Flow through Porous Media

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Lecture – 15
Transport Mechanisms (Introduction)

Welcome you to this course on Flow through Porous Media. The lecture that we are going to concentrate on would be this Transport Mechanisms.

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We have discussed about; we have discussed about Darcy's law. If you look at it more carefull, this is basically Poiseuille's flow; I mean in fluid mechanics, it is it is basically a viscous flow. There is a pressure it is it is it is so called pressure driven flow and with viscosity considered; that means, one layer sliding against the other and there is a pressure drop as it travels through the pore. So, this is more of a viscous flow.

Now, there could be other types of mechanisms by which there could be flow through the pores. For example, if let us say this is the pore . So, we are talking about a viscous flow; that means, viscous flow means one layer I have I have a flow taking place. We talked about this laminar flow and then we have talked about one layer, next layer; next layer and the layers are sliding against the other; velocity at the wall is 0, here the velocity at the wall is 0. And, one layer sliding against the other and how much it can slide how easily it can slide this is defined by the viscosity. So, this is this is the type of flow we have considered when we talk about Darcy's law. Let us say we call this viscous flow.

So, we have a resistance to these viscous flow when the flow takes place there is a resistance to flow and this resistance is given. So, this is this is basically viscous flow resistance. So, when we have a flow going on this is this is the direction at which the flow say let us see if I conceptualize these as the porous medium. So, there is the flow going in coming out. So, it has to overcome some viscous flow resistance, resistance of this kind and this is amply taken care of by so called Darcy's law.

What could be other mechanisms by which there could be flow? The molecules they have a tendency to diffuse; I mean if I keep let us say I have a beaker full of water and then I put a drop of ink and I see that I have put a drop of ink and gradually I see that the color is traveling and this entire liquid becomes if I put drop of ink of blue color if this entire beaker would be this water would be blue.

So, what is that how the color is traveling from this place to this place? It is not a pressure driven flow. You have this is anyway the static condition it is held. So, it is basically the molecules they diffuse, they have a mean free path, they have their diffusing and because of that there would be they would be traveling to other corner of the beaker. So, by that same token I have some mixture of a mixture of gases and I can assume that this could be or some dyes I can assume that they will be traveling downstream.

So, this is known as this process is known as bulk diffusion. There could be other ways of doing things. For example, when particularly this will happen when these two are close to each other this characteristic dimension of the channel is small instead of a large channel when the pore size is small. So, when pore size is small one thing can happen is that these molecules that are undergoing bulk diffusion, these are essentially colliding with the wall and traveling.

So, one can have a diffusion which is not exactly bulk diffusion because bulk diffusion means the wall does not feature in there, but there could be a situation where they are colliding with the wall and this is known as loops and diffusion and if this di- characteristic dimension is even smaller when the particle is at the say let us say the characteristic dimension is this much. So, that means, when the when the molecule is at the at the center of the channel, center of the pore it is still dominated by the wall effects ok. So, the interactions with the wall is still dominating.

So, in that case it will not be a bulk diffusion, but still the particle will travel from this side to the other. So, then that is referred as surface diffusion; that means, it is it remains within the effect of the wall ok. So, the wall effect is not interaction with the wall is still continuing and at the same time it is diffusing through the pore. So, there is this surface diffusion.

So, when the pores are large I can expect these viscous the this Darcy's law would be dominating definitely when the pores are large and bulk diffusion can always take place ok. Now, obviously, you can say that if I have water on this side, water on that side water is inside. So, naturally there is no question of diffusion can happen when I have a lower concentration here, higher concentration there, then only for example; I have the ink here, it is at higher concentration, it is a lower concentration, there is a concentration gradient; so there is diffusion taking place.

So, if you do not have that you do not have bulk diffusion if there is no concentration gradient as such, but Knudsen and diffusion you what ok, so you may not have the bulk diffusion in that case or there would be definitely diffusion where there will not be any change in concentration. There would definitely be molecules moving from here to from this side to that side, but there is there is not any net flow because of this.

So, now given this fact so, now, this is when the pores are larger this is what is what would be dominating and when pores are smaller, then probably these surface when the pores are very small the surface diffusion would be dominating.

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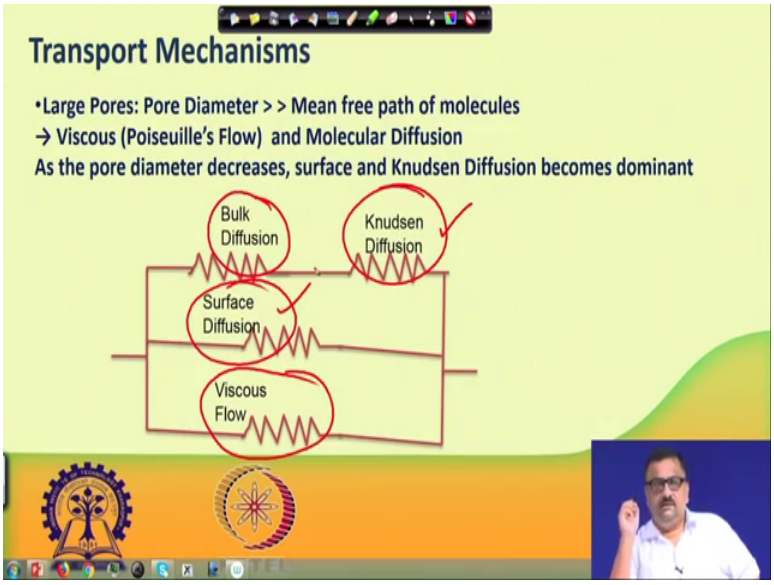

So, now with this understanding let us look at these. So, we had talked about bulk diffusion already, we had talked about bulk diffusion already, we have talked about Knudsen diffusion, we have talked about surface diffusion and we have talked about viscous flow. So, this is how the flow takes place.

So, when you have large pores what; that means, is pore diameter is much greater than mean free path of molecules in that case you have viscous Poiseuille's flow; this is the viscous flow as I mentioned one layer sliding against the other and molecular diffusion very much you one can have. Whereas, as the pore diameter decreases, as the pore diameter decreases surface and Knudsen diffusion becomes dominant ok. So, then viscous flow is not dominant, but these are dominant.

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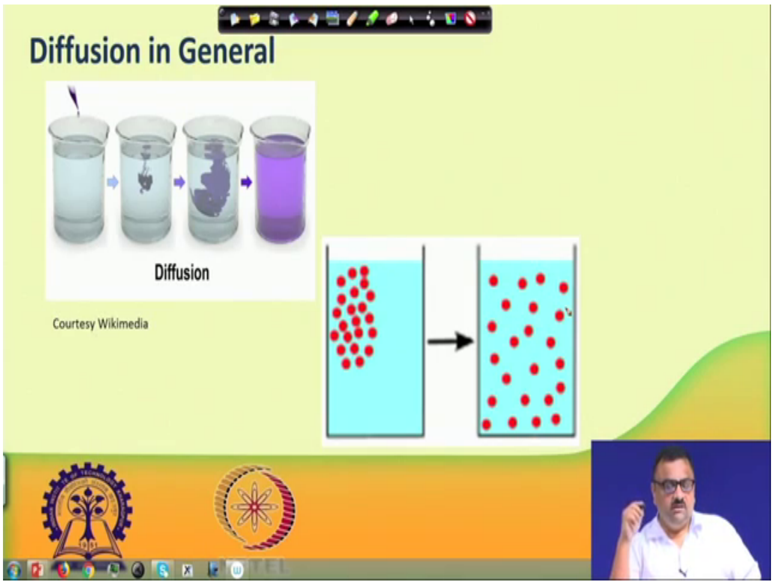

This is the definition of this is some picture of diffusion, you can see here that this water with a drop of ink as I said and the ink is traveling all over the beaker. So, there is a higher concentration and given some time you find they are being distributed everywhere. So, this is commonly referred as diffusion.

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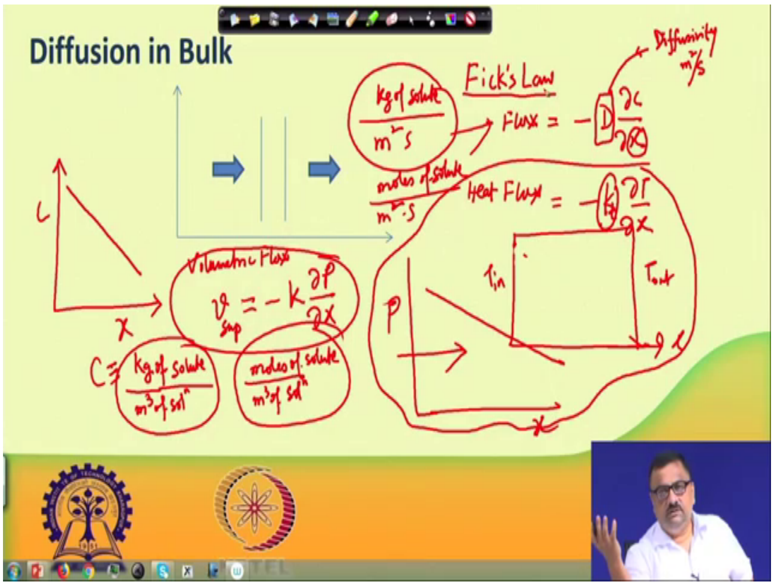

