Flow through Porous Media
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Lecture – 14
Mass Continuity (Superposition of Elementary Flow) Contd.

I welcome you to this lecture on Flow through Porous Media. We were discussing Mass Continuity and in that context we talked about complex potential and then how we can take two elemental complex potential and superpose them to arrive at some meaningful results, meaningful conclusions.

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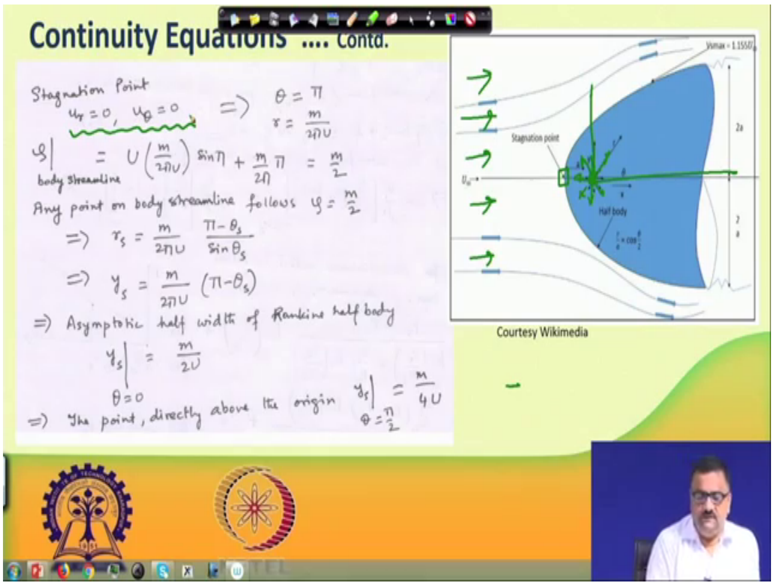

So, what we were discussing at the end of last lecture was that we said so, let us say I have a source here. So, this is the source; this is our origin and the source is located at the origin. So, source is emanating fluid like this.

So, then we are talking about in the last class that an envelope will be generated like this. There would be an envelope generated and this envelope; so, these are the stream lines how they will turn. There were uniform flows and these uniform flows would be then there would be this uniform flow and then because of the source there would be the twisting of the stream lines. Uniform flow had all this is the uniform flow, but now we see that these uniform flow they are being twisted because we have a source also acting here.

So, now, if we try to find out what are the coordinates of this stagnation point, we must point out here we said that here these two stream lines they meet and then the velocity is 0; velocity is 0 means we are talking about u r and u theta. So, u r and u theta they have to be separately 0. So, if u r and u theta they have to be separately 0, you can see that the our earlier expression there we have couple of choices I can quickly go to that expression here. We can see that we had talked about u theta as minus u sin theta and u r as u cos theta plus m by 2 pi r.

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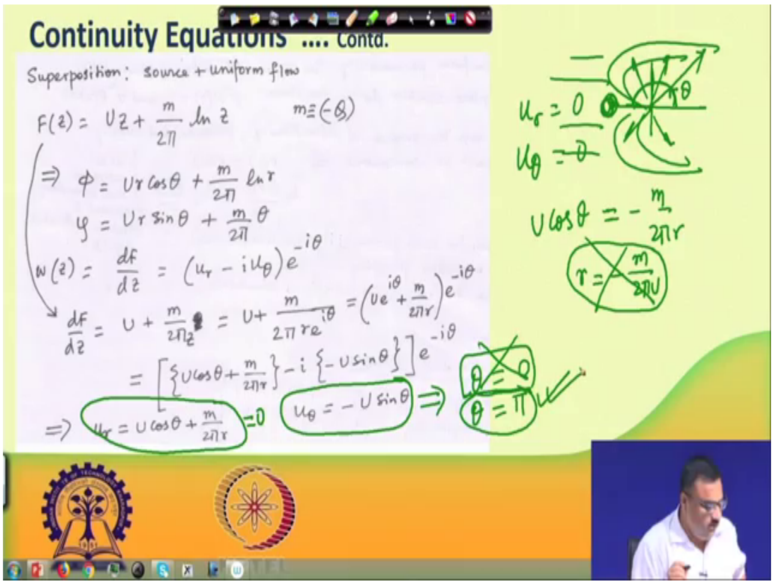

