

Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 53
Solar Radiation and the Concept of View Factors

So, therefore, we have now a fair idea of what is emissivity and what is absorptivity. The same way we have obtained the expressions for absorptivity, the expressions for reflectivity and expressions for transmittivity can be obtained. Exactly the same relation except that the spectral absorptivity values are to be replaced by the spectral reflective spectral transmittivity values and so on.

In all 3 cases; that means the absorptivity, the transmittivity, and the reflectivity in all these cases the incident radiation from a source is to be taken into account, unlike the case of emissivity whereas, the surface temperature itself is of primary concern. So, that is the only difference that we have in the definition of emissivity and the 3 quantities which characterizes the which are the absorptivity reflectivity and transmissivity these 3 depend on incident radiation whereas, the emissivity depends on the temperature of the surface itself.

There is a vast literature available on the values of these properties for a number of surfaces under different conditions. So, if you ever come across a problem it is almost sure that reflect the radiative properties of that surface of that material will be available in the literature. And, we also have seen when and how to make the assumption, that the grey surface behavior can be assumed, and if we can make the grey surface assumption, then the hemispherical emissivity would be equal to hemispherical absorptivity, which will reduce complexities in our subsequent calculations as we would see later.

But, there is one more part of radiation which is very useful, which has practical applications in which we receive almost every day we I am talking about the solar radiation. So, on the earth's surface we have an atmosphere, in through the atmosphere the radiation the suns radiative energy comes to the earth surface without which we any life form on earth will not be able to sustain.

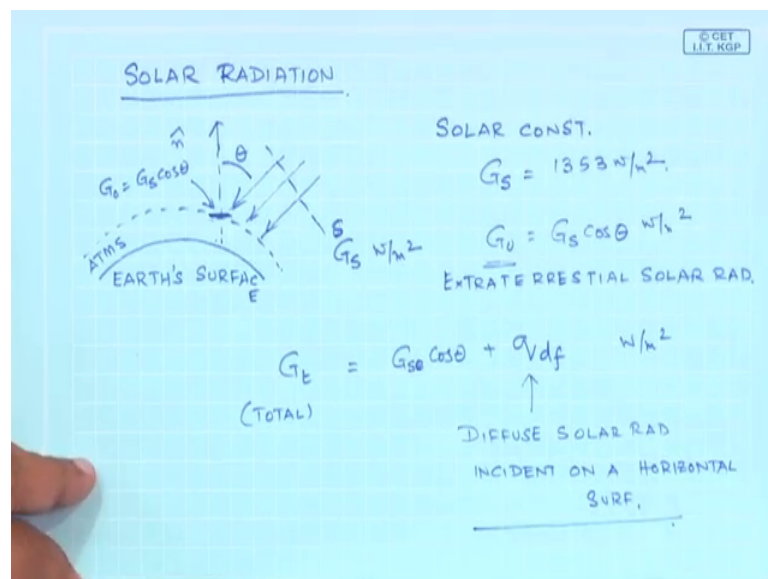
So, it is important to know; what is the value of this radiation, which is coming from the sun just before it enters the atmosphere. And, once it enters the atmosphere there is going to be some absorption into the sun's rays, the absorption would be spectral in nature so; that means, some of them would be absorbed at a specific wavelength more as compared to another wavelength.

And, it is also going to get reflected; re-reflected and come will come back to the surface again. So, these factors we will have to be kept in mind while designing many of the useful devices for example, a solar collector. So, how much of energy you are going to get based on the angle of the sun, based on how much of diffused sunlight you are going to get from other sources, which are reflecting the suns radiation and so on.

But, fundamental quantity of importance is how much of radiation you are going to get, if you have place a plate just outside the earth's atmosphere in a direction perpendicular to the ray of the sun, that is a constant.

So, when you place an a surface just outside the earth's atmosphere and measure how much of radiative energy, it is getting from sun that that is a constant and that is generally denoted by G_S which is a constant. And, if you look at the way it is so, this is where the radiation from the sun.

