# Flow through Porous Media

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## Lecture – 19
## Flow Equation (Introduction)

I welcome you to this lecture on Flow through Porous Media. We have been discussing about different mechanisms by which the transport takes place in porous media. Now, we were going to in this module we are going to discuss about the flow equations. We will look at this Darcy's law that we talked about that cause and effect relationship in a bigger detail. We are going to look at it and also what other ways other than Darcy's law what other ways viscous flow can be theorized.

So, that is something which we are heading to.

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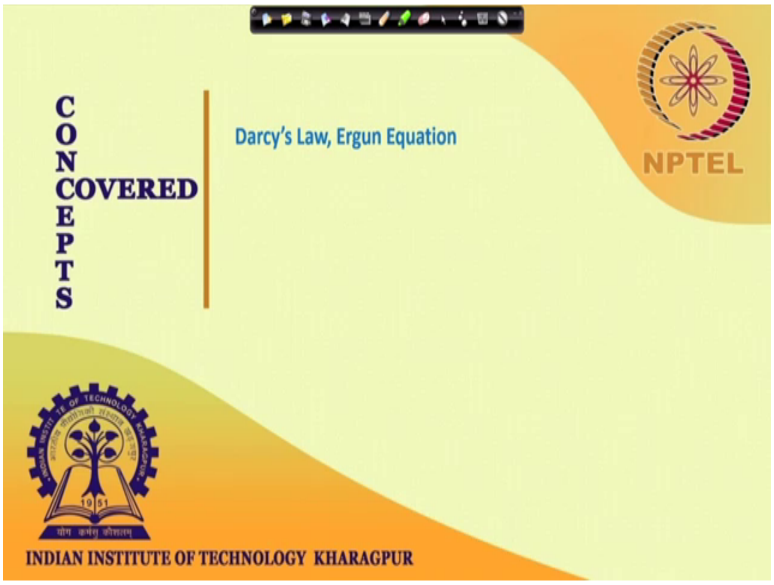


Here in this section you will be focusing mostly on Darcy's law and also further an equation which is known as Ergun equation.

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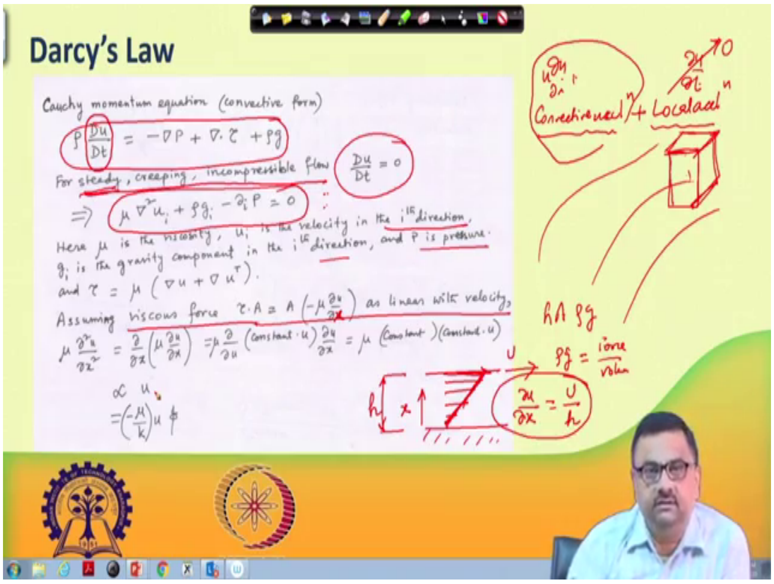


If we look at the Cauchy momentum equation in convective form it is written like this. You must be familiar with Navier stokes equation in fluid mechanics, if not you must think of this whole exercise as an extension of Newton's second law.

So, what Newton's second law said is that force is equal to mass into acceleration, in case of fluid mechanics mass into acceleration can be in different forms. Acceleration can be that you put some liquid in a tank and then you halt that liquid using a truck. So, there also that liquid can accelerate depending on the acceleration of the truck ok.