So, we if u r has to be 0, we are saying that at stagnation point u r is 0 and u theta is 0 separately. So, u theta is 0 this implies we can have two values of theta right; one is at theta is equal to 0 and another is at theta is equal to pi. And, here we have again in this case this u r has to be equal to 0. So, if u r has to be equal to 0, then we see U cos theta has to be equal to minus m by 2 pi r.

So, now, if we see here theta if we treat as 0, I have two choices from this expression. So, now if we take theta is equal to 0 here, then cos theta would be 1 and then this U would be equal to minus m by 2 pi; U would be equal to minus m by 2 pi r or we can in that case write r is equal to minus m by 2 pi U. Now, we are not comfortable with r being negative because our r is this is the origin and r is always positive. So, that is why this is not taken; that means, this theta is equal to 0 cannot be worked with. So, we are only left with the other choice theta is equal to pi.

So, where is theta is equal to pi located? This angle is theta and then theta is equal to pi means all the way here and which makes perfect sense because you remember if these were the radial flow outward I mean, this was the uniform flow here. We said that the stagnation point is located somewhere here because these streamlines were carving like this and stagnation point is located here. So, basically it is theta is equal to pi. So, this makes perfect sense.

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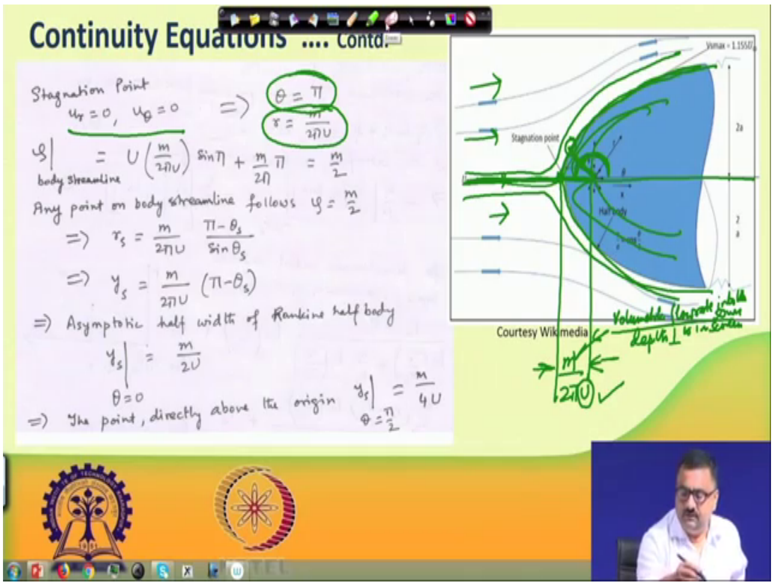

So, now this is what we were discussing where u r is equal to 0 and u theta is equal to 0 this means theta is equal to pi that is one thing and moment theta is equal to pi, r is equal to m by 2 pi U. Now, r need not have to be earlier r this was becoming minus m by 2 pi U which we could not work with, but now it is r equal to m by 2 pi U.

So, now, I have a way to find out how far is this distance. From the origin where the source is located and to the stagnation point this distance is basically m by 2 pi U because this is the point. At this point r is this much, this much is r and theta is this much; this angle is theta. So, at this point r is this much; this much is r and the theta is 2 pi. So, that is exactly so, this r is m by 2 pi U. So, now, we know that at m by 2 pi U distance to the left there would be the stagnation point.

Now, m by 2 pi U, what is m? m is the strength of that source; m is called strength of that source which is volumetric flow into the source; volumetric flow rate into the source into the source divided by depth of perpendicular to the screen right. So, this and what is U? U is the uniform velocity which you are submitting it to.

