

Heat Transfer
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Lecture - 55
Radiosity Blackbody Radiation Exchanges, Relevant Problem

We are going to start with new concept in this class which is known as Radiosity. So, what is radiosity? When you see a surface, let us say when you are looking at a hot surface which is in an enclosure, but the side walls are also hot. So, when you are looking at the surface, if you do not have the enclosed side walls, you are going to get some amount of energy which is coming out of the hot surface. This is due to emission an emission is a volumetric phenomenon.

So, because of its temperature there would be some amount of heat flux, some amount of radiative heat, which is coming from the surface and that is the emissive power of the surface, depending on whether it is a blackbody, or not a blackbody by the introduction of emissivity you can predict what is the heat flux emissive heat flux coming out of the surface.

So, using emissivity and the Stefan Boltzmann law you can say that the emissive power of the surface, which is at an absolute temperature of t would simply be equal to $\epsilon \sigma t^4$, if this is a blackbody ϵ is equal to 1 and the emissive power is simply σt^4 . Now, let us say this hot surface is enclosed by another surface which is even hotter than the surface that, we were considering. So, that is going to be some amount of the second surface is also going to emit the emission will take place from the second surface. Part of which is going to fall on the first surface and we know how to calculate that part of energy which comes from surface 2 that falls directly on 1 by the incorporation of view factor, but that is besides the point.

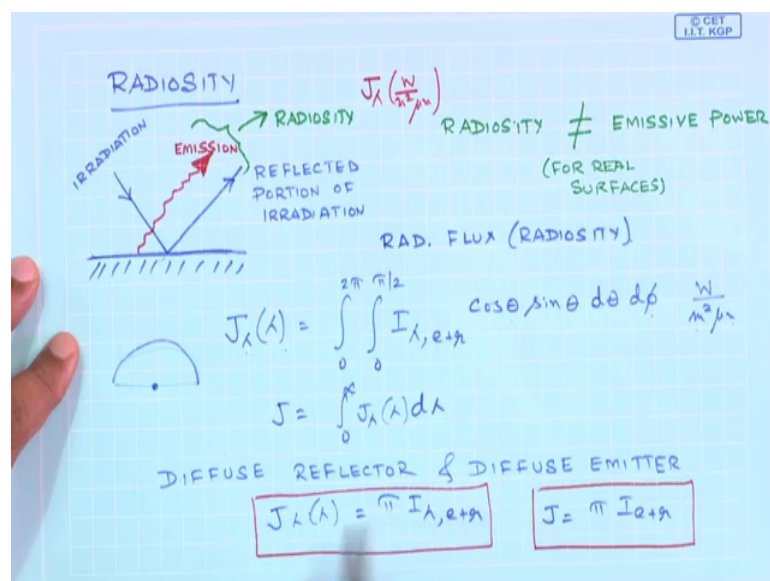
Let us say that we have the surface 2 which is emitting and this emission is going to strike surface 1, which is emitting anyway when that happens, if this is a real surface then and let us assume that it is an opaque surface. So, part of the energy which is incident from 2 on 1 is going to be absorbed and part of it is going to be reflected. So, if you are an observer and you are standing just outside of surface 1, what you see coming

out as radiative energy from surface 1 is not the emissive power alone, you also have to consider whether or not the surface is reflecting some amount of incident radiation.

So, what you see while standing outside of the surface is different from the emissive power or emissive power of the surface. So, what you see on the outside, just outside of the area which is emitting and reflecting all at the same time is called the radiosity. So, the radiosity refers to emissive power of the body and how much of incident emission it is reflecting.

So, it is going to be a function of what is the value of the; what is the value of the incident radiation and what is the value of the reflectivity of the surface together, they would give you an idea of the emissive potential of the body, that is the emission potential of the of the surface, which is a sum of its inherent emission and the reflection of incident radiation that is what is known as radiosity. So, whatever have described so far.

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If you look at the figure I think it is it will become even more clear to you is I have a real surface which due to its temperature is emitting so, this is the emission and there is some irradiation, which is coming on to the surface from another nearby surfaces and its going to reflect it reflect portion of the irradiation. When you combine this together if you are the person who is standing over here, you not only sense the emission you also sense the reflected portion of the irradiation and this is what is called the radiosity.

So, radiosity normally generally is not equal to the emissive power, for real surfaces. But if it is if the case is such that the surface is not reflecting anything, whatever is irradiated on the surface it is going to absorb that without reflecting any portion of it. So, which; obviously, I am not disturbing a blackbody in this case. So, for a blackbody for the special case of a blackbody there would not be any reflected portion of radiation and emissivity will solely be constitute the radiosity of the surface.