On the other hand when a fluid flows through a pipe or fluid flows through any conduit that time one has acceleration, but that acceleration is not simply D u D t, where u is the velocity it is not simple derivative of velocity instead we talk about the something called a substantial derivative which is capital D u capital D t.

So, this substantial derivative these gives an acceleration which is a combination of convective acceleration, convective acceleration plus local acceleration. So, this convective acceleration and local acceleration we are familiar with these del u del t term as acceleration, but that we are calling local acceleration; Why, we have to get into this convective flux local acceleration?

Because these velocity the u that we are talking about, u is not velocity of a particle had it been a Lagrangian framework; that means, had it been a velocity of a particle u is a velocity of a particle we could have simply we could have taken a derivative of it with respect to time and we could have called this acceleration, as we have studied in particle mechanics.

But here in this case u is not velocity of a single particle; u is the velocity field. What that means, is flow is taking place here and I picked up a box attic or some coordinate I picked up a box. And in that box in that location let us say x y and z location, in there are multiple there are several number of particles that are entering into the box several particles leaving the box.

If I look at the average velocity of all these particles at that particular location that is the u that we are talking about here. So, this is basically a velocity field which is a function of space and time, space means this is the xyz coordinate where this box I have located and t is the time at which I am doing this averaging.

So, this is a velocity field. So, when we for try to find out what is the acceleration in this velocity field we need to take a substantial derivative. So, this is the mass into acceleration term and on the right-hand side we have the force term, and the force is basically force per area.

So, we have now; so we have from the force term this rho g, you remember h rho g was a hydrostatic head, h rho g is a pressure right. So, I have h rho g used to be the pressure. So, this is rho g term is giving me so, pressure so, when it comes to force it is h into rho g into A. So, h into A gives me the volume right, h into A into rho into g h into a is the volume into density is mass, mass into g is the acceleration due to gravity. So, this is the force.

So, rho g is basically; rho g is basically the force per volume. So, this is the force per volume. So, you are similarly minus grad of P, similarly the shear component is contributing here. So, this is basically the Newton's second law applied to fluid mechanics in case you are not having exposure to this Navier stokes equation. And if you have then you must reconcile this quickly I mean this equation you must have studied.

Now, for steady creeping incompressible flow, steady means these del u D u D t is the convective acceleration plus local acceleration. Local acceleration was basically del u del t and the convective acceleration is basically u del u del x plus there are a few other terms. So, now this del u del t time derivative is 0. So, steady means this part is gone. Creeping; creeping means convective acceleration part is gone and then incompressible; that means, rho is constant incompressible means density is constant.

So, and D u D t is already equal to 0 so, you are left with this equation. This is basically the Navier stokes equation with this convective term D u with these substantial derivative is equal to 0. So, under these assumptions this is the equation for this is the momentum balance equation. So, now, you can see here we put a subscript i so, i gives the; that i is as an index it gives me the direction it could be x y and z.

So, for example, a del i P; that means, if we are talking about x direction then it would be u subscript x this is rho g x direction gravity this generally does not work gravity works with y direction or gravity works in z direction, if you have x y and z three directions and this would be del p del x. Similarly, when you are talking about the y direction then it be u i. So, that should be mu del square u del y square plus rho gy, if y direction gravity in the direction that you have chosen as y if gravity does not apply to that direction then this term will be automatically equal to 0 and minus del P del y.

So, you have three equations; so, these three equations we had put here in one notation with these index i ok. So, in one case it is del P del x in other case it is del P del y in other case it is del P del z. Here it is 0 0, and if z direction is the gravity or maybe z is opposite to gravity lot of times you work with z which is going up. So, if it is opposite to gravity then you would be minus g, and here in this term this term becomes mu del square u del x square in the second case it becomes mu del square u del y square and third case it.