So, now, you have basically two – two parameters here; one is the strength of that source how at what flow rate you are putting in fluid here in this at the origin source and at what rate the fluid is flowing in the x direction. So, this U is already there, m is already there; m by 2 pi U is the distance at which you would see the stagnation point.

And, now if you want to find out what is the body streamline; that means, these are called body streamline. The streamline that follows the I mean this is the envelope all the streamlines are within this. So, this streamline has to; so you think of it, one line was coming so, the next one was following this. The next one on the north side is following this and the middle one is going there and it is blocked.

So, these streamlines; so, this is the basically this is a streamline. This line has come all the way here and traveling and since there is a symmetry in the problem we can only focus on the upper half considering this to be just a mirror image. So, if I just look at the upper half this is basically is referred as body streamline. So, this body streamline which is passing through the stagnation point you can write this psi body streamline that is equal to U into m by 2 pi U sin pi plus what exactly is happening here? We had what was the psi earlier we worked with?

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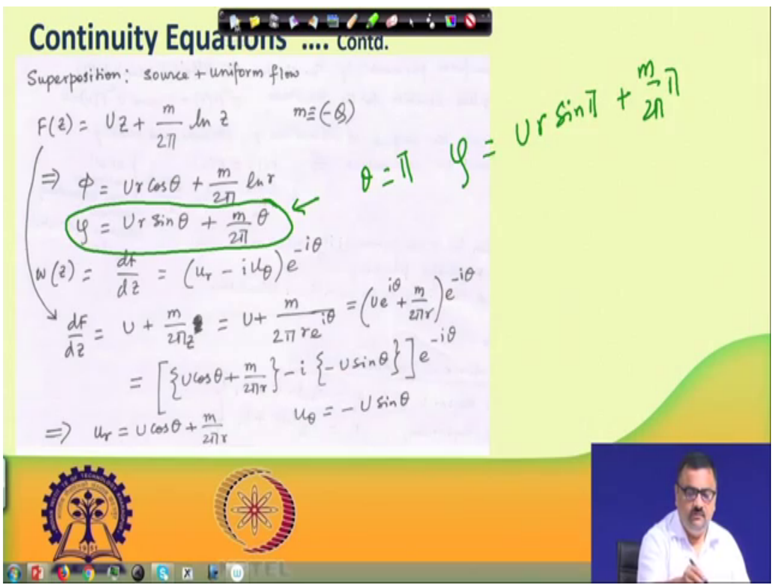

If you recall what; if we recall what we worked with as psi at that time was, what was the psi at that time? U r sin theta plus m by 2 pi theta. So, U this was the psi; U r sin theta plus m by 2 pi theta ok so, this is the equation. So, here if we put theta is equal to pi, what do we get, then psi becomes equal to U r and sin of pi plus m by 2 pi into pi. So, when if you. So, this is this is the equation for the stream function.

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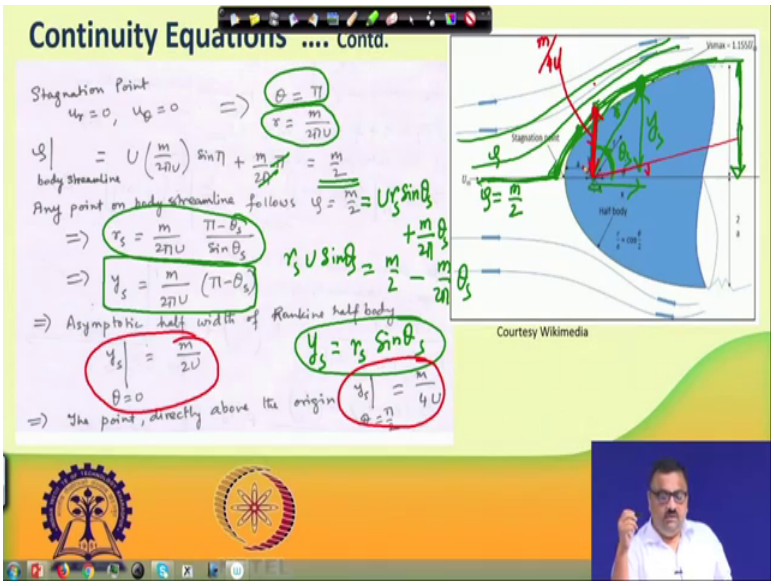