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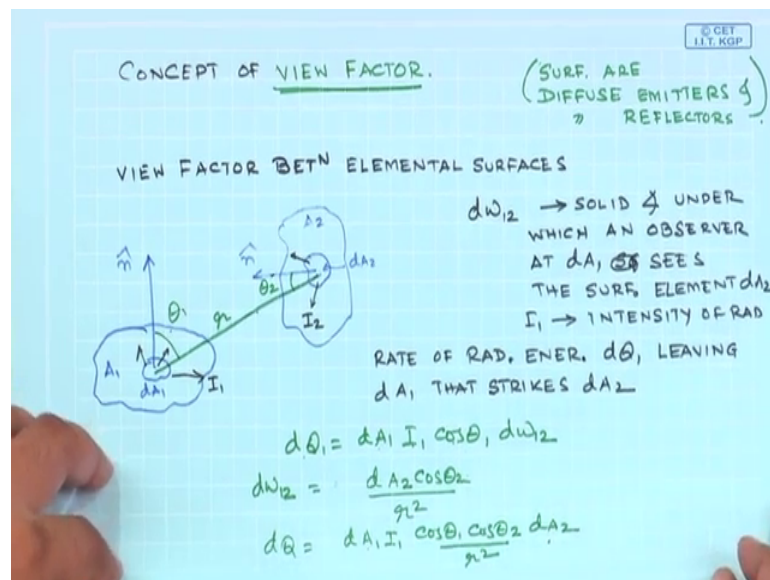


Is coming from and it is hitting the earth's atmosphere and I have placed the surface which is perpendicular to the direction of the sun's rays. And, this G_s it is also known as the solar constant the solar constant is measured and the value of G_s is 1353 watt per meter square ok. So, this is the solar constant.

But, the solar energy which is falling perpendicularly normal to the outer surface of earth's atmosphere G_0 , which I have denoted here must be equal to G_s times $\cos \theta$ also in watt per meter square. So, this G_0 is the extra-terrestrial solar radiation. So, this is what it is and the total radiation radiative flux. This is the total that you are going to that is the total radiation, that reaches the earth surface from all directions over the sky is called over the sky is a sum of what we get over here G_s times $\cos \theta$ plus q_{df} all in watt per meter square.

So, this is the direct solar radiative flux per unit area normal to the direction of the solar beam in so, this is this is G_s is this one and $G_s \cos \theta$ is on this surface, but this q_{df} is the diffuse solar radiation incident on a horizontal surface. So, that is all about the solar radiation and the other radiative properties.

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Next, we are what we are going to do is something very interesting it is known as the concept or the of view factors. So, what is a view factor? But, the first thing is we what we are going to assume is that the, whenever we are talking about the surfaces are diffuse emitters and diffuse reflectors.

So, view factor is a very interesting property, very interesting concept in radiation. Now, when a body is when radiation is coming out of a body and you place another one close by not all radiation coming out of body one would reach body 2 ok. So, if you keep them close together the fraction of energy emitted by surface 1 which is intercepted, which falls directly on body 2 would keep on increasing. As, I move the surface to away from surface 1, the fraction of energy emitted from surface 1, that directly reaches surface 2 will decrease. The part of the energy is going to go out in the space in between in between the 2 objects.

But, as I bring them close together the fraction increases. So, it is directly a function of how much of surface 2 is visible to surface 1. So, that is known as the view factor. So, view factor is the fraction of energy emitted by a surface that is going to fall directly on surface 2. So, it is expressed as capital F with a subscript 1 2. So, F_{12} denotes the fraction of energy emitted by surface 1, that is directly incident on directly intercepted by surface 2.

And, as I said let us think about these as 2 plates, if I bring the 2 plates close together this fraction would increase, if they touch each other then this fraction has a value equal to 1. That means, all energy emitted by 1 is going to be intercepted by 2. If, I move them away this fraction the value of this fraction decreases.

So, if you think a bit more in this then you can clearly see that it all depends if we if you are an observer on surface 1, then whatever be the solid angle subtended by 2 on 1, 2 as seen from 1 is going to denote what fraction of energy from surface 1 is going to reach surface 2. If, I bring them to close together the solid angle increases, if I put them back away the solid angle decreases.

So, therefore, the view factor is going to be a strong function of solid angle. Now, these 2 surfaces besides that these 2 surfaces are parallel to each other. Let us say I have one surface at an angle and the second surface is also at an angle. So, what I need to do is if this is the case then the view factor would be different.

So, the view factor not only depends on the solid angle subtended by 2 on 1 or 1 on 2, it also depends on what is the what is the angle with which they are inclined with respect to the this parallel these parallel lines. So, in order to do that first have to take a projection

of this surface on to this plane, and a projection of this surface onto this plane. And, then I evaluate what is the solid angle subtended by 2 on 1.

So, it is not only the solid angle the angle of these surfaces will also play an important role. And, you know that the solid angle is expressed in terms of area divided by r^2 and we can extend that to find out what is the view factor from 1 to 2, when both of them are at an angle and the distance between the 2 is known to us.

So, whatever be the energy that is going out of one what fraction of it is going to reach 2 is the view factor. And, this is very important industrially, because if you think of let us say you have a cylindrical furnace ok. Where the 2 ends are one end is heated the other sides a simply working or the let us say the top is heated circular top is heated and you are keeping the keeping the material, which you want to drive, which you want to expose to high temperature at the back.

So, you would like to know, what is the amount of energy provided at the top is going to hit the material directly. Because some of it is going to hit the cylindrical side walls, and some of it is going to go opposite and hit the material which you would like to treat.

So, the concept of view factor is extremely important especially when you are dealing with enclosures ok. Special enclosures for example, furnaces. What is the fraction of energy? That you are actually going to get based on your direction, based on your distance and so on. So, we would like to go a bit deeper into the view factor and get to and try to find out, what is the, what there are literature available on the values of view factors for common geometries. We can utilize and manipulate them in order to obtain view factors for unknown or view factors complex geometries, view factors for complex orientations is a method which is known as view factor algebra. And, we would see; what are the relations, which exist for view factors between surfaces and so on.

So, let us first start with the simplest possible case is view factor between elemental surfaces. So, you have a surface, which is A_1 , on that I have a surface which is dA_1 and this is the direction of the area vector. Over here I have another surface which is A_2 , this surface of the elemental surface over here is dA_2 , this is the elemental surface over dA_2 .

And, let us say this is the area vector direction of area vector over here. So, first of all I connect these 2 and when I connect these 2 let say the distance between them is r , this angle is θ_1 , this angle is θ_2 and you have emission coming out of these surfaces the intensity I call this as I_1 and this call as I_2 .

So, what I need to do is let us first say that $d\omega_{12}$ is the solid angle under which an observer at dA_1 sees the surface element dA_2 ok. And, I_1 is the intensity of radiation from intensity of radiation from surface 1 ok. And, it is going out diffusely in all directions in the hemispherical space. So, the rate of radiative energy the rate of radiative energy let us call it as dQ_1 , which leaving with leaves dA_1 and strikes dA_2 would be dQ_1 is equal to $dA_1, I_1 \cos \theta_1 d\omega_{12}$.

. So, let us see let us understand this a bit more carefully. This is the radiation intensity. Radiation intensity is energy per unit area per unit solid angle ok. So, dQ_1 is the rate of radiative energy, then per unit are would be $dA_1 \cos \theta_1 dA_1 \cos \theta_1$. And, the solid angle subtended by 2 on 1 is $d\omega_{12}$. So, if you consider I once again, it is the amount it is the intensity is defined as the energy as the energy, which go which goes out of a surface per unit area per unit per unit area which is $dA_1 \cos \theta_1$ per unit per unit solid angle and so on.

And since it is a diffuse surface, so there we do not need to invoke the we do not need to invoke the spectral nature of the surface. So, the only thing which remains to be evaluated is what is $d\omega_{12}$, $d\omega_{12}$ is area divided by r square. So, what is the area $d\omega_{12}$ is the dA_2 what is the solid angle dA_2 is making on is creating on dA_1 . So, I first have to take a projection of dA_2 , which is perpendicular to the distance vector between dA_1 and dA_2 . So, dA_2 I take it like this. So, $dA_2 \cos \theta_2$ is the area of dA_2 , which is perpendicular to the area vector r and the distance between them is r .

So, by definition of a solid angle it is area by distance square. So, $d\omega_{12}$ is simply this. So, therefore, dQ_1 would be equal to $dA_1 I_1 \cos \theta_1 \cos \theta_2 dA_2$ by r square. So, this is the this is the radiative energy which is leaving dA_1 , that strikes dA_2 in terms of the areas, the in terms of the elemental areas, the distance between them and the 2 angles the areas make with the corresponding normal θ_1 and θ_2 .

Now, so, this in order to find the view factor this the rate of radiation energy dQ_1 leaving dA_1 that strikes dA_2 must be divided by the total rate of energy which is which is originating from A_1 into the dA_1 into the hemispherical space. So, in order to obtain the fraction this is the numerator. So, the denominator would be the total energy emitted by the dA_1 into the hemispherical space. So, how do I find that from our earlier discussion?

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RATE OF RAD. ENERGY Q_1 LEAVING THE SURFACE ELEMENT dA_1 IN ALL DIRECTIONS OVER THE HEMISP. SPACE

$$Q_1 = dA_1 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_1 \cos\theta \sin\theta d\theta d\phi$$

$$Q_1 = \pi I_1 dA_1 \quad dQ_1 = \frac{dA_1 I_1 \cos\theta_1 \cos\theta_2 dA_2}{r^2}$$

$$dF_{dA_1-dA_2} = \frac{dQ_1}{Q_1} = \frac{\cos\theta_1 \cos\theta_2 dA_2}{\pi r^2}$$

$$dF_{dA_2-dA_1} = \frac{dQ_2}{Q_2} = \frac{\cos\theta_2 \cos\theta_1 dA_1}{\pi r^2}$$

$$dA_1 dF_{dA_1-dA_2} = dA_2 dF_{dA_2-dA_1} \quad \text{RECIPROCALITY RELATION.}$$

We know that the rate of radiation, rate of radiative energy let us call it as Q_1 , on leaving the surface element, dA_1 in all directions over the hemispherical space hemispherical space. This we have done before this Q_1 would be dA_1 phi the azimuth angle from 0 to 2π theta 1 to be 0 from 0 to pi by 2, the intensity $\cos\theta_1 \sin\theta_1 d\theta_1$ times $d\phi$.

So, this expression we have obtained before as well ok. So, do for a diffused so, there is nothing new in here. For a diffusely emitting and diffusely reflecting surface I_1 is not going to be a function either of theta and phi. So, I_1 can be taken outside and the integration without I_1 can be performed. So, once you perform that it simply going to be phi times $I_1 dA_1$.

So, this Q_1 is known to me I already have from my previous slide, the expression for dQ as $dA_1 I_1 \cos\theta_1 \cos\theta_2 dA_2$ by r^2 . So, therefore, the view factor dF , it is an elemental it is a view factor between elemental area dA_1 to dA_2 would be the

amount of energy released by emitted by 1, that strikes dA_2 directly which is dQ_1 divided by the total energy emitted by surface 1 in the hemispherical space.

So, if you divide this by this what you are going to get is $\cos \theta_1 \cos \theta_2 dA_2$ by πr^2 . So, this is a very important formula which tells me tells us, what is going to be the elemental view factor between 2 surfaces A_1 to A_2 . So, the amount of energy released amount of energy emitted by 1 that strikes 2 directly, which is this part and the total energy emitted by surface dA_2 which is Q_1 that we have evaluated. So, this is the 1. If I have $dF_{dA_1 \rightarrow dA_2}$, then $dF_{dA_2 \rightarrow dA_1}$ can simply be obtained by changing the 1 to 2. So, this is simply going to be $\cos \theta_1 \cos \theta_2 dA_1$ by πr^2 .

So, the elemental view factor from 1 to 2 has this expression from 2 to 1 can simply be obtained by interchanging the subscripts 1 and 2. So, what this if you look at these 2 expressions? They are the same except dA_2 appears over here and dA_1 appears over here. So, if you divide this by this, then what you get here is dA_1 the view factor the elemental view factor between dA_1 to dA_2 would be equal to the area, $dA_2 dF_{dA_2 \rightarrow dA_1}$ to dA_1 .

This is a very important relation in view factor this is a very important relation involving view factor, it is known as the reciprocity relation. What this reciprocity relations is that product of the area, multiplied by a product of the area and the view factor from the area to another area is equal to the product of the other area multiplied by the view factor from the second area to the first area.

So, this is the reciprocity relation related to view factors. The same type of analysis can also be done for areas of finite size A_1 and A_2 . If, that is the case then the view factor from 1 to 2 is expressed as F_{12} and the view factor from 2 to 1 is expressed as F_{21} . So, F_{12} stands for the fraction of energy emitted by 1 that strikes 2 directly. And, similarly for F_{21} , we have to keep in mind that all these surfaces are assumed to be diffuse emitters and reflectors.

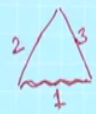
So, there is no directional dependence of the intensity of radiation coming out of these surfaces. So, once you extend these relations for finite surfaces the reciprocity relation which we have obtained for the elemental surface can simply be written in following way.

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PROPERTIES OF VIEW FACTORS

1. RECIPROcity RELATION $A_1 F_{A_1-A_2} = A_2 F_{A_2-A_1}$

2. SUMMATION RELATION $F_{11} + F_{12} + F_{13} = 1$



$F_{12} + F_{13} + F_{11}$

3. $F_{A_i-A_i} = 0$ — IF A_i IS PLANE OR CONVEX
 $F_{A_i-A_i} \neq 0$ — IF A_i IS CONCAVE

So, the reciprocity relation for a finite surface reciprocity relation for a finite surface can simply be written as $A_1 F_{A_1 \text{ to } A_2} = A_2 F_{A_2 \text{ to } A_1}$. So, this is the reciprocity relation which is important in view factor geometry. So, these are essentially the properties of view factors.

So, the first property of view factor is the reciprocity relation. The second property of view factor is known as the summation relation. So, what the summation relation tells us is that, let us say a surface a surface 1 is enclosed like this where this is 2 and this is 3. So, there would be view factor from between 1 and 2, which is F_{12} that is the fraction of energy emitted by 1 which directionally falls on 2.

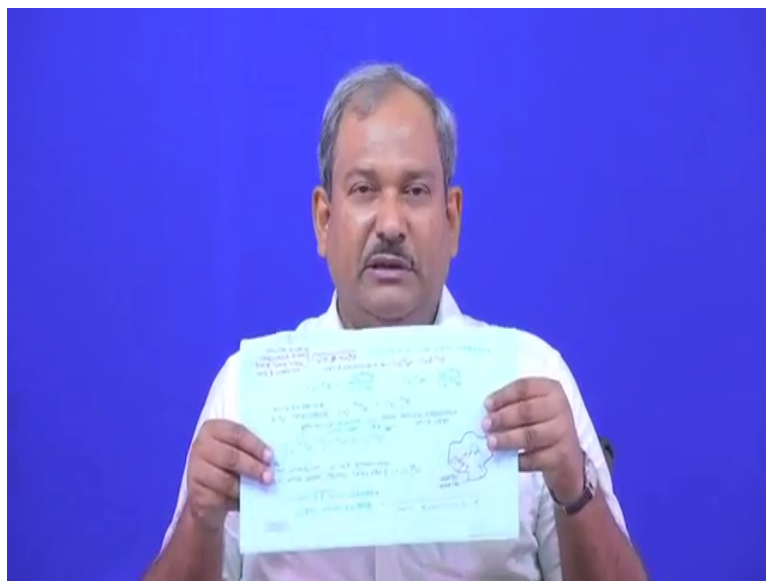
There would be another 1 which is F_{13} the fraction of energy emitted by 1 which falls directly on 3. And, there could be because of the nature of this; this area some amount of energy some amount of emission which is coming out of 1 may fall on 1 itself, in which case the view factor of that can be denoted as F_{11} . So, F_{12} for A 3 zone enclosure so, to say the some energy from 1 can fall on 2, some energy fraction can fall on 3, and some can fall on itself.

