**Heat Transfer**
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# Lecture - 51
# Kirchhoff's Law

In this class, we are going to establish the relation between two of the very important properties related to radiation from earth to a surface or to radiative properties of a surface which are known as the emissivity and the absorptivity. Emissivity gives us the idea of how close a surface is to a blackbody for a blackbody, it is emissive power is maximum and the value of emissivity is equal to 1. But for most of the real surfaces, the it is their emissive powers are not as good as that of a blackbody. So, the value of emissivity for a real surface varies between 0 to 1 for maximum for the case of a blackbody.

On the other hand, knows real surface can observe every say everything all energy which is incidental on it, unlike a blackbody a blackbody absorbs all energy that are incident on it. But for a real surface, it is going to reflect part of the incident energy, it may transmit a fraction of the incident energy as well the rest is going to be absorbed by it. So, the absorptivity of a real surface is less than 1, it is equal to 1 for a blackbody.

So, what is the relation between these two the emissivity and the blackbody and emissivity and the absorptivity of real surfaces. Now, for that we are going to do a thought experiment, what we would think is that an enclosure which is maintained at a temperature of some absolute temperature t and a small object is going to be placed inside the enclosure. So, the small object is going to receive energy from all directions because it is inside the enclosure and initially the temperature of the small object could be different than that of the enclosure temperature which is assumed to be very large the size of which is going to be very large.

So, you let it inside let it receive energy from the enclosure, let it emit energy as well and throughout this process the the temperature of the object will slowly come towards the temperature of the enclosure. We will assume that it is only radiation which is taking place. So, when perfect thermal when thermal equilibrium has reached the temperature of the body and the temperature of the enclosure they are going to be the same and we are going to start our analysis from that point onwards.

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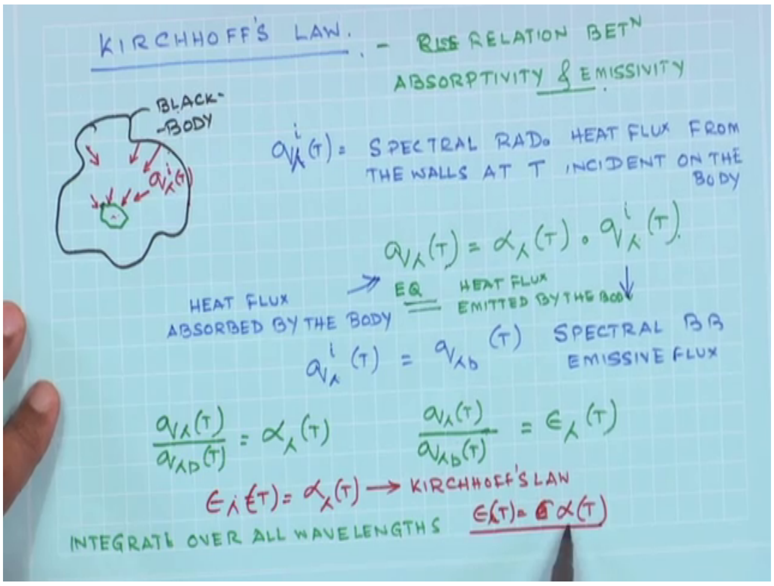


So, if you look at the what I have said, if you look at the pictorial description of this. So, this is an enclosure which is a blackbody it is emitting radiation which is emitting radiation and this is the object which is placed inside the enclosure and the amount of energy which is incident on it is q i lambda T; i refers to the incident, lambda shows the spectral nature and T is the temperature of the object. So, q i lambda T is the spectral radiative heat flux from the walls which are at a temperature T in which are incident on the body.

So, the this what it reaches on the body is essentially q i lambda T. Now, part of it is going to be absorbed by the body. So, what is going to be the amount which is absorbed by the body, let us denote it as q lambda T. Again the lambda is kept here to underscore to highlight the spectral dependence of radiation phenomena and that is why we keep lambda over here. So, this is alpha which is the spectral absorptivity multiplied by the incident radiation on the object.