So, these are three different equations right hand side is zero and left-hand side is taking those forms. So, three different equations will generate. So, that is the purpose of putting a subscript i. So, here mu is the viscosity u i is the velocity in ith direction, g i is the gravity component in ith direction and P is the pressure we have already talked about it. And the definition of tau is given here. Now, here another assumption one needs to take if one wants to arrive at Darcy's law, one assumption is this and density was treated as constant.

So, these are the assumptions by which you are reaching their; steady, creeping, incompressible flow. So, this is one thing and second thing is you are assuming that the viscous force is as linear with velocity. Actually the exercise that I am doing is these are mostly in a approximate and dimensional sense, I mean it has to be done more rigorously.

But I just wanted to point out that if one considers viscous force as linear with velocity, viscous force viscous force; that means, A into minus mu del u del z or tell you what is the del u del x mu into del u del x; mu into del u del x if we if we assume this to be linear with velocity. When would you assume del u del x to be linear with velocity think of it? We have to let us say we have two parallel plates, one plate is fixed and the other plate is moving at a velocity let us say u.

So, in between these layers you will find that this here the velocity is 0, here the velocity is little bit, little bit further, little bit further and here the velocity is all the way the velocity u. So, the velocity profile will take a shape like this. Now this is and this is supposed to be non-linear, but if this gap is small one can and some other putting some other assumptions one can see that del u del; let us say I have this is the direction of y or let us say this is the direction of x.

So, del u del x the velocity gradient can be written as let us say this distance between two plates is h is U by h. So, in this case one can say that this del u del x the gradient is equal to u by h one can make an assumption. Similarly, if one can assume here the viscous force as linear with velocity. So, in that case one can go here and place this instead of mu del square u del x square which is del x of mu del u del x, these del u del x can be written as constant multiplied by U since its linear with velocity.

So, one ends up with these as proportional to U if you follow this logic ok, and then this proportionality constant if someone writes as minus mu by k into phi then one ends up with truly the Darcy’s law. Because rho g i let us say we are ignoring if we are having a flow in horizontal direction if. So, this would be ignored and this term would be simply minus mu by k u into phi this term and this goes to the right-hand side these becomes del p del x.

So, then del p del x is equal to minus mu by k u into phi. So, then u would be then u if you take everything to the right-hand side u end up with the Darcy’s law. So, this Darcy’s law I mean though we placed it as a cause and effect relationship, but if somebody wants to you know derive it from the basic Newton's second law. So, one can do that with the assumptions that I mentioned just now.

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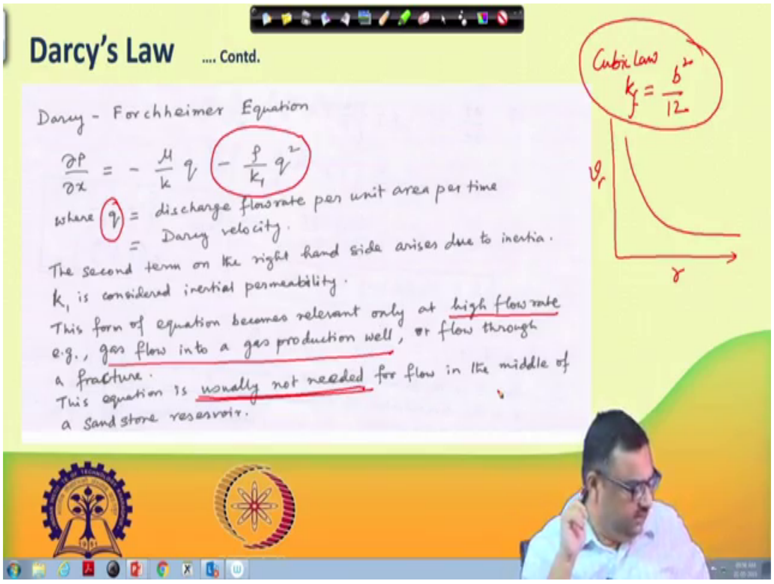


There are other variations to these Darcy's law possible, one such variation is mentioned here and here you can see del p del x is not just a function of velocity rather it is also there is a component given as velocity square ok.