So, for a blackbody the emissive power and the radiosity are equal, but for real surfaces the blackbody real surfaces the radiosity and emissive powers are different. So, the radiation flux which is radiosity is based on the actual surface area. And radiosity is generally denoted by J , if it is spectral its watt per metre square per micron and from the spectral, you can find out what is the overall what is the total value of radiosity, since this λ this denotes the spectral property. So, that is why its emissive power watt per metre square power per unit area per micron, since I have the spectral property over here.

So, this J_λ for any λ is equal to 0 it is connected with the intensity, as I the intensity the spectral and this contains both emission and radiation that is the significance of e and r over here. So, I am trying to relate the spectral radiosity to the spectral radiative in intensity and since it is the radiative intensity it should contain both the emissive part and the reflected part. So, when we are trying to find out the so, this is directional and spectral; this is directional and spectral intensity.

So, now, I we know how to do this for; how to find out how to perform this, somehow I need to take this out of this and once I have this J_λ this the total emissivity is going to be 0 to infinity J_λ which is the function of λ times $d\lambda$.

So, once again I will explain what I have written over here, this is the intensity of radiation, I refers to the intensity. Since I have the λ over here, this is the spectral intensity of radiation coming from a surface. And since it is coming from a real surface I need to consider both the emissive intensity and the radiative intensity of this. So, this is the spectral directional radiation intensity from a surface.

From this I would like to convert this to spectral only. So, I need to take care of the direction part over here and as we have done before this J_λ should denote the emissive the radiate radiosity of a surface, when of a surface of unit area, when it is placed at the centre of a hemisphere at the centre of a hemisphere. So, how do I do that? I

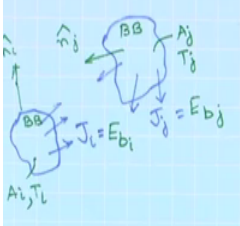
integrate over all possible theta and phi. And therefore, this should give me the emissive power per unit wavelength that is why I have the spectral.

Once I have the spectral one, then I should be able to find out by integrating it from 0 to infinity, what is the overall radiosity of a surface by integrating over all possible wavelength. The integration the first integration becomes easy to perform, if we assume that its diffuse reflector and diffuse emitter, if the surface that we are talking about is a diffuse reflector and if it is a diffuse emitter; that means, after the whatever comes out of the surface it does not have any directional property and whatever is incident on it after reflection, the radiation has no directional property, then $I_{\lambda} e_{\lambda} + r_{\lambda}$ does not depend on theta and phi.

So, this can be taken outside of the integration sign and the integration can be performed for $\cos \theta \sin \theta d\theta d\phi$ between the limit 0 to 2π and 0 to $\pi/2$. So, once we do that we know that the result for that we have said before the result for that would be equal to π . So, therefore, J_{λ} the spectral radiosity is simply going to be equal to π times I_{λ} , this is first relation and the second relation; obviously, is going to be J to be equal to π times $I e_{\lambda} + r_{\lambda}$, this is the overall radiosity and this is the spectral radiosity, but more importantly is the concept that what radiosity is that it is some of these two.

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BB RADIATION EXCHANGE



$q_{i \rightarrow j} = (A_i J_i) F_{ij}$
 RATE AT WHICH RAD. LEAVES 'i' & IS INTERCEPTED BY 'j'
 $q_{j \rightarrow i} = (A_j J_j) F_{ji}$
 NET RATE OF EXCHANGE BETWEEN 'i' & 'j'
 $q_{ij} = q_{i \rightarrow j} - q_{j \rightarrow i} = (A_i J_i) F_{ij} - (A_j J_j) F_{ji}$
 $J_i = E_{bi}, J_j = E_{bj}, A_i F_{ij} = A_j F_{ji}, E_b = \sigma T^4$
 $q_{ij} = A_i F_{ij} \sigma (T_i^4 - T_j^4)$ FOR AN ENCLASURE $q_i = \sum_{j=1}^n A_i F_{ij} \sigma (T_i^4 - T_j^4)$

The next step which we are going to do is blackbody radiation exchange, what is blackbody radiation exchange? Let us say I have two blackbodies. So, these are all

blackbodies and this has an area A_i , which is at a temperature of T_i , this has an area A_j which has a temperature of T_j . So, this is let us say the area vector and the area vector here is n_j , it does not matter whatever will be the area vector.