So, if we try to understand is diffusion a bit because viscous flow we have already you know cause and effect relationship and all we understood. Though I will touch upon again the genesis of this Darcy's law but, these diffusion part which we are now getting into slowly, I think we need to get some more feel about what we mean by diffusion. What we see here is that diffusion is there is diffusion takes place there is a famous Fick's law of diffusion Fick's law. The Fick's law says that the flux is equal to minus D del c del x in what way? We had we were familiar with these cause and effect relationship right. So, what cause and effect relationship we have studied so far? We have studied in case of heat transfer, we are writing heat flux as heat flux we write it as equal to minus k del T del x right.

So, these k I can put up since we are working with permeability all the time I put a subscript t so that this k t is the k t is the thermal conductivity. So, heat flux is equal to minus k t del T del x. So, del T del x is the temperature gradient. So, I have let us say a metal plate, I have this is at a higher temperature and this is at a lower temperature let us say T in and T out and conduction is taking place.

So, this is at lower temperature we will find that the temperature gradient would be something like this or in other words temperature gradient would be something like this; this is T and this is x, this is the direction x, so this is the direction x. So, we see that the del T del x is negative; that means, the temperature decreases with x and that makes sense because del T del x negative means heat flux is positive. So, heat will flow in the direction in which temperature decreases.

So, we have already seen this case of heat flux. We have already seen this case of see in this case of supervision velocity in Darcy's law, the volumetric flux, so we call it volumetric flux right. Superficial velocity is volumetric flow per unit area; volumetric flow rate per unit area ok. So, that is volumetric flux which is same as V superficial; V superficial is equal to minus k del P del x. So, here also I have a pressure gradient instead of T we can think of P, so pressure decreases we takes and the flow we takes place in that direction. So, in case of porous media we have seen this kind of Darcy flow for Darcy flow this is the case.

So, whenever we have similar cause and effect relationship we have one gradient and against that we have flow. So, similarly in case of mass diffusion we have an equation where flux is equal to minus D del c del x. So, here in this case we have concentration as a function of x. So, we have the concentration going down; that means, I have saline water and I have plain water. So, I can see salt from the saline water will go to the non saline water.

So, salt will gradually travel and at one point of time you will find the entire beaker would be saline, but it will less concentration. So, salt will travel from higher concentration to lower concentration, so this is so, del c del x here in this case is negative. Unit of concentration I can have two ways to work with unit of concentration; in one case it could be kg per meter cube of solution or it could be moles per meter cube of solution. But, this is not kg per meter cube means kg of solute not kg of water.

I mean suppose I say salty in water I am not talking about kg typically density of water is 1000 kg per meter cube 1 gram per cc. So, 1000 kg per meter cube is the density of water we are not talking about kg of water. We are talking about kg of solute kg of salt that you have added. So, that is probably 0.001 kg of salt you have added and so, that little bit of salt divided by meter cube of the solution.

So, either so, kg of solute per meter cube of solution this could be 1 unit or it could be moles of solute per meter cube of the solution, so this could be the other unit. And the flux is in that case you will have kg of solute per meter square second eta square meter square is the area flux means flow per flow rate per unit area; flow rate means flow per time per area. So, this is the unit of flux; so this is the unit of flux and I have the concentration, this is the unit of concentration I have already shown and D is refer D is called diffusivity or diffusion coefficient. So, diffusivity and this has a unit of meter square per second.

So, flux could be either kg of solute per meter square second or moles of solute per meter square second. So, it depends on which you need to choose for the concentration. So, the accordingly you have to place it and D remains as meter square per second because x has a unit of meter and here we have, so if you work out here it is per meter cube. So, if you work out with the meters you will find diffusivity, you will have a unit of meter square per second. So, this is how the flux is defined by Fick's law for diffusion. Diffusion means molecules are traveling due to Brownian motion and you will see from higher concentration the these solutes are traveling to lower concentration.