So, this is basically this q is equal to Darcy velocity here which is flow rate per unit area per time. The second term on the right hand side which is nu because this two they come they constitute the Darcy's law, but this term is nu the second term on the right hand side arises due to inertia, where k 1 is considered inertial permeability.

This form of equation becomes relevant only at high flow rate because inertia will dominate only at high flow rate. Such as gas flow in a gas production way gas production will means near the wellbore you will have much higher velocity you have already seen this right; you have already seen that the velocity if I put v r as a function of r and this is proportional to 1 by r.

So, naturally near the wellbore the velocity is much higher and so if it is a gas flow even it is higher. So, or flow through a fracture, flow through a fracture means fracture is a porous medium they can have some channels I mean porous media if there is a fracture there is a crack through that the flow will take place at a much higher rate. We will discuss about how the permeability of a fracture can be obtained from again Navier stokes equation. But this fracture can be treated I mean one can have a permeability and one can treat these just as equivalent to Darcy's law.

Just using the Darcy's law one can treat them and that similarity of that treatment with flow between two parallel plates as we have done in fluid mechanics that can be established. In fact, there is a very famous cubic law; cubic law for flow through a fracture which relates these two flow between two parallel plates and flow through a fracture whereby they have shown that the fracture permeability k we put a subscript f.

So, fracture permeability can be written as b square by 12, where b is the distance between these parallel plates. So, this is something which we can touch upon later. So, this is; so these are so, when it comes to flow through a fracture this kind this type of equation may be may be valid. Now, this equation is usually not needed, frankly speaking this equation is usually not needed for flow in the middle of a sandstone reservoir.

So, when you are working with a working with a regular flow in the reservoir this type of equation is not needed it is a very special treatment. But just for information that such type are we accounting inertial terms and through an inertial permeability that exists.

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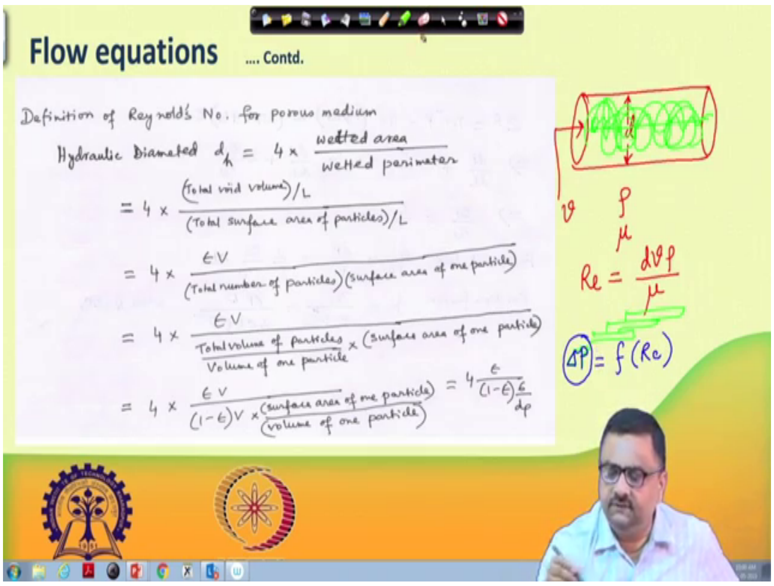

Now, here comes the other way of doing things. We have been talking about flow through porous medium through Darcy's law and all these things. So, now suppose someone goes to try to relate more to the concepts of fluid mechanics flow through a channel, if someone wants to wants to relate the flow through porous media with flow through channels. So, the most obvious thing they will do is they will immediately go to Reynolds number. Flow through a pipe; flow through a channel it is typically defined at the pressure drop; pressure drop for flow through a channel is given by the most fundamental dimensionless number in this context is Reynolds number. You all are aware of this, must be aware of this is the most basic thing in fluid mechanics.

When you have a pipe of diameter d and fluid is flowing at a average velocity v and the density of the fluid is rho, viscosity of the fluid is mu then a Reynolds number is given by d v rho by mu this is what is the definition of a Reynolds number. So, Reynolds number defines what in a pipe, generally Reynolds number if it is less than certain threshold number let us say 2100 or so then one considers the flow to be laminar. In fact, there is a classical example of this scientist Reynolds, what he has done is; he has performed an experiment where he introduced some color in the flowing in the flow of water in a pipe.

And what he found is that as he continue to increase the velocity he found initially it was just; initially it was just a line it was just a line wherever he introduced the color it was just a line. Then as he continued to increase the velocity he found that this line is becoming wavy and then if it at a very high velocity he found that this line is basically churning like this. So, he put this; so, put some numbers he assigned that this dimensionless number when it crossed when it crosses certain threshold then this type of churning starts getting there.

So, these number the higher the threshold number which if Reynolds number is beyond that threshold value that is considered the turbulence. And if it is less than the threshold value it is considered laminar, laminar means one layer sliding against the other. That means, we this is one layer the other layer is sliding, the other layer is sliding one is sliding against the other whereas, in case of turbulent there are ad packets which is moving randomly inside the conduit.

So, now, this Reynolds number what they did is now related this pressure drop delta P now what they did is they related this pressure drop delta P as a function of Reynolds number. So, not only it is not just Reynolds number so, there would be other parameters also. So, this; so it is how; what would be the dependence our pressure drop on Reynolds number.

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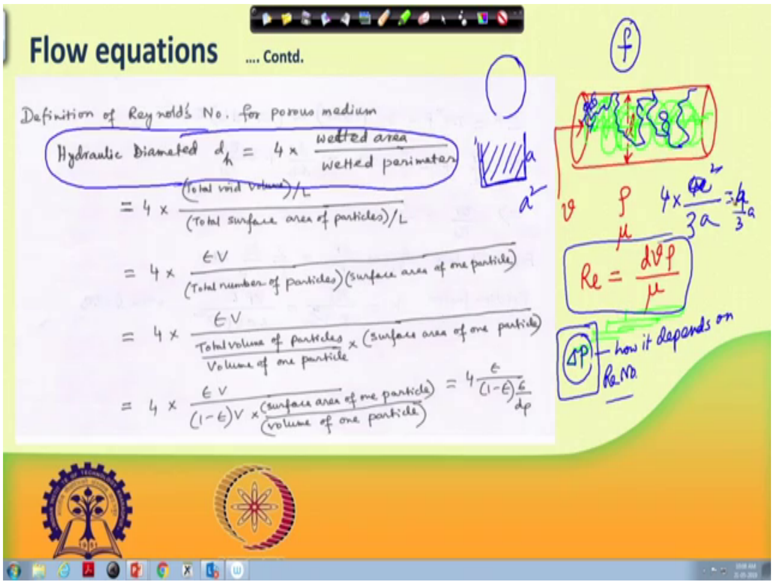


So, rather I should put this as delta P how it depends on Reynolds number. So, if one wants to go to that logic there in fact, the framework that has been developed there is by the use of a term called friction factor. Friction factor is a term which is that when a fluid flows through these pipe there would be frictional loss. In fact, that is why if we have a cross country pipeline, we have to put some pumps in between over regular intervals so that pressure can be maintained.

So, where is pressure going because pressure is getting lost pressure is lost due to what it is lost due to frictional drop. So, to overcome the friction one has to put additional pumping effect. So, what is this frictional loss? So, if this frictional loss is causing this delta P is arising due to the frictional loss. So, this frictional loss the one has equated this frictional loss by a term called friction factor.

The friction factor is so called a factor suppose a fluid is flowing it has certain kinetic head; kinetic head means it has certain kinetic energy of if you look at half mv square is a kinetic energy. So, per unit volume if we look at it would be half rho u square. So, then you; so out of this kinetic energy which fraction is lost due to the friction. So, friction factor is one such indication, one such parameter that defines how much is getting how much of energy is getting lost due to friction.

