these Reynolds number which was d v rho by mu d is the diameter. So, this Reynolds number is applicable for a pipe; pipe of diameter pipe of diameter d velocity average velocity is v, rho is the density of the fluid and mu is the viscosity of the fluid.

So, now this is this is true for a pipe when you do not have a circular cross section, then one can find out an equivalent to this d value; equivalent to d equivalent to the diameter which is known as hydraulic diameter and the definition of hydraulic diameter is 4 into weighted area by weighted perimeter. So, what; that means, is let us say we talk about the circular cross section, the weighted area in this case would be pi r square that would be the weighted area, weighted perimeter; perimeter of a circle which is 2 pi r and if you multiply this by 4.

So, now, the 4 into weighted area by a weighted perimeter these becomes this equals to 2r, so 2r is the diameter. So, the hydraulic diameter of a circular cross section is the diameter of the pipe itself. So, you can see that this concept of hydraulic diameter that works generally if one does not have a circular cross section. So, if one can one can find the equivalent diameter.

Now, if we try to find out what is the hydraulic diameter for a porous medium? So, what would be the weighted area? Weighted area means there is a flow taking place. So, what cross sectional area through which the flow is taking place, so that is why we said it is pi r square. So, if we have a porous medium; that means, we have some part we have void and some part we have solid part.

So, now, if I think of this porous media as a bundle of capillaries I mean that could be a good conceptual understanding of this. So, in that case the weighted area would be first of all weighted area is total void volume; total void volume. What is the void volume? Say if I if I think of this porous medium comprising of several capillaries; porous medium consisting of several capillaries.

So, then the total void volume would be the void volume of that or the volume of those capillaries right. So, total void volume divided by [Laughter]; total void volume divided by L that gives you the area through which the flow is taking place, total void volume void volume means the volume of those capillaries. So, volume of these capillaries is that is what? A area of those capillaries and the length is length of the porous right. So, this is the total this is equal to the total void volume. So, this total void volume if I divide it by L, so L will cancel out.

So, in this case you are left with area of the capillaries. So, area means these open areas we are talking about; these areas we are talking about. So, these areas these open areas that is basically the weighted area because through those open areas the flow is taking place. So, it is basically total void volume by L the upper part weighted area whereas, weighted perimeter is total surface area of particles divided by L. What is the weighted perimeter in if we consider these to be capillaries? It would be the total see weighted perimeter.

So, we are talking about the perimeter of these capillaries multiplied by L right. So, that is going to be the total surface area of particles right you if you if you if you look at if you think of.

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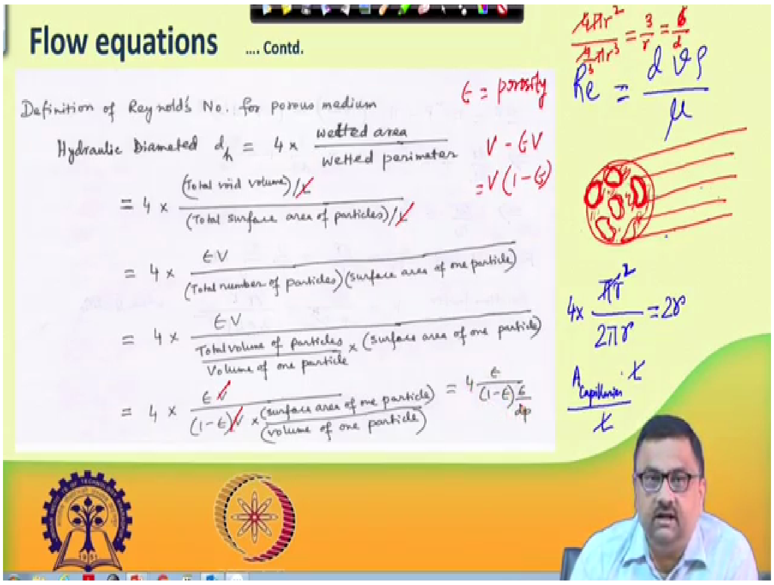

Let us say I let us say I have this as the I look at the these are the grains located, if I assume that the grains are not touching with each other they are just isolated grains present. So, now, I have these are the; so this is the area this is these will all constitute suppose these are the pathways I am talking about this part is all solid. So, these solid part is arising from the particle. So, this is the pathway through which the flow is taking place.

So, now, this is the perimeter this is also this will be constitute to the perimeter; this will constitute to the perimeter; this will constitute to the perimeter and now think of this extended to all the way to length this all these pathways are extended to the length. So, what would be the total surface area of particles; total surface area of particles would be these perimeter which has a unit of length if it if they had it been circulated be 2 pi r; r is this radius of this small you know circle and then multiplied by this length just like think of a cylinder 2 pi r L is the surface area.

So, this cross sectional area multiplied by L gives you the surface area. So, now; so total surface area is this cross sectional area multiplied by L plus this cross sectional area multiplied by L plus this cross sectional area multiplied by L. So, if you divide it by L sorry this was the total volume is this area multiplied by L and total surface area is this surface area multiplied by L, these surface area multiplied by this perimeter multiplied by [Laughter]; this perimeter multiplied by L; these perimeter multiplied by L this perimeter. So, all their sum that gives you the surface area.

So, these that surface areas divided by L now, so that gives you this perimeter plus this parameter plus this parameter. So, that is exactly what is done here total surface area particles divided by L. So, that gives me these perimeters. So, 4 into total void volume by L divided by total surface area particles divided by L. Now, void volume is now this L and L they will cancel out total void volume is epsa into V where epsa is the porosity.

So, here epsa is equal to porosity. So, if the total volume is V and epsa multiplied by V gives you the pore volume or total void volume. So, that is how it is epsa into V and total surface area of particles when it comes to total surface area particles one can write these to be total number of particles multiplied by surface area of one particle ok.

So, so you are now we are now going to this instead of this concept of cylindrical capillary you now we are talking about particles constituting these pathways I mean gap between particles. So, total surface area particles would be total number of particles that are present in the system. So, now, we are consists we are considering this to be number of particles instead of capillaries and total number of particles present in the system multiplied by surface area of one particle.

So, you are assuming the two particles there is no common areas shared by the particles. So, now, with that now total number of particles can be written further as total volume of particle divided by volume of one particle, that is total number of particle. So, what is total volume of particle see if the void volume is epsa into V total void volume is epsa into V total volume was V. So, then what is the volume of the solid this total medium is consisting of solid volume plus void volume.

So, solid volume plus void volume is V out of that void volume is epsa into V. So, solid volume would be then V minus epsa into V. So, or in other words V into 1 minus epsa. So, that is exactly what we have done here 1 minus epsa to V which is the which is the total volume of particles or total volume of the solid part. Now, these has to be multiplied by surface area one particle which is which we are carrying all the time surface area one particle divided by volume of one particle because this is this is coming from this denominator volume of one particle.

Now, this in this case you have surface area of one particle divided by volume of one particle, if we look at if we consider these particles to be all spheres. So, in case of a sphere the surface area is known to be 4 pi r square; and the surface area is known to be 4 pi r square and the volume is known to be for a sphere 4 4rd pi r cube. So, if you if you if you take care of this you see this 4 and 4 they will cancel out pi and pi will cancel out.

So, you are left with only 3 divided by r or if you want to write it in terms of diameter of the particle it would be 6 divided by d. So, that is exactly what is written here dp is the diameter of the particle. So, to differentiate this dp with d h hydraulic diameter is d h and dp here is the diameter of the particle. So, 6 divided by dp is basically surface area of one particle by volume of one particle considering these particles to be spherical.

So, 6 by dp and then this V this V and this V they cancelled out and you are left with epsa divided by 1 minus epsa they are coming from here and multiplied by 4. So, you have 4 into epsa divided by 1 minus epsa and 6 by dp in the denominator. So, this is essentially the hydraulic diameter.

So, hydraulic diameter is a function of what all; one is the particle diameter considering the particles to be spherical this is a function of the porosity which is given here as epsa. So, this defines the hydraulic diameter and on the basis of this hydraulic diameter one can define the Reynolds number.

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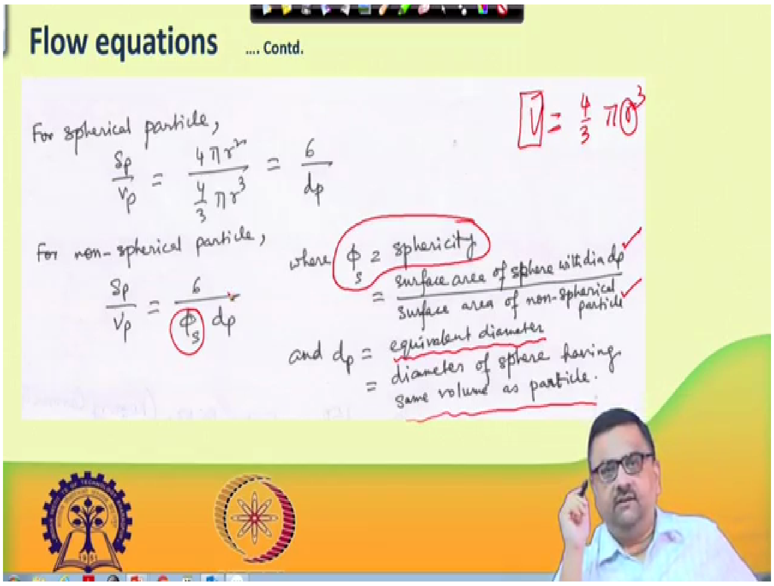

Now, for spherical particles we said that S P by V P, S P is the surface area and V P is the volume that is equal to just now I mentioned 4 pi r square for a sphere and 4 3rd pi r cube which comes to 6 by dp. Now, it is very likely that porous media will not be consisting of perfectly spherical particles these would be just random particles.

So, for non spherical particles one can introduce a concept known as sphericity which is given by phi subscript S and it is written as this S P by V P term is instead of 6 by dp you introduce 2 dp you multiply the dp by a term called phi S. What is phi S? Phi S is surface area of sphere with diameter dp divided by surface area of non spherical particle and what is dp in this case? dp is the equivalent diameter which is diameter of sphere having same volume as particle.

So, what essentially we are talking about here. I have some a particle which is not spherical. So, we have to find out what is the equivalent diameter of that particle first. How would you find out the equivalent diameter? The equivalent diameter would be diameter of sphere having same volume as particle. How will you find out diameter of sphere having same volume of what same volume as particle? One what one needs to do is find out what is the volume of the particle first of all; what is the volume of the particle, how will you find a volume of the particle? The best way to find suppose some irregular particle is there and we want to know the volume of the particle.

If the particle is not having any reaction with the water or whatever liquid we wear it does not have any reaction and need you drop the particle and let it fully some get fully submerged in that liquid. So, the amount of volume that would be now if you do it in the measuring cylinder you will see that the volume of the liquid level goes up. So, that gives you that that is that is the volume of the particle.

So, moment you know the volume of the particle, then you equate that volume that volume that you measured as 4 3rd pi r cube. So, find out what is the radius equivalent radius of the particle; that means, you calculated you may you is you measure the volume of the particle by submerging it into some liquid or water and then from that volume now you are equating with a sphere. So, if a sphere will have the same volume as that of the particle what would be the radius and correspondingly what is the diameter? That is the equivalent diameter.

Now, surface area of sphere with that with diameter dp that dp that we said equivalent diameter divided by surface area of non spherical particle. So, if you have this information of the surface area of non spherical particle. So, this ratio surface area of sphere with diameter dp divided by surface area. So, this ratio gives you this sphericity phi S and that phi S has to be multiplied to this dp if to get this ratio S P by V P when you have a non spherical particle.

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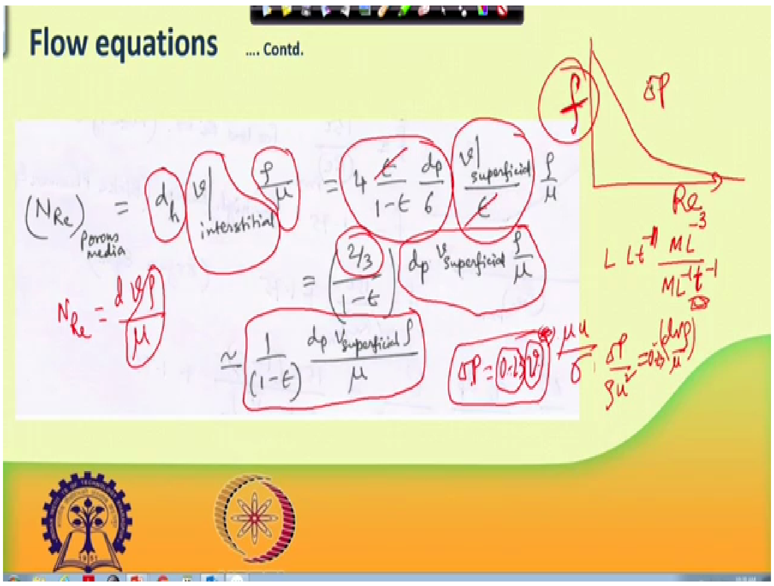

So, once you have this hydraulic diameter part taken care of then what you do is, then you have to define the Reynolds number for porous media and the Reynolds number for porous media would be hydraulic diameter multiplied. So, originally our Reynolds number was what? Our original Reynolds number was d v rho by mu in a pipe. So, in a pipe d is the diameter of the pipe in a diameter of the pipe v is the average velocity, rho is the density and mu is the viscosity. So, rho and mu these are viscosity and these are the properties of the fluid.

So, this is rho and mu these are characteristic property of the fluid. So, this rho by mu term is remaining as it is we do not have any issue. Here we have this hydraulic diameter which we have calculated just now with 4 into weighted by weighed perimeter and everything. So, for porous medium we have this hydraulic diameter now as 4 into weighted area by weighted perimeter and now this is function of 3 things, what are those? One is epsa the. So, this hydraulic diameter depends on epsa the porosity, it depends on the equivalent diameter if it is spherical wave the porous medias consist of spherical particles it is then equivalent it is just a diameter of the sphere.

If it is non spherical particle you have to look at the equivalent diameter and also a terms sphericity. So, this is this is all hydraulic diameter will depend on and the velocity when it comes to this superficial velocity is not going to work here because we are talking about the velocity that is operational inside the pore. So, naturally this v this has to be v interstitial.

So, v interstitial you remember you must be remembering v interstitial is v superficial divided by epsa in earlier lecture we derived how it is how we are arriving at this. So, this is my this is the hydraulic diameter we do not have this v phi s term ideally if you have phi s term then this phi s term would be appearing here, if this is this is without the sphericity if you have this phericity then dp has to be associated with phi s term ok. So, this is the v interstitial, this is the hydraulic diameter and rho by mu remains as it is.

So, now if you if you simplify this it becomes v superficial as it is these epsa and these epsa they will cancel out. So, essentially you are on this part you clubbed dp v superficial rho by mu. See all these three they will have similar dimensions right dp would be same as any diameter so; that means, this is a these are the dimension of length, v superficial will have a dimension of length and time inverse and time inverse and this rho by mu rho by mu will have rho will have ml to the power minus 3 mass divided by length cube.

And mu will have M L inverse t inverse; t inverse. So, then you will you will end up you can see here it would be it would be just perfectly all terms they are cancelling out. So, this part is a perfect dimensionless number. In fact, dv rho by mu, so it has to be, but here in this case it is not the diameter of the pipe it is the diameter of the particle that we are talking about. Particle equivalent diameter of the particle which is which the porous media is comprising of, v supervision is not the velocity at which the flow is taking place inside the pore or any pore for that matter. It is the superficial velocity; that means, volumetric flux and rho by mu these are the properties of the fluid. So, it has nothing to do with the porous media.

So, now, outside what do you have here is 2 3rd divided by 1 minus epsa. Now, epsa is as such dimensionless it is it is just a fraction it is it is it is a this is porosity; porosity is dimensionless because it is volume by volume. So, this part is dimensionless anyway. So, now, so it you are essentially having a term which is I mean which we can relate to Reynolds number because these are somehow dv rho by mu and into this 2 3rd divided by 1 minus epsa.

Now, when it there are several I mean if you if you look at these dimensionless numbers they have their own significance ok. So, these dimensionless numbers first of all they these compares the two different you know influencing factors ok. For example, inertia by viscous force, surface tension contribution of surface tension against contribution of viscous force.

So, this kind of this kind of dimensionless numbers people would be continuously working on. For example, since a you would be when it comes to two phase flow we would be talking about you know surface tension or interfacial tension much more you will find mu u by sigma this is a dimensionless number, this number is known as capillary number.

So, this is basically contribution of viscous component to the contribution of surface tension. So, which part is dominating? If you have a large capillary number you can say that viscous part is dominating over surface tension whereas, if you have a small capillary number you know that surface tension is dominating over there. Similarly, here also this Reynolds number means it has it is it is basically supposed to be a ratio which term which gives me the effect of inertia to the viscous force.

So, now, this 2 3rd does not feature there 2 3rd I mean nobody talks that what I did. So, its a I mean it does not make sense ok. So, when it comes to a dimensionless number you take some parameters clump them together make them dimensionless. So, that first of all they should make some there should be some meaning out of it that this should they should they should compare the two contributing factors that is one thing. Second thing is this dimensionless number use of these dimension dimensionless number should make the process easier for us. The for example, I mean I suppose somebody wants to know what is the pressure drop for a flow through a pipe ok.

So, they could have found out how pressure drop depends on diameter, how pressure drop depends on velocity, how pressure drop depends on density of the fluid, how pressure drop depends on the viscosity of the fluid separately they could have found out. And then with that information they could have they could have they could have come up with some expression and that could have been just fine.

But the fact is if they want to establish that dependence of pressure drop against let us say the diameter of the pipe to verify them experimentally one has to conduct experiments with different diameter pipes. Similarly one has to conduct experiment with fluids of different viscosities all other parameters remain same. Similarly, fluids of density where all other all other parameters they remain same.

So, so one has to do you know until then one has to do several such experiments to arrive at these relationships first of all. Second thing is whenever you write such type of suppose if one does some experiment and come up with a correlation of delta P is 0.23 into velocity to the power say something some number, if the other terms are remaining same if one comes up with this expression. First thing would be then moment you write such kind of an expression what is the unit of velocity we look because these equation becomes unit specific.

So, if what is the effect of unit specific equations say these velocities in meter per second. So, 0.23 is valid 0.23 to the power something, so 0.23 is valid or these to be these number to the power that that exponent is valid only if velocity is given in meter per second and delta P in so and so. Now, if somebody does one has to remember this somebody does the velocity instead of meter per second it does centimeter per second. So, then these whole these constants will change.

So, this constants will be then that this constants will depend on what unit you have chosen. Whereas, if somebody does this with say delta P by rho u square that is a dimensionless number which is known as Euler number, these delta p by rho u square is dv rho by mu and somewhat somebody comes up with 0.23 into dv rho by mu to the power something some exponent.

So, in this case whether you see whether you are using SI system or PS system or any other unit whatever you do they will cancel out this is after all a dimensionless number there is no dimension involved in these; there is no dimension involved in these this by the way this dimensionless number is known as Euler number. So, this there is no dimension all dimensions are cancelling out.

So, whatever you need to choose as long as the units are canceled cancelling out you do not have to bother. So, this 0.23 and this exponent that we are talking about these are universal these are not unit specific. So, there are always advantages of working with this dimensionless number. So, that is why people have gotten in gone into this ok.

So, now; so naturally here this two third does not feature, but this is alright this is this as a definition of Reynolds number this is alright because this part we can relate very well to that this is this is truly a Reynolds number. And this part this part we can see that it is this 1 by 1 minus epsa is simply a it is a dimensionless factor. So, Reynolds number for porous media is given by some Reynolds number definition in terms of pore diameter superficial velocity rho by mu and multiplied by 1 by 1 minus epsa. So, this is the new Reynolds number.

Now, we have to use this Reynolds number to find out what is the pressure drop through this definition of friction factor that we have studied in fluid mechanics. So, I am going to stop here in this lecture when you come to the when you when you start getting into this next lecture I expect that you revise a bit, the concepts of this pressure drop and friction factor as we have studied in fluid mechanics at least at least try to try to get some firsthand idea of what these terms are.

So; that means, for example, there is a very famous friction factor chart which says friction factor depends on Reynolds number; friction factor depends on this is if this is the friction factor and this is the Reynolds number there is a friction factor chart something like this. So, this the what is this chart, what is this friction factor and how this chart is used to calculate pressure drop? This is a very standard fluid mechanics assignment this is covered include mechanics class.

I will definitely discuss this in detail, but it would be helpful if you if you and if possibility or end please go through these concepts once before you get to this next lecture. So, this is all I have on this particular module of the lecture.

Thank you very much.