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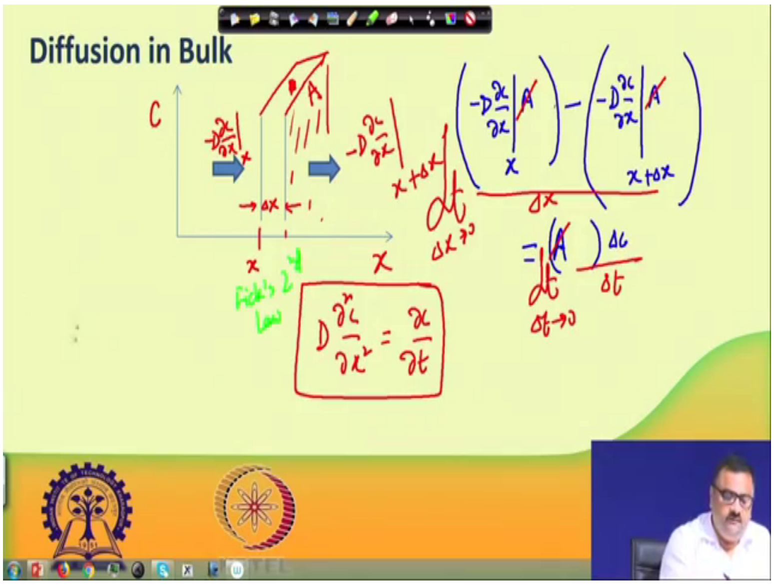

So, now if we if we look at case of say I have a concentration here as a function of x. Now, suppose concentration is flow the diffusion is happening from higher concentration to lower concentration. In that case the flux at this point, flux is minus D del c del x at this location which is let us say x and flux will be; so, this is minus D del c del x at x.

So, if I pick up a differential element; if I pick up a differential element which is delta x; delta x is the width of this differential element and it must be there must be area A perpendicular to the screen, so that area is let us say A. So, this area this there is an area here, this area is let us say A. So, there is a flux minus D del c del x at x and there would be a flow out which is minus D del c del x at x plus delta x.

So, this is how the flow is taking place and in this case you have. Now, if you if you want to draw some kind of mass continuity in this case how much of flow is going in from the left and how much is coming out from the right and the rest is the remaining part is the accumulation. So, then what you do in this case is what is going in from the left is basically minus D del c del x at x multiplied by A.

So, what do you what did you arrive at? We are talking about this as the flux minus D del c del x. So, minus D del c del x is the flux; flux means so many kg per unit area per unit time. Now, you are multiplied by area; area is perpendicular to the screen per that I have I have put a try to put a third dimension to show the area A and let us say we are talking about a time interval delta t. So, over time interval delta t this much of this many kg of solute not fluid kg of solute; kg of salt kg of solute that has traveled from the left phase.

And how much has left how much has gone from the right phase? It is minus D del c del x at x plus delta x A delta t. So, that would be the amount of solute that is traveling from the right phase. So, the remaining part, so, this is in minus this is out and that has to be equal to accumulation and the accumulation would be the volume of this differential element which is A delta x, this is the volume of this differential element multiplied by the change in concentration I can expect that as the salt is traveling from left to right as the salt is traveling from left to right. There is accumulation and that is how the concentration rises that is how the concentration rises.

So, let us say the change in concentration over time interval delta t; change in concentration in this differential element over time interval delta t is delta c; that means, to start with the concentration over c after delta t the concentration was delta c. So, what is the additional amount that has come into this differential element? This is the total volume and this delta c is the additional kg per meter cube.

So, this is unit of a delta x is meter cube, this is the additional kg per meter cube. So, this meter cube and this meter cube will cancel out, so you have additional kg. So, kg in minus kg out is the additional kg inside the differential element. So, in minus out is equal to accumulation. So, now, if you take this delta t to the right hand side so, if you take this delta t to the right hand side first of all I see that the A will cancel out; the A will cancel out ok.

Next thing is I would say take these delta x to the denominator of the left hand side and take this delta t to the denominator of the right hand side. So, if you do that; if you do that means, if we take this delta t out here and delta x out here and instead bring on the left hand side this delta x and here right hand side delta t and then we impose this limit delta x tending to 0 and here on this side we have this limit delta t tending to 0.

So, if we do this then you will end up with D is a constant which goes outside when you run this limit these becomes D del square c del x square that is equal to del c del t this is the master equation you arrive at. So, this is referred as Fick's second law; this is referred as fixed second law and this is an this is the equation that you that one needs for you know if one needs to find out the unsteady state change in concentration.

So, this is so mind one thing here I mean you just have to remember one thing that this the Fick's law of diffusion when you apply and when you apply it this way one assumption here is that the concentration is very dilute, I mean the if the concentration is much higher, then there are certain corrections that needs to be done it is it was that we that has with the assumption of dilute concentration we are working with this; working with these equations.

So, this becomes an important equation when it comes to buildup of concentration. Where we need this kind of equation? Suppose, I have a situation here as I said if we put a drop of ink the problem that we are talking about.

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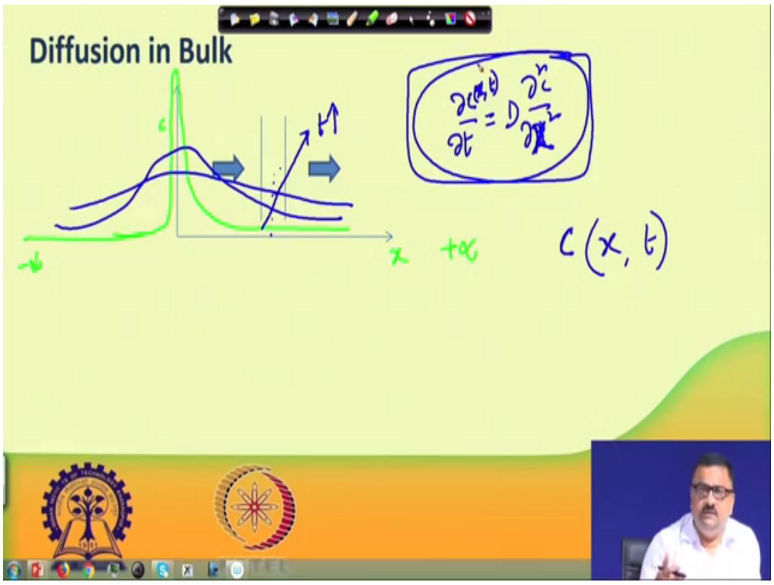

So, in that case if we put a drop of ink we are looking at if you think of the drop of ink problem we are looking at a Dirac delta type function; that means, at x equal to 0, you are giving a very high concentration and then and this side also it is the same thing. It is extended from I am treating these as a one-dimensional problem let us say an area perpendicular to the screen you have some A ok.

So, if we treat this as an one dimensional problem do you have put a concentration which is equivalent to a Dirac delta function, it is a very high concentration at x equal to 0 and then as time progresses you would see gradually there would be the change in concentration in the sense. This side the concentration would increase and the and the concentration at x equal to 0 is decreasing like this. So, if we look at this particular differential element which we are focusing on gradually we see that the concentration is rising as time progresses the concentration is rising.

So, if one needs to solve this kind how at what rate the concentration is going to increase, if one needs to solve one has to solve this equation of del c del t is equal to D del square c del z square. So, these equation one needs to solve or so this equation with appropriate boundary conditions will give you how concentration changes because here the concentration is a function of I am sorry we had written it as x naught z this our length unit is x. So, del c del t equal to D del square c del x square. So, here it is x and here it is concentration.

So, here this concentration is a function of space and time, concentration here is a function of space and time. So, if you; so here these equation if you solve this equation with appropriate boundary conditions you can get concentration as a function of position and time. So, if you fix your position; so, let us say I fix my position here, so the concentration will continuously increase with time as we see the concentration at this location we will increase with time right. At the cost of what? At the cost of concentration at x equal to 0 decreasing.

So, I have to find out what is the concentration as a function of x and time and this is the equation one needs to solve with appropriate boundary condition. So, now I am trying to get give feel that how these diffusion is handled because now we are going to club; now we are going to club the viscous flow, the Darcy's law, the Poiseuille's flow that we are talk talking about with the viscosity term featuring there one layer sliding against the other. And now, we are saying that we have some parallel mechanisms by which molecules can travel and these are the mechanisms and then we started talking about it. So, this so, here I this is where this is what I am then; so this equation one needs to remember when it comes to transport by diffusion.

I will continue this lecture in the continue this in the next session. So, here the take-home so, probably before you attend the next lecture, if you browse these concepts of diffusion, browse these concepts of diffusivity and Fick's first law second law it would be good because this would be used in the subsequent lectures . So, that is all I have for this lecture, I will continue this exercise in the next lecture.

Thank you very much.