So, this friction factor and now this delta p treating this delta P as a function of this friction factor; now, naturally this friction factor will depend on whether you are flowing too fast or whether you are flowing too slow right.

So, if you are flowing too fast then these there are ads, the ads are there would be there would be lot of cross movements and that will contribute to pressure drop in a different way, as against when the when the velocity is lower the treatment would be would be different. So, naturally this friction factor will be a function of; friction factor would be a function of this Reynolds number when Reynolds number is high then we expect the friction factor to be low. Whereas, when Reynolds number is low the friction factor would be high.

So, this is a treatment that is already existing in fluid mechanics for flow through a pipe. So, what we would like to do at this point is we have to define somehow these Reynolds number for porous media. The objective is that if we can define these Reynolds number for porous media, then if we can define the Reynolds number for porous media then we can come up with some kind of friction factor for porous media. Then if we can find out a friction factor then we can come up with an equation that will relate delta P the pressure drop with the Reynolds number through this friction factor term.

So, that is something which we are interested in, that could be an alternative treatment and that would be true if we want to follow what is exactly followed in fluid mechanics for a flow through a pipe. We can follow the same thing for flow through a porous media and the outcome of these is, there are three equations possible Blake Plummer, Kozeny carman equation and a combination of these which is known as Ergun equation.

So, these equations are arising from this idea that we exactly the way fluid mechanics is handled the Reynolds number, friction factor and then corresponding pressure drop. So, that same thing will be applied for porous media.

Now when you are going to draw when; we are going to apply this Reynolds number for when we are going to find out the Reynolds number for porous media the first problem would be that for a pipe we know the diameter it has a fixed diameter. But when it comes to a porous media it has flow is taking place through nook and cranny of this you know tortuous it is forming a tortuous pathway or not traveling like this. So, then how would you find out what is the diameter, here it is straightforward I have the overall diameter. And of course, if I take the overall diameter of the porous media that does not make sense because someone will have higher porosity someone will have lower porosity so that will also contribute to the Reynolds number.

So, where to start in this regard there is a concept called hydraulic diameter. Hydraulic diameter lot of times in civil engineering and in other applications people do not, say where we do not have a circular cross section. If you have a circular cross section, you can work with that d and you can find out the corresponding Reynolds number using that d. If one does not have a circular cross section say let us say they have a square cross section, it could very well be the water is flowing through a square the some conduit of square cross section.

So, in that case what would be; how do you calculate the Reynolds number? In that case what essentially is done, is that hydraulic diameter or it is so called equivalent diameter one finds out and that equivalent diameter is four into weighted area by weighted perimeter. So, weighted area by weighted perimeter so, this is the concept of hydraulic diameter that is invoked in this case for example, in case of a square weighted area would be these, the area that is weighted when as the fluid flows the weighting is taking place by the fluid.

So, weighted area is a into a so, weighted area is a square and weighted perimeter this would be the perimeter. So, perimeter is 4 a. So, it would be 4 into 4 a sorry, a square weighted area by weighted perimeter 4 a so, this is equal to a. So, hydraulic diameter in this case would be 4 into a. On the other hand if I have a case where I have a open drain so, the upper part is open, it is just a drain with three sides. So, in that case it will not be the area would be then weighted area would be a square.

But weighted perimeter will not be 4 a, the weighted perimeter would be 3 a. So, then it will not be a it would be 4 by 3 a. So, that would be the hydraulic there 4 by 3 a would be the hydraulic diameter. So, we have to apply these concepts; so, you may brush up; so, I am going to close this discussion now and when I continue in the next lecture the same subject.

Before we start that I suggest you brush up your concepts of hydraulic diameter and for various geometries how hydraulic diameter is computed. Because these hydraulic diameter will be used now to find out the Reynolds number; Reynolds number will be expressed in terms of hydraulic diameter in this case. Ok, that is all I have for this lecture module.

Thank you.