So, whatever be the incident radiation on the object which is denoted by q lambda i T multiplied by the spectral emissivity would give you the amount of energy which is absorbed by the body. But when you think of the thermal equilibrium, the temperature has the temperature is steady now so, therefore, whatever amount of radiation it is getting from the enclosure walls and what whatever be the fraction that is absorbed by the object, the same amount must be emitted by the object in order to maintain the constant temperature.

So, the energy which is actually absorbed by the surface must be equal to the energy which is emitted by the surface that is what thermal equilibrium demands. So, if we will look at the previous expression now that we have written this q lambda T which is the heat flux absorbed by the body must also be equal to the heat flux which is released which is emitted by the object. Furthermore, we note that this incident radiation is coming from a blackbody ok, the incident radiation is coming from a from the enclosure walls which in itself is is is a black body.

So, therefore, this q lambda i which is at some temperature this is q lambda b at T so, this is the spectral blackbody emissive flux. This we are familiar with two observations we have made over here. The first observation is this is coming from a blackbody so; this must be equal to the spectral blackbody emissive flux. So, this can simply be replaced by this. Secondly, your thermal equilibrium our thermal equilibrium demands that whatever is absorbed must be emitted in order to maintain thermal equilibrium.

So, if I work on this little bit more, what I would get is that q lambda T by q lambda b T is equal to alpha lambda T. I have only replaced this by the spectral blackbody emissive flux. Now, if you think about the definition of epsilon then q lambda T whatever is emitted divided by whatever is emitted by a blackbody at that specific temperature is the emissivity. Emissivity is the amount emitted by an object divided by amount emitted by a blackbody at the same temperature that is the definition of spectral emissivity.

So, if you look at these two now, since q lambda T is equal to the amount of heat flux absorbed by the body and to maintain thermal equilibrium, it is also the amount of energy emitted by the body. So, this in order to maintain equilibrium, this must be equal to the heat flux emitted by the body. So, if that is the case and if you look at these two definitions now, what you can simply say is that epsilon lambda is equal at a given temperature is equal to alpha lambda at a given temperature. This is what is known as the Kirchhoff's Law.

So, the Kirchhoff's Law, simply tells that the spectral emissivity is equal to the spectral absorptivity provided and that is a big one provided the emission is coming from a blackbody at the same temperature. Remember, we have assumed that this has reached thermal equilibrium with this. So, whatever be the temperature of the blackbody since this is inside the enclosure for a long time it has also reached the same temperature as this one. So, when we when these conditions are met we can say that the spectral emissivity is equal to the spectral absorptivity.

So, the so, the assumptions involved are that the radiation is coming from a blackbody as the same temperature as that of the object itself. Only when these two conditions are met we can say that Kirchhoff's Law is valid that is the radiation is coming from a blackbody and the temperature of the blackbody from where the radiation is coming is the same as that of the body itself. So, these are two major restrictions on the use of Kirchhoff's Law.

It may so happen also that if we integrate this relation over all the wavelengths then, it may happen that it may you epsilon is going to be epsilon at a given temperature is going to be alpha at a given temperature. So, not only spectral emissivity is equal to the spectral absorptivity, the overall emissivity is also equal to the overall absorptivity provided and this is a big one provided that the incident and the emitted radiation are have same spectral distribution.

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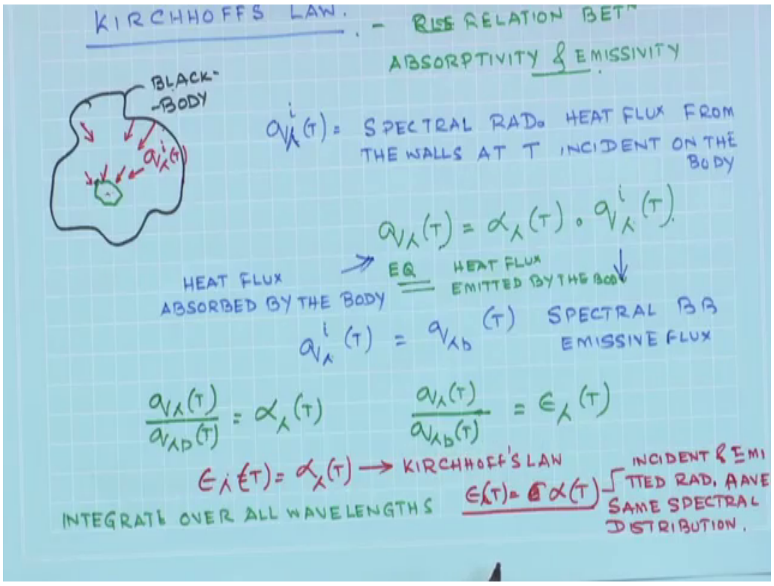


This would require some more analysis on our part before you appreciate the whole meaning of the incident and radiation incident and the emission are having the same spectral distribution, what exactly we do mean by that. So, I will I will go into this in the next slide.

But this simple thought experiment tells us that if two objects are in thermal equilibrium such that the temperatures are the same and if the emission to the object is coming from a blackbody. Then the spectral emissivity would be equal to the spectral absorptivity or epsilon lambda at a given temperature is going to be alpha equal to alpha lambda. But the two major constraints for this are the emission has to come from a blackbody and the temperature of the blackbody must be the same as that of the object.

So, of course, you can see that these two are quite restrictive and we we really have to think is there any other way by which we can make a statement about the values of epsilon and the values of alpha. Now, since these are spectral quantities one would be tempted to integrate them over the entire wavelength to see what is the overall what is the relation between the overall emissivity and overall absorptivity? So, that is something which we are going to do next in this part of the class.

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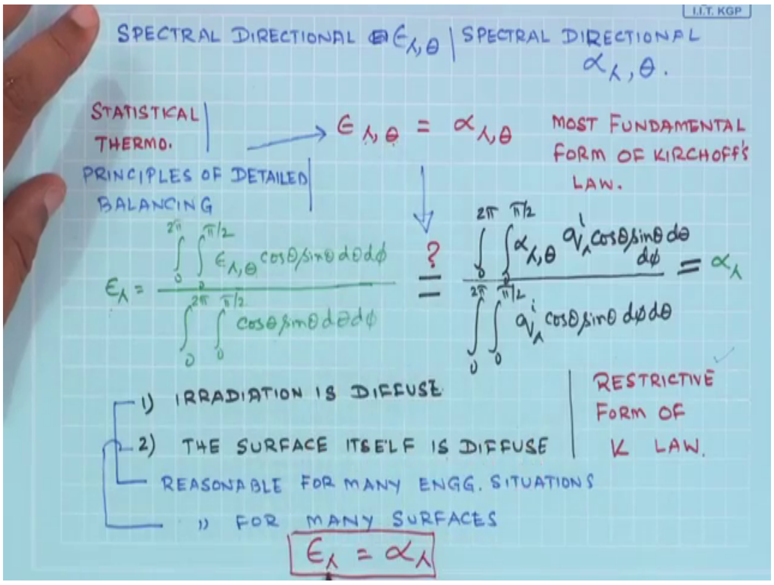


So, what we start with is a relation which is spectral directional emissivity. I will call it as simply emissivity since it is spectral and it has it is directional. So, lambda and theta are given and are that these two subscripts must appear with emissivity and the other one is spectral directional absorptivity which is alpha, since it is spectral I have lambda; since its direction I have theta.

So, what is we I we do not we cannot prove that in this course, but what statistical thermodynamics tells us tells us that these two epsilon lambda theta will always be equal to alpha lambda theta. So, this comes from statistical thermo and this is this follows from something which is called principles of detailed balancing, which is beyond the scope of this course.

I will not introduce it here but, this is what you can obtain from the statistical thermodynamics and this is known as the most fundamental form of Kirchhoff's Law which is always valid. Irrespective of what you have whether where the radiation is coming from what it is emitting what is the temperature, this relation will always be valid ok.

Now, what we are going to do is we are going to put on restrictions to this relation and try to see the more restrictive form of the Kirchhoff's Law. So, I am from this I am going to find out what is the spectral emissivity and what is the spectral absorptivity using just the definition of the spectral from the expression for spectral and directional.

So, since it is directional in order to get rid of the direction, I need to integrate it over the entire hemispherical space. So, the same way we have done it for the blackbody when it is placed inside a hemisphere, I am simply going to find out this is as cos theta sin theta d theta d phi. This is what we would the expression and 0 to 2 pi, 0 to pi by 2, then the value this is for the blackbody and for the blackbody the emissivity has a value equal to 1.

So, what I will have then is cos theta, sin theta, d theta and d phi. The this one this is the directional one so, the definition of absorptivity is again 0 to 2 pi as before 0 to pi by 2, then alpha lambda times theta q lambda i cos theta sin theta d theta d phi; this is the incident radiation and if we do it for the case of a blackbody. Then this absorptivity will have a value equal to 1 therefore, I will only have this as cos theta sin theta d theta d phi. When these two are going to be equal that is the big question? So, we need to find out under what condition epsilon lambda is going to be equal to alpha lambda.

So, let us look at the possibilities here. The first one is what we can say is that the irradiation is diffuse, if the irradiation which is q lambda i which is this one, if this is diffuse then, it will have no directional dependence. It will be a constant with respect to the direction that is what diffused is all about a property is diffuse a radiative property is diffused when it does not depend on the direction.

So, if the irradiation is diffused, then it is not going to depend either on theta or on phi. So, therefore, this can be taken outside, this can be taken outside, they will cancel and after that if you look at these two. Let us say this q i terms are not there and you are looking at only this and this except the q and since epsilon lambda theta is always equal to alpha lambda theta in absence of q this epsilon lambda must be equal to alpha lambda. So, that is the first condition which needs to be maintained. Second is the surface is diffused surface itself is diffused.

So, what is the advantage of making this assumption? If the surface is diffused, then epsilon lambda theta does not depend on theta. So, this can be taken outside and alpha lambda theta does not depend on theta so, that can also be taken outside. So, since epsilon lambda theta and epsilon lambda theta and alpha lambda theta are equal and if they do not depend on this, then they would simply give you these terms will cancel what is left is epsilon lambda theta and alpha lambda theta. So, epsilon lambda is equal to epsilon lambda theta and alpha lambda is equal to alpha lambda theta and since these two are equal, you are going to get epsilon lambda is equal to alpha lambda.

I think you should look at the expression carefully and imagine what would happen when the directional dependence of the property is not there. So, these two can be taken outside alpha lambda theta and epsilon lambda theta. Whatever is left inside the integration sign are simply equal so, they can be cancelled out from numerator and denominator for both cases.

And therefore, you would get alpha lambda is equal to epsilon lambda; that means, the spectral values of emissivity is equal to the spectral values of absorptivity. So, this is known as the restrictive condition of Kirchhoff's Law what are the restrictions? It can either be that the surface is a diffuse or the incident ray the incident radiation is diffused. In both cases what you are going to get is that the spectral emissivity is equal to the spectral absorptivity.

So, once again if you look at this is known as the restrictive form restrictive form of Kirchhoff's Law. Now, one of the advantages is the this one is reasonable for a many engineering situations. That means, it is safe to assume that the irradiation is diffuse and number 2 is the it is reasonable for many surfaces that the surface itself can be assumed as they are diffused surface. So, it tells us that the restrictive form of Kirchhoff's Law is not too restrictive a condition which is followed by many surfaces.

So, therefore, what I write is that epsilon lambda is equal to alpha lambda. This is the relation which we have obtained subject to these 2 conditions. So, from this pic from the spectral directional, I come to spectral and from spectral, now I would go to the overall. So, what is the definition for the case of overall?

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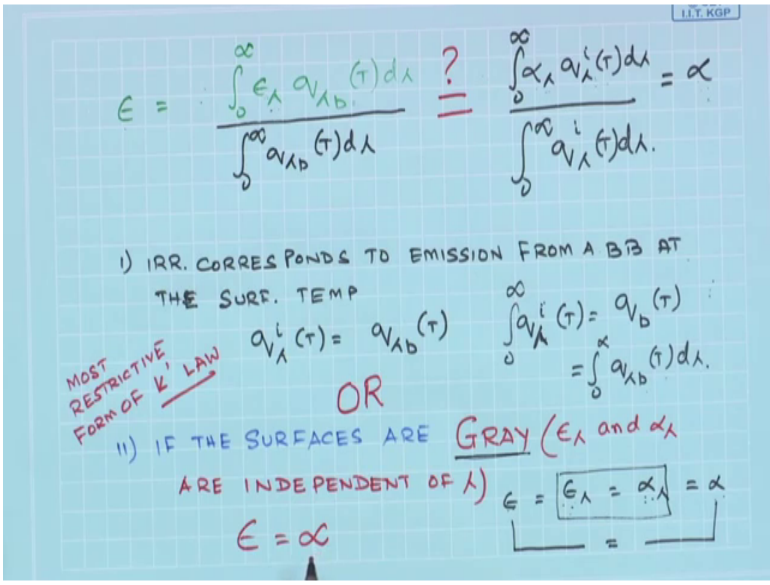


The overall emissivity is from the spectral one, 0 to infinity epsilon lambda q lambda b T d lambda divided by 0 to infinity q lambda b T d lambda; are they going to be equal? That is the question mark. On the other side, I have alpha, the absorptivity which is equal to 0 to infinity, alpha lambda the incident radiation divided by 0 to infinity q I lambda T d lambda.

So, these are just the definitions of overall emissivity from spectral emissivity. So, the spectral absorptivity and this is the overall absorptivity. So, when these two are going to be when I am going to put this equality sign. So, the first one is irradiation corresponds to emission from a blackbody at the surface temperature T. So, if that is the case, then q lambda i at T is equal to q lambda b from a blackbody at T and therefore, 0 to infinity q i q lambda i at T is equal to q b T which is nothing but saying that this is equal to 0 to infinity q lambda b T d lambda.

So, if the radiation corresponds to emission from a blackbody at the surface temperature, then this is to be replaced by q lambda b T d lambda. So, once you replace that these two would become identical because, we have already assumed that epsilon lambda is equal to alpha lambda. Since, we have assumed the spectral properties are the same and now, we are assuming that the irradiation is also coming from a blackbody at the same temperature.

Therefore, this is going to be same as this and this part as I shown here is going to be equal to this part. So, if this is the condition even though it is a very restrictive condition, if this is the condition then alpha is going to be equal to epsilon or this is a major statement or if the surfaces are gray if the surfaces are gray.

So, what is a gray surface? A gray surface is the one for which this epsilon and alpha lambda are independent of lambda let us see what I am what we mean here. If a surface is gray then they do not depend on alpha anymore. See, if they do not depend all on alpha anymore, they can be taken out of the integration sign and here alpha can be taken out of the integration sign. So, what you have if you do not have epsilon, if you do not have this one, then the numerator and the denominator are the same. If you do not have alpha inside the integration, then the numerator and the denominator are the same.

So, what you have in that specific case is that epsilon lambda is equal to alpha lambda and this is equal to epsilon and this is equal to alpha. This is true since our starting point was the spectral emissivity is equal to the spectral absorptivity and what we are saying here is that since the surfaces are gray, then epsilon is equal to epsilon lambda and alpha is equal to alpha lambda which shows that these two must be equal that means, epsilon is equal to alpha.

So, even though this is a very restrictive condition which says that the this the equality of emissivity and absorptivity overall not the spectral overall can be obtained only the emission is coming from a blackbody at the surface temperature. There is a different way of expressing the same thing which tells us that the overall values would be equal, if we assume a surface for which epsilon lambda and alpha lambda are independent of lambda.

If you do assume that which defines a special class of substrate as the gray substrate, for a gray substrate since these two are independent of directions. So, these two can be taken outside of the integration sign numerator and denominator in that case will cancel out.

So, what I would left with is epsilon equals epsilon lambda equals alpha lambda equals alpha which essentially means epsilon is equal to alpha, the overall emissivity is equal to the overall absorptivity.

So, this is the most restrictive form of a Kirchhoff's Law. But this is for the gray surface gray surface is so; this is the most restrictive form of Kirchhoff's Law which says that for a gray surface, these two are going to be equal. So, the total hemispherical emissivity is equal to the total hemispherical absorptivity. So, calculations for radiation exchange between surfaces are greatly simplified if this equation can be if this can be assumed to be true.

So, many calculations most of the calculations of radiation between two or three surfaces when they are exchanging radiation among themselves should be greatly simplified, if we can assume that they are gray surfaces. Fortunately, many of the surfaces many of the some of the many of the engineering surfaces the assumption of a gray behaviour, assumption of gray behaviour this does not incorporate significant errors.

So, the moment we can so, if we can we should always try to see if gray behaviour can be assumed for the engineering surfaces for the surfaces which are exchanging radiative heat among themselves. So, if we can do that we would show next is how this would simplify the calculations significantly.

But, if it is not if the spectral values the spectral the if the values of the alpha and epsilon, if they depend on wavelength then this assumption cannot be used and I would just give you an example which would show that why we cannot use that approximations for surfaces which have spectral distribution of emissivity and absorptivity.

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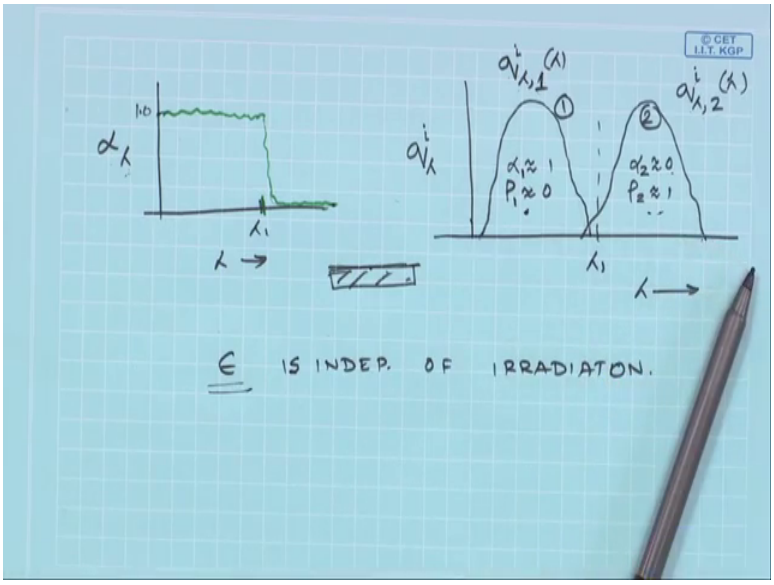


So, for that I am going to draw 2 curves which would show how would the values of alpha and epsilon are varying with lambda. This is alpha which is a function of lambda and let us say this is equal to 1 and this is lambda 1. So, the value the behaviour of lambda alpha is something like this it is close to 1 from this one, it sharply falls and becomes almost like 0 so, this is the value of lambda 1.

On the other hand the amount incident 1, this is my lambda 1; the incident 1 is such that the spectral distribution of this 2 incident rays. So, this is lambda, this is lambda and let us call this as q lambda type of emission 1 this is incident type of incident 1 which is a function of lambda and this is q lambda 2 incident 1 times lambda. So, this is case 1 and this is case 2. So, what we would see from here is alpha is almost equal to 1 in this case and the reflectivity is 0. These are opaque substrates so, reflectivity is 0, alpha 1 is equal to 1, rho 1 is equal to 0 and in this case alpha 2 is about 0 so, rho 2 would be equal to 1.

So, what is the above surface, if this is the surface the surface which we are talking about; this surface has the property where the absorptivity varies from this is roughly 1 up to a wavelength of lambda 1 and beyond lambda 1, it comