

Flow Through Porous Media
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Lecture - 02
Introduction (Conceptual Flow Models)

I welcome you once again for this Flow Through Porous media; we are discussing about this Introduction chapter. So, we are in the topic introduction where we are discussing about the most basic features of porous medium and how their they are going to help us in different applications. We are discussing about definition of porous medium; what are those major characteristic parameters of a porous media and what are the major applications?

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The slide, titled "Definition of Porous Media", features a diagram of a porous medium with a pressure gradient $\frac{\partial P}{\partial x}$ and a flow rate Q over a length L . Handwritten notes include:

- Superficial velocity: $V_{\text{Superficial}} = \frac{k}{\mu} \frac{(P_a - P_b)}{L}$
- Interstitial velocity: $V_{\text{Interstitial}} = -\frac{k}{\phi \mu} \frac{\partial P}{\partial x}$
- A note stating: "Effect \propto Cause (Pressure Gradient)"
- A thought bubble: "Restrict to flow; Superficial and interstitial velocities"

Courtesy Wikimedia

So, in my last lecture I was discussing about these restrain to flow and superficial and interstitial velocities and we discussed about this Darcy's law. What we said the Darcy's law says that $V_{\text{superficial}}$; $V_{\text{superficial}}$ is equal to k by μ . We just now said it is P_a minus P_b ; in our next in our last lecture we showed that k by μ P_a minus P_b divided by L . So, this superficial velocity it has P_a ; P_a minus P_b ; P_a minus P_b is basically the pressure at position a minus pressure at position b divided by L .

So, we have a pressure gradient; so we said that this is this relationship is more like a cause and effect relationship. So, we have a cause, this is the effect; and effect is

proportional to the cause. What is the cause? Cause is the pressure gradient here. So, we have cause is basically the pressure gradient; I have a pressure difference P_a minus P_b and over a length L . So, the gradient is; is equal to P_a minus P_b divided by L and what is the effect? I have a velocity, I have a fluid flow taking place. So, so the velocity is the effect.

So, and this the constant that we have here is basically k by μ ; μ is the viscosity, it depends on what type of fluid we are running through this porous medium. If it is water it is close to 1 centipoise; if it is honey the viscosity would be much higher and k is the permeability.

Now, one can write these instead of in algebraic form, one can write this in differential form and that differential form would be V superficial; V superficial is equal to minus k by μ del P del x . So, this is the velocity in differential form. So, it is minus k by μ del P del x ah; why minus sign?

Because I expect that when there is a flow taking place; if there is a flow taking place and if I plot pressure versus distance x , I would definitely see that the pressure will decrease; flow will take place from high pressure to low pressure. So, pressure will decrease as you go from inlet to outlet. So, what; that means, is the slope of this line which is given by del P del x itself is negative, but you have; if x is the velocity in the x direction is positive, so one needs to have this minus sign.

In fact, if it is if you convert this to algebraic form; if you assume the pressure profile to be linear in that case this del P del x would be you will write it as P_b minus P_a . So, this minus sign will make it P_a minus P_b which is the algebraic form that I mentioned here. So, these form of equation we are more we are going to use more often that V superficial is equal to minus k by μ del P del x .

Now, this here; obviously, if we want to bring in the V interstitial; V interstitial would be, interstitial would be then you have to simply divide one way we have to have one extra term; ϕ in the denominator. So, this would be this is going to be the interstitial velocity.

So, this is; this is one good thing because why when we work with this kind of a; so we have we are seeing a sort of a structure here how we can relate this flow through porous medium, just like the way we do move just with what we did in fluid mechanics.

Because in case of fluid mechanics also we have a velocity against a pressure gradient; when there is a flow of fluid through a pipe. So, we can somehow we can start thinking in those lines. Of course, there we do not have permeability as such that is something which we need to dig into a bit. But essentially this pressure gradient and this cause and effect relationship is gelling with what we had already learnt, what we had already studied in fluid mechanics in the context of fluid flow through a conduit through a pipe.

So, that is one good thing and we have this porosity and how the porosity relates superficial velocity with the interstitial velocity. Or in other words I have this Darcy's law which is taking care of the overall; in an overall sense what is the velocity which we will get directly from q ; q directly by dividing directly from q divided by a we call this superficial velocity and how we can relate these to the local velocity interstitial inside the pores.

So, how they relate we can see from these equations; at all these two equations these equations are you mind it; these equations are valid only when your only when see we have to pick up a dx right. Whenever we whenever we write this kind of an equation we essentially our x starts from here, so x starts from here., so this is x . So, we pick up a differential length and we call this differential length as dx right. So these differential length that we have here which is which is basically dx , this dx differential length has to be large enough, so that we do not get into the trap of that; you know sample sampling volume below that threshold volume for definition of porosity.