So, now if we go to this expression here, we see that U into m by 2 pi U into sin pi right, U r sin theta plus m by 2 pi theta that is what we learnt as psi. Now, instead of theta we are putting it as pi and instead of r we are putting it as m by 2 pi U. So, U into m by 2 pi U sin pi plus m by 2 pi into pi. So, that becomes equal to this pi and this pi will cancel out and sin pi is equal to 0. So, you are ending up with psi for body streamline as to be equal to m by 2.

So, that means, when we talk about this upper symmetric half and when we are talking about this particular streamline which is called body streamline so, every streamline has some stream function value right. So, this is at some psi value, this is at some other psi value because along the stream line the stream function has to be constant. Now, for this body stream line the value of stream function is m by 2.

So, now, if so, at any point let us say this point, this point will also have this same stream function of m by 2 and let us say this we are referring as at any point we are referring these as r s which is the radius on this streamline. So, r s and let us say this angle is referred as theta s. So, then in that case we can write these r s is equal to m by 2 pi U pi minus theta s by sin theta s. How we are ending up with this? We are writing on one hand psi is equal to m by 2.

So, this has to be; this psi has to be satisfied for this point, for this point, for this point any point on these streamline. So, let us say I am I picked up this point. So, this point the psi is supposed to be what psi is supposed to be U into m by U into, sorry, one second. What is what is the what is the psi at this place? It would be U into r s sin theta s plus m by 2 pi into theta s that is supposed to be just value of this stream function here.

And, now I am forcing it that though I have already seen that this psi is equal to m by 2, the psi the stream function for this particular stream line is m by 2 and for any arbitrary point if this is the equation of stream function that we have already seen earlier so, that has to be satisfied. So, what; that means is, what is the relation between r s and theta s we must satisfy if psi everywhere has to be m by 2 for this particular stream line. So, then m by 2 has to be U into r s sin theta s m by 2 pi theta s.

Now, if this is the r s theta is combination we have, we could have this point then we will have some r theta; either we can have this point we will have some r theta, we can have this point we have theta as 2 pi and r s m by 2 pi u. So, all these the stream function we has to be m by 2. So, by equating this we can find out r s is equal to m by 2 pi U pi minus theta s divided by sin theta s. If we equate this to m by 2 then if we simplify this, what we have is r s U sin theta s that is equal to m by 2 minus m by 2 pi theta s. So, if you now work with; now if we work with you if you find out what is r s you will have r s is equal to m by 2 pi U pi minus theta s divided by sin theta s.

So, now if this is the r s; if this is the value of r s what would be y s? y s is what; y s is this quantity this height this is y s and this is x s right. If we if we from r theta if we travel to x y then this becomes y s. So, then y s becomes equal to m by 2 pi U into pi minus theta s, because y s is simply y s would be r s sin theta s ok. So, if you take this r s and then multiply with sin theta s this will can this sin theta as will cancel out and you will have m by 2 pi U into pi minus theta s.

So, you can find out here if this is the y s, and if this y s comes to these; y s is m by 2 pi U into pi minus theta s, then how much would be the y s for this point? How much would be the y s for this point and how much would be the y s at infinity? So, at this point the stretch is y s at theta is equal to 90 degree, theta is equal to pi by 2. So, we can see the point directly above the origin, y s theta is equal to pi by 2 is m by 4U.

What that means is, these particular height these particular height is m by 4U; m is the strength of the source and U is the, that U refers to the uniform velocity. So, y s at theta is equal to pi by 2 that is equal to m by 4U. So, these height is m by 4U. Whereas, if we look at infinity we are looking at infinity, so, at infinity you will see that theta tending to 0 when you are approaching infinity. When you are approaching infinity theta tends to 0 so, this asymptotic half width of Rankine half body here is y s at theta is equal to 0.