If this is a curved surface, some amount of energy emitted by the surface may fall on the surface itself, in which case it has a view factor to itself. So, it has a view factor with respect to surface 2 A, view factor to surface 3, but it may also have a view factor to itself.

So, if you add all these view factors add all these fractions then they must be equal to 1. So, the fraction of energy emitted by 1 which falls on 2, fraction of energy emitted by 1 which falls on 3, and fraction of energy emitted by 1 which falls on 1 if you sum them all together what you are going to get is simply the new simply equal to 1. So, which is known as the summation relation and which is expressed as $F_{11} + F_{12} + F_{13} = 1$. So, that is the second relation of view factor which would be useful while dealing with such dealing with such cases.

And, there are other things which are the third type of relation is $F_{Ai} / A_i = 0$ if A_i is plane or convex F_{Ai} / A_i is not equal to 0, if A_i is concave. Let us think about this for some time ok. What it tells us is that, if a surface is a flat surface, if a flat surface is like this page.

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Then, in any emission which is coming out of this page of this surface, there is no chance of it coming on to the surface directly. So, if it is it can go to the wall gets get it may get reflected and come may come back onto the surface, but there is no chance of emission from surface directly falling on surface 1 itself. Which is true, if it is a plane 1 or if it is a convex 1 If, it is a convex surface then of course, there is no chance of emission coming out of 1 falling on 1 itself.

But, on the other hand, if this is a concave surface, then the emission from this point may hit the other points on the surface. So, for a concave surface you are not going to

you cannot say, that F_{11} or F_{AIAI} is equal to 0. So, the third condition on the view factor would be that it is equal to 0 for a plane or a convex surface, but it will not be 0, if it is a concave surface. So, we have 3 relations then for evaluating the view factors, one is the summation rule, the second is reciprocity rule and the third which depends on the nature of the surface itself. And, you can be you can you can find out different-different values I mean you can find out the values of the view factors for several such cases.

Now, the other part is there are a vast majority of literature, which is which deals with the values of the view factor, which are evaluated analytically ok. The analytical expressions are quite complex I will not go into those expressions in this course, but there are graphical methods. There are graphical there are many graphs which are available based on different geometries what would be the view factor.

For example, you can consider of an enclosed cylinder, what is the view factor of the top valve with respect to the bottom the top circular plate with respect to the bottom circular plate which; obviously, depends on what is the distance between the cylinder and what is the radius of the cylinder?

. So, there are there are charts available which would tell you what is the view factor for such a case. You may have a cube in you may have a cube or a cuboids and the view factor from 1 of the flat surfaces to any one of the other surfaces can all are also documented in the form of charts. So, you would be you would be able to obtain you would be able to use that as well.

So, these various geometries and their corresponding view factors are provided in many in many of the books in your text book itself, you would see that the view factors for such surfaces or combination of such surfaces in an enclosure are provided. So, while solving for while trying to find out the unknown view factor, first try to see if information about such a configuration is available in the literature, in your text book.

And, if it is so, then directly use the values of those view factors for your calculations. If not then maybe sufficient inputs given in the problem itself, which when you visualize the system appropriately, the value of the view factor would be apparent. And, if 1 value of the view factor can be obtained, the other values of the unknown view factors can also be obtained either by the use of summation rules, reciprocity relations and so on.

So, in the next class we will solve some problems on finding out the view factors either using chart or using any of the relations that I have just described.