So, that that you must keep in mind that; we are assuming that this material is homogeneous or even if it is a heterogeneous; we have we have that the length scale is different. We are not going to that sampling volume which is less than the threshold value; for which we need to consider discrete you know changes in velocity. Here you can see V superficial velocity; that means,; here we are considering uniform velocity in this direction.

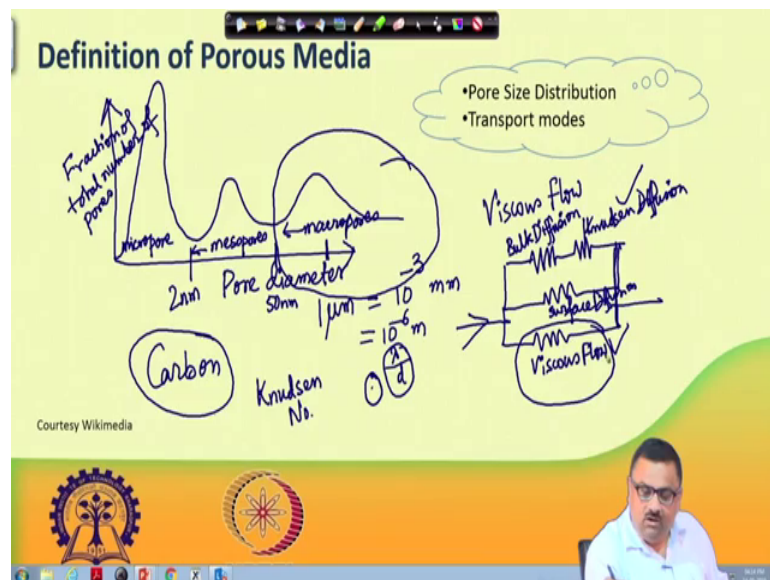
So, we are assuming see there is no function of z . So, we are assuming all these they are having the same velocity and that velocity is given by this, but if you have porous medium with these grains. So, how the flow takes place? It is taking a turn here, then turn there and going round and round and doing all those things right.

So, what we are assuming is that as if I have a continuum existing; my flow is being restrained, but that restraint is all taken care of by this permeability term k . So, there that will take care of how much would be the pressure drop etcetera. But I will assume that a continuum exists and the velocity is uniform be actual velocity is like this.

So, this is a major assumption that you are treating, you are considering this porous medium to be homogeneous you are you are your reference volume your reference sampling volume is large enough to draw this kind of continuity. Now, this is the trend this is this is what we are going to do in case of porous flow through porous medium; depending on I mean we there are some ways to get into these discrete modeling though one is to one is to get.

Basically this type of continuum approach is what excellently well or what worked work very well; for various applications that we are going to design. So, that is why we have I can tell you that I mean; I have full confidence on this continuum assumption in this case. Because this has this has been used in almost all applications and people have done the people have done their design very efficiently with these continuum assumption.

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So, what we have next is; what we have next is so called pore size distribution. This is also another important thing we must keep in mind; we have been talking about permeability and porosity and all this, but this pores, they are not of uniform size.

So, if we try to plot fraction of total number of pores; let us say I or number of pores or fraction of total number of pores either way; versus the let us say I write pore diameter and here I write fraction of total number of pores. So, what we expect here is some pores will be large, some pores will be small some are not so large.

So, there would be various sizes of pores present in the porous medium because it is a random structure. Had it been a monolith of uniforms uniform you know we have we have approximated, we have approximated porous medium as a bundle of capillaries.

There we can think of; you know all capillaries of same diameters they are running through the rock, they are running through that solid. So, that kind of porous medium they will have a fixed structure, the pore diameter is all a single; pore diameters diameter is not varying it is just a single value single diameter. But in a real porous medium in a random porous medium you will have all diameters present.

So, in this context you can have something like; you have some pores of diameter let us say 1 micrometer; 1 micrometer; 1 micrometer means, it is equal to 10^{-6} meter or you can write 1 micrometer as equal to 10^{-6} meter. So, one can have pores of this dimensions, one can have pores in the nanometer range, one can have pores even less. So, in this case you can define the porous media depending porous medium; depending on what kind of what type of pores are dominating in this.

For example, if we look at look at a sand pack or a sandstone which we get from mother earth sandstone where most of the oil is held up many a times you get water. So, this sandstone these are so called porous rock; there the diameter for pore diameter or pore width pore opening is to the tune of 1 micrometer or even larger; whereas, there are porous medium which are having much smaller pores.

For example, this carbon; carbon material they are highly porous what I understand is that a very small piece of carbon can hold a football field worth of surface area tucked inside. So, they are highly porous.

So, this carbon porosity the pore size is a much smaller range. Carbon has mostly so called micro pores; micro in this in this context I must point out that you can have if you are in this range; let us say I have 2 nanometer; 2 let us say about 50 nanometer; this

range is referred as mesopores ok; 2 nanometer 250 nanometer. And higher generally we call it macropores and below 2 nanometer; we refer them as micropore ok.

So, you will find that your that the porous bar medium that is provided to you that has substantial amount of these; then some these some this like this. So, a porous medium could be; so you will find fraction of total number of pores there that there may be good number of micropores present, there are some mesopores present, some are macropores are also present in this. So, you will have a pore size distribution like this; generally it is; it is generally the whenever you have a micropore and mesopore; this is this kind of combination one can get in carbon.

In fact, carbon which has some amount of mesopore; let us say 80 percent or 90 percent of pores are micropores, 10 to 20 percent are mesopores that kind of combination is very useful. You can one can access the that one can access that area and this can be this can be used; I am I will discuss this can be used in many in many different applications ah.

Whereas, in case of macropores and even larger 1 micrometer and even larger; they are used for application such as you know oil recovery from underground reservoir, water you know from underground reservoir one collects water also or maybe you know sequestration of various chemicals inside mother earth. So, in these a these are; these are basically the macroporous regime.

Of course, there are engineered products which uses all these pore sizes, but essentially these this is this kind of a distribution one can have; this micro meso and macro pores and their combinations. Now, when it comes to flow through macropore when we are talking about these Darcy's law and this V superficial as $\frac{k}{\mu} \frac{\Delta P}{\Delta x}$ and all these; these are primarily meant for this part.

So, this part we are more there you will find these to be the applicable more. Whereas, as you are going down as the pore size decreases you will find that this; there would be some other mechanisms governing the transport process. What are those? For example, what did we consider when we talked about Darcy's law? We talked about basically the Poiseuille's flow; I mean if you refer fluid flow if you refer the fluid mechanics the in this context; it would be Poiseuille's flow.

Simple flow through a pipe flow through a channel that will have a pressure gradient and you will have some resistance. And this so these are these are these are somethings which we had done earlier. Probably we are more familiar with something called friction factor; in case of flow through a pipe here we see permeability.

So, this part is the flow through macro pores there you will have you know this kind of Poiseuille's flow or you can call it in very simple term you can call it viscous flow. I think the viscous flow because you have a viscosity term k divided by μ after all.

So, this viscous flow is governing when we are working in this region; when the pores are primarily macro pores. But as you go down when the pore diameters are small you will find there will be other mechanisms that are coming into play. In fact, down the line we will discuss this in detail; a transport can be thought of as few other types of. So, the a flow that takes place through a porous medium; they can have this kind of parallel you know circuits. So, let us say the bottom one we can write these for; this is for the viscous field ok.

So, here you have macro pores and viscous flow is taking place, here the top one I can write these as bulk diffusion. What is bulk diffusion? Bulk diffusion is that if you put some ink in a beaker full of water, you will find that the ink percolates in quickly travels and fills the weaker completely. So, you put a drop of ink in a beaker full of water and the ink travels; it is because the process is known as diffusion you and it has its own Brownian motion and it travels into this; you are not you are not imposing any pressure gradient to the ink for travel.

So, you can have this kind of bulk diffusion for the, so you can have as some molecules can travel through pores by the method of bulk diffusion. There is another process which goes by the name Knudsen diffusion; Knudsen diffusion. Knudsen diffusion I believe you have studied in the in physical chemistry; Knudsen diffusion is that you there is molecules are traveling through the pore, but molecules are colliding with the wall.

Generally when the pore diameter if in this pore diameter let us say pore diameter is d and the mean free path that length is λ . So, it depends on what is the value of λ by d ; what is the value of this λ by d ? When, this λ by d they are close then you will see that the collision of molecules with the wall that, governs the flow that that influences the flow.

So, one has so this is basically the see as you decrease d from the macropore range to on the lower side, you will at one point of time the mean free path length of these molecules that are traveling and the diameter of the pore; they would be close to each other. In fact, that there is a dimensionless number in this context which goes by the name Knudsen number.

So, these so when this is this number is close to 1 or for. So, so depending on some threshold number you will find that this kind of transport mechanism they start dominating. Similarly here you will find that there would be a process known as surface diffusion.