So, if you put theta equal to 0 in this term, in this equation y, in this equation for y s, you will get m by 2U this pi theta s is 0 pi n pi will cancel out. So, you have m by 2U. So, what that means, is at infinity the stretch that you have is twice the stretch that you have at this location. So, this is something an observation you have in this case, that y s is equal to m by 2U here and this is equal to m by 4U.

So, you can see how far I mean, if somebody is disposing some water in the through this well and there is a uniform flow. So, in that case how far this disposed water will travel at the location where you are putting it. So, that means, this how far it will be stretched in this direction, how far it will be stretched in this direction and this side bottom side is just a mirror image. And at infinity if you continue how far would be this would be traveling?

Obviously, this here we are talking about flow we are not considering a diffusion a lot of times the fluid that is outside in the fluid that is inside there could be you know Brownian motion and there could be molecules traveling to the other side. So, this layer would become fuzzy, if diffusion is permitted and if it is not very much miscible then this situation does not arise. So, you will have this height is equal to, this height is double the height that you see at the origin. So, this is what we have when we have this Rankine half body.

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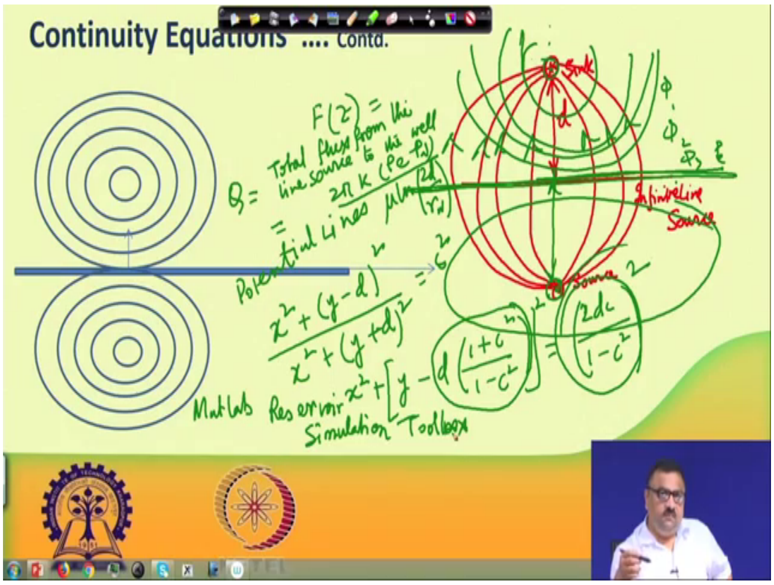

There could be other such situations for example, you can have let us say we have a situation where you have a infinite line source and you have a sink; you have a well from which you are drawing fluid ok. So, here is a infinite line source maybe a river, maybe some line source is there and here you have you are drawing the fluid at this location and this is a porous medium.

So, if we try to find out what would be the stream lines and what would be the velocity, how much would be the flow rate and all these things. So, in that case you would be doing what you will doing is generally what you do here this is a sink. This line source is not sink we can understand that is m by 2 pi ln z here it that we have shift if you put this as the origin, then z minus z 0 has to be considered. But, this infinite line source generally this infinite line source is not considered instead you put a mirror image here.

If this distance is d if this distance is d; then at d distance from here that means, here you put a source. You put a source at d distance from here and then you consider these the flow from source to sink. So, what would be the flow then? From source to sink there would be a flow like this. So, now, you remove this part, forget about this part. This part is completely hypothetical this part is completely hypothetical; this part is completely hypothetical. This part is hypothetical, but using a source; so, basically you have to superpose two flows now. One is for the sink and the other is for the source.

So, here in this case you have F z so, now, you have one case it is m by 2 pi another is case it is the same, but with minus m. So, sink is minus m and this is plus m and there has to be one is z minus; that there has to be you have to consider is z minus z 0 here because one is d distance apart another is minus d distance apart ok. So, if you put this now if you superpose these two flow, then you can generate the type of streamlines that one would have seen if you had a infinite line source and a sink ok.

In other also it could very well be possible you are working with an with an electrode and in that in the electrode you are the flow for the fluid is put here and then you, it is there is a porous electrode and the flow and there is this channel where it is being collected. So, you can have a source and the infinite line sink, in that case you have to consider a sink and there would be a flow in this direction.

So, this is the way it is this superposition will work out. You can even come up with different complex situations there. And then here you have if you really look at the type of equation you will end up with it would be something like this, x square plus y minus d whole square divided by x square plus y plus d whole square that is equal to c square. I am not giving you the derivation I just give you a feel for what it that is. And, now this equation can be converted to x square plus y minus d into 1 plus c square by 1 minus c square whole square that is equal to 2 dc divided by 1 minus c square whole square.

So, this is giving you this if this is the equation c is some constant so, then in that case you can see the type of streamlines and potential lines that have been generated because of these is so, this is the equate; this is the equation for potential lines. This is the equation for potential lines the equations that the equations that I mentioned. So, the potential lines in this case would be then circles, but circles with origin being shifted as I change c; c refers to see I have phi is equal to phi 1, phi 2, phi 3 like this right. So, this the c will also change c 1, c 2, c 3.

So, what you would see is that as it is changing you will see that you are looking at the potential lines are like this. These are the potential lines in this case, and the potential lines will have the radius of the circle is this; radius of the circle is 2dc mind by 1 minus c square and the origin of these circles that defines these potential lines is this quantity, 1 plus c square by 1 minus c square d into 1 plus c square into 1 minus c square. These term is this is the this is the origin of the circle shifted from our reference point.

So, this is how these; so, you will have circles so, as you go the circles they will not have the same origin the origin is being shifted; origin is being shifted and the radius also being shifted. So, you will have these are the potential lines and streamlines will be perpendicular to this. So, one can work out and then finally, with these information these basically potential lines are same as isobaric lines the pressure it is basically the lines along which the pressure is constant.

But, if somebody the when they worked with this they found that Q in this case, Q is total flux. For total flux from the line source to the well the total flux is given as 2 pi k P e minus P w divided by mu ln 2 d by r w; r w is the radius of the well, d is the distance between the line source and this sink and P e is the equilibrium; P e is the pressure far away. So, sorry the P e is the pressure of the line source.

So, now the; so, this is the way you can you know put superpose two flows and you can generate various situations. These are the analytical way of doing things one can use simulators for doing the same thing. For example, I can give you an example here of a simulator which is MATLAB reservoir simulation toolbox. So, it is simulation toolbox; it is downloadable freely, but one needs to have access to MATLAB to do to run this reservoir simulation toolbox.

And, MATLAB reservoir simulation tool box will do similar things. You have multiple wells, you have patterns and their how the streamlines are how the flow what would be the flow at various places. So, these reservoir simulation toolbox they do very similar things, they have tutorial problems etcetera. So, this is other than analytical ways one can resort to of course, one can solve these equations numerically that is one way of doing things and the other is resorting to reservoir simulation toolbox.

So, in this lecture I tried to summarize what all ways one can address the continuity and what are the ways one can extract all the experience all the wisdom, all the knowledge people have with the continuity and apply them in flow through porous media, and get some good visual, get some quick analysis.

So, I am going to close this lecture at this point and in the from next lecture I would be looking at a few other things not exactly continuity. Now, next what do you do is we would look at the flow equations we have only talked about Darcy's law, if there are any other way to explain flow through porous media. So, then we will get into that, but in two-dimensional, three-dimensional, cylindrical, Cartesian so, that part I think I have given you a good enough of exposure.

So, that now you can carry on with this and try to solve these equations numerically or get to this MATLAB reservoir simulation tool box or there are several other in fact, petroleum engineering and the reservoir engineering, hydrology. There are good number of simulators available to simulate this kind of flow. So, this is all I have as far as the continuity mass continuity section is concerned.

Next I will be talking about all the fundamentals of all the other I mean alternative to Darcy's law what all we have, we will get to that now from the next lecture. So, that is all I have for this lecture.

Thank you very much.