So, when an observer sees the blackbody it sees not only its emissive power, it sees also whatever emission coming from J getting reflected from b and therefore, it is what it see is coming out as emissions as J_i , which should be equal to, we are going to find out what this J_i is similarly the emission which is coming out from over here is going to be the radiosity of the black of the object.

For the special case where the body is a blackbody it is not going to reflect anything. So, for this special case of a blackbody the radiosity and emissive power are the same. Now, these two are blackbodies so they are only going to emit energy based on their inherent temperature, not on any reflection from incident from any nearby from any object from any other surface.

So, for surface i and for surface j the two blackbodies, who are interacting with each other, we know that we have to take into account the radiosity, because that is the actual potential of the surface in terms of its radiative power. So, it is not just the emission it is a reflection as well. So, if you think of the emission, emissive power of an object of a surface, it is the radiosity and not the emissivity, but since we are dealing with a blackbody the emissive power and the total radiative power which is the radiosity they are equal, because the absorption is going to be total and reflection is going to be equal to 0.

So, for this case only since we are dealing with blackbody this J_i is simply J_i is simply going to be equal to E_{b_i} and J_j is going to be equal to E_{b_j} . So, which tells us that the radiosities are going to be; radiosities are going to be replaced by E_{b_i} and E_{b_j} so, $q_{i \rightarrow j}$; that means, rate at which radiation leaves surface i and is intercepted by j , what is this going to be? It is first of all the amount of energy which comes out is A_i times J_i , because J_i is unit, J_i is it has units of energy per unit area so, I multiply it with corresponding area and I get the total amount of energy which comes out of it, comes out of surface i or these the area of the object having area A_i , but not all of it is going to strike j the fraction which is going to strike j directly is $F_{i \rightarrow j}$.

So, this entire thing; obviously, gives me the rate at which radiation which leaves i is intercepted by j . Similarly you can write $q_{j \rightarrow i}$ as $A_j J_j$ times $F_{j \rightarrow i}$. So, this should be the radiation at the rate at which radiation which leaves j and is intercepted by i . So, this is the view factor of j to i and I think these are must be clear to you now. So, the net exchange between 1 and 2, net rate of exchange between i and j , if I call it as $q_{i \rightarrow j}$ would be $q_{i \rightarrow j}$ minus $q_{j \rightarrow i}$ that is the net rate of exchange which we would like to find out.

So, what is this? This is $A_i J_i F_{i \rightarrow j}$ minus $A_j J_j$ times $F_{j \rightarrow i}$, now $A_i F_{i \rightarrow j}$ is equal to we also we know that $A_i F_{i \rightarrow j}$ is equal to $A_j F_{j \rightarrow i}$ which is nothing, but the reciprocity relation and we also understand that J_i is simply equal to $E_{b \rightarrow i}$ and J_j is simply equal to $E_{b \rightarrow j}$. Since this is a blackbody and we also understand that E_b for a blackbody is simply equal to σT to the power four this is Stefan Boltzmann law. So, if E_b is equal to σT to the power 4 J_i is $E_{b \rightarrow i}$ and $A_i F_{i \rightarrow j}$ is equal to $A_j F_{j \rightarrow i}$. So, when you put all of them together what you are going to get is $q_{i \rightarrow j}$ is $A_i F_{i \rightarrow j} \sigma T_i$ to the power 4 minus T_j to the power 4.

So, if you look at this I have simply substituted $A_j F_{j \rightarrow i}$ by $A_i F_{j \rightarrow i}$ which I can write from the reciprocity relation J_i and J_j I have taken to be $E_{b \rightarrow i}$ and $E_{b \rightarrow j}$ and I know that E_b is equal to σT to the power 4. So, this is a very important relation which tells us what is the net rate at which the i th surface, the surface i is going to receive energy from the net energy exchange between surface i and surface j , when both of them are blackbodies.

So, if I apply this for an enclosure the energy which we i surface i is going to exchange is going to be j equal to 1 to n . So, this is what; obviously, you can see this rule to be an extension of this one. So, this is extremely important relation which we have done it for the blackbody, but we will see how real surfaces can be handled in the remaining time I will quickly solve a problem.