Surface diffusion means that molecule that is supposed to travel molecule will get adsorbed to the inner wall of the pore and then the diffusion is taking place through the surface. That means, it is it is not a bulk diffusion; that means, it is not molecules colliding with each other in the bulk and traveling through the center of the pore; molecule first gets you know attached to the inner wall of the pore and then by means of surface diffusion it is travelling.

So, there are various mechanisms that are there various; various transport modes by which molecules can travel through porous medium. And when we talk about Darcy's law; when we talk about superficial velocity that we discussed in the last lecture that is because of viscous flow ok; it could very well be that that the pore size that one works with you are at this viscous flow and Knudsen diffusion.

These two parallel lines are working surface you are not bothered with the surface diffusion. In fact, we will show that when somebody works with flow of gas through shell. Shell is some subsurface material in which lot of you know; hydrocarbons are stored and nowadays people are extracting hydrocarbon out of those that those shell layers. So, these there you will find that this diameter is very small and it is the λ which is the gas; the mean free path. So, when you are talking about flow of a gas through shell layer, one may have to consider viscous flow that is fine and on top of that may be Knudsen diffusion.

So, there could be combination of this transport mechanisms also possible; combination of this transport modes. So, what I mean we though we discuss this viscous flow though we talked about Darcy's law and it cause an effect and pressure gradient just be aware

that there are other transport modes available. And there could be combination of transport modes of course, it depends on what pore size distribution you have here you are working. So, with this understanding; now let us see what we have next we talked about this permeability; permeability was in case of Darcy's law right.

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Definition of Porous Media

- A medium with a permeability of 1 (darcy) permits a flow of 1 cm^3/s of a fluid with viscosity 1 cP (1 mPa·s) under a pressure gradient of 1 atm/cm acting across an area of 1 cm^2 .
- Typical values : 100,000 darcys for gravel, to less than 0.01 microdarcy for granite.
- Approximate permeability of sand is 1 darcy.
- 1 darcy is equivalent to $9.869233 \times 10^{-13} \text{ m}^2$ or $0.9869233 (\mu\text{m})^2$

Courtesy Wikimedia

In case of Darcy's law he had this definition of permeability what was the Darcy's law? We had written there as v superficial is equal to is equal to minus k by μ del P del x . So, this k is what we said is the permeability. So, a medium the definition of permeability is something like this; a medium with a permeability of 1 darcy. Mind it darcy's is the this darcy is 1 unit of permeability just like meter is a unit of length. So, permeability of 1 darcy, permits a flow of 1 centimeter cube per second of a fluid with viscosity 1 centipoise.

1 poise means gram per centimeter second, so 1 centipoise would it be 10 to the power minus 2 gram per centimeter second; which comes to 1 mili Pascal second. So, of; so a viscosity similar to water when it flows through a porous medium under a pressure gradient of 1 atmosphere per centimeter; So, pressure gradient is 1 atmosphere per centimeter; that means, if this is the length of the core; if this is the length of the core and if this is 1 centimeter; the pressure drop over this lengths 1 centimeter is 1 atmosphere.

So, pressure gradient of 1 atmosphere per centimeter acting across an area of 1 centimeter square. So, this area the area here is 1 centimeter square and a pressure drop

is 1 atmosphere. So, under such condition if one gets flow rate of 1 centimeter cube per second, then one call this porous medium is having a permeability of 1 darcy.

So, that is the definition of one darcy; now typical values for gravels; gravels are you know what gravels are gravel pack we have seen. For gravels it could run up to 100000 Darcy's; that is too high I mean I would say. If you work with the sand pack it would be of the order of darcy to less than 0.01 micro

So, darcy is a unit of permeability and approximate permeability of sand is 1 darcy. So, 1 darcy is equivalent to some meter square you have written here; it is close to 10^{-12} meter square. So, 1 darcy is equivalent to approximately 10^{-12} meter square. So, you must note that in SI unit; this unit of permeability is meter square which would be obvious.

V superficial would be meter per second, you put this $\frac{\Delta P}{\Delta x}$, ΔP would be Pascal and Δx the length would be meter. So, Pascal per meter and these viscosity; you put it as kg per meter into second. So, put viscosity as kg per meter into second, pressure drop as Pascal, length as meter and superficial velocity is meter per second you will find that unit of permeability will come as meter square.

So, darcy and meter square this is the relation 1 darcy is equal into 10^{-12} meter square. So, this is this is how the basic- this is; this is what is the basic definition of 1 darcy. So, with this understanding I mean we have more or less you know got some glimpses of how this flow through porous media is going to and you know what all we need to touch upon at least in this course.

And next what I am going to discuss is the applications, but that would be the next lecture. So, with this understanding I am closing this lecture here and I will continue this lecture with the application; the application section of introduction to flow through porous media.

Thank you.