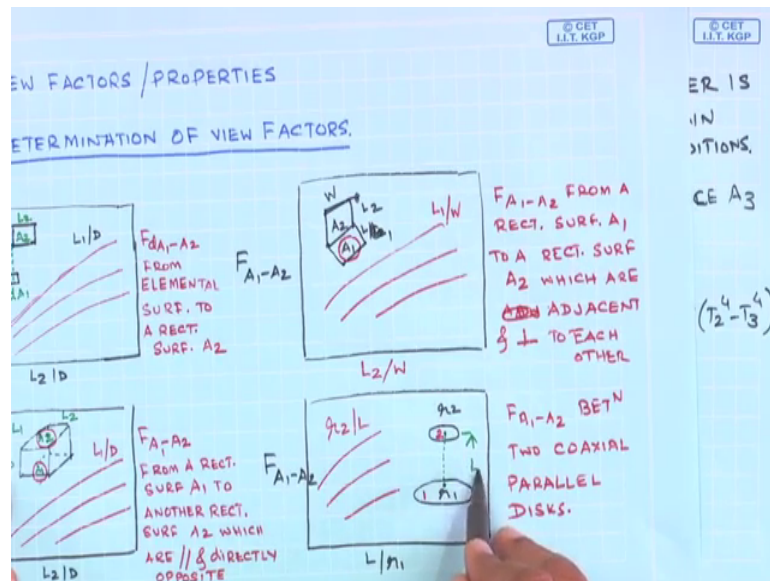
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$T_{SUR} = 300$
 CYLINDRICAL FURNACE $L = 0.15$
 $D = 0.075$ m
 $A_1, T_1 = 1350^\circ C$
 $A_2, T_2 = 1650^\circ C$
 ALL SURF. ARE TREATED AS BLACK
 FIND HOW MUCH POWER IS REQUIRED TO MAINTAIN THE FURNACE CONDITIONS.
 HYPOTHETICAL SURFACE A_3
 HEAT LOSS
 $q_v = q_{13} + q_{23}$
 $q_v = A_1 F_{13} \sigma (T_1^4 - T_3^4) + A_2 F_{23} \sigma (T_2^4 - T_3^4)$
 FROM FIG $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$
 FOR $g_{12}/L = 0.0375/0.15$, $L/g_{11} = 0.15/0.0375$
 $F_{23} = 0.06$ FROM SYMM. $A_1 F_{12} = A_2 F_{21}$
 $F_{23} + F_{21} + F_{22} = 1 \Rightarrow F_{21} = 0.94$ $F_{12} = 0.118$, $F_{13} = F_{12}$
 $q_v = (\pi \times 0.075 \times 0.15) \times 0.118 \times \sigma [(1623)^4 - (300)^4] + \frac{\pi (0.075)^2 \times 0.06 \times \sigma [(1350)^4 - (300)^4]}{4}$
 $q_v = 1844 \text{ W}$

So, this is a cylindrical furnace whose length is 0.15 metre, diameter is 0 point this is open to the surrounding temperature the curved area is 1, this is your A 1 then which has a temperature 1 3 5 0 centigrade and A 2 which is this surface its temperature T 2 is 1650 degree centigrade and this is 300.

So; obviously, some you know energy is to be supplied to 2 some energy is to be supplied to wall to maintain them at these two temperatures and they are going to lose energy through this enclosure. Let us say I am putting as hypothetical surface which is A 3, I need to find this is what we have to find out. So, hypothetical surface A 3, this is the one that, we have obtained, the heat loss this is clearly I am going to lose the some heat from 1 to 3 and some heat from 2 to 3. So, this is the heat loss and q then q 1 3 would be A 1 between 1 and 3 A 1 F 1 3 sigma T 1 to the power 4 minus T 3 to the power 4, because of the relation which we have obtained just the in just recently plus q 2 3 would be A 2 F 2 3 sigma T 2 to the power 4 minus T 3 to the power 4.

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Now, from the figure that we have done in the last class, it is a case like this, where r_1 and r_2 are same and you know what is the length, what is the distance between the them. So, here r_2 and r_3 are the same and you know what is the distance between these 2 as 0.075 metre so, you should be able to obtain using the figure from figure for r_j by L to be equal 0.0375 by 0.15 0.15 over here and L by r_i to be equal to 0.15 by 0.0375 you find out F_{23} to be equal to 0.06.

And F_{23} plus F_{21} is this surface plus F_{22} is equal to 1, since it is a plane surface this is flat surface, this is equal to 0. So, you can find out F_{21} to be equal to 0.94. So, F_{21} is known to you now, from reciprocity relation $A_1 F_{12}$ is equal to $A_2 F_{21}$. So, therefore, F_{12} can be calculated after you put the values of A_1 and A_2 , this to be calculated as 0.118. And from symmetry from symmetry F_{13} to 3 must be equal to 1 to 2. So, when you do all these so therefore, you know what is F_{13} from here F_{13} is F_{12} , F_{12} is 0.118 F_{23} , you have calculated from here F_{21} you have also calculated.

So, therefore, the total q would simply be the area over here. So, π times 0.075 into 0.15 which is πd_1 times F_{13} , F_{13} is 0.118 into σ which is the Stefan Boltzmann constant and 1623, you have to be very careful that always the temperature is going to be in absolute temperature in kelvin minus 300 to the power 4 plus the second area which is a circular area. So, its π by 4 0.075 square times the view factor which is 0.06 times σ times 1923 to the power 4 minus 300 to the power 4 and the value of σ ; obviously, you can see its 5.67 into 10 to the power minus 8 watt per metre square Kelvin to the power 4.

So, when you do this the value of q turns out to be 1844 watt. So, this is a simple example which shows how much of heat is lost by a cylindrical furnace, but if you think of the utility or application of this, this is very significant so, any cylindrical furnace let us say this side is not properly insulated it is going to lose some energy. And at this high temperature the energy the mode of exchange of energy is going to be radiation. So, using the formula and we I have assumed all the surfaces the assumption here is all surfaces are treated as blackbody.

So, all surfaces are treated as blackbodies. So, I have in the first part of the class I have found out what is the exchange of heat between two blackbodies at different temperature. So, when you introduce the fraction the fraction energy; that means, a view factor concept, this is what you have get we have obtained, then from the figure of view factors calculate what is F_{23} , $F_{2 \rightarrow 3}$ and once you have $F_{2 \rightarrow 2}$ to $F_{2 \rightarrow 3}$ using the summation relation, you can find out what is $F_{2 \rightarrow 1}$, because that is going to be important not only $F_{2 \rightarrow 1}$ is going to be important $F_{1 \rightarrow 2}$ is also going to be important $F_{1 \rightarrow 3}$ is also going to be important as well.

So, using reciprocity relation and the known value of $F_{2 \rightarrow 1}$, I calculate what is $F_{1 \rightarrow 2}$ and from symmetry I find out what is $F_{1 \rightarrow 3}$. Therefore, my q I have obtained all the parameters required for finding out the total loss of energy A_1 and A_2 from geometry $F_{1 \rightarrow 3}$ and $F_{2 \rightarrow 3}$, $F_{1 \rightarrow 3}$ and $F_{2 \rightarrow 3}$, we have obtained from these relations the last thing I need to mention stress again and again that all temperatures to be used must be in kelvin in radiative heat exchange. So, you plug in the values and what you get is the total heat loss from the furnace, through the opening, through the top opening which is exposed to atmosphere at 300 Kelvin.

So, this is a very simple case of an enclosure problem in which all surfaces are black and therefore, you do not need to introduce the concept of radiosity, but always keep in mind that radiosity is the correct potential; correct radiative potential true radiative potential of a surface, because it takes into account the emissive power as well as the part of the energy which is reflected from the surface itself.

So, one emission which is a volumetric phenomena, depends on the temperature inherent temperature of the object. And the second is a surface phenomena which depends on the radiative property reflectivity of the surface, together they form the radiosity. Calculate

what is the radiative heat exchange between two blackbodies at different temperature? Invoke the view factor concept you get a compact relation.

So, to apply those relations into real problems, you need to know the view factor which you can read from the graph for such enclosures. Once you have that view factor and if once you have used the summation and the reciprocate relations, then you will see that you could find all the view factors which are to be used to calculate the net heat exchange from one or more of the surfaces to another. And that is precisely what we have done in this specific problem, where all the enclosed areas are assumed surfaces are assumed to be blackbodies such that radiosity is equal to the emissive power.

What we would take up next class from the in the remaining two classes or three classes is that, when we are dealing with real surface, where the surface potential or the radiosity is different from the inherent emissive potential of the surface due to its temperature. So, we will take the cases of non-blackbodies to be more precise, we will take the cases assuming the bodies are gray, such that ϵ is equal to the α this this relation the Kirchhoff's law, the most relaxed form of Kirchhoff's law is valid.

And from there we try to form a network of potentials and resistances and try to see what is the current; that means, what is the heat which flows between these nodes, each node representing an area of an enclosed surface. So, those are very interesting calculations and you would see lot of similarity with electrical technology, the way these radiation problems for an enclosure are treated which we will do in we will, which we will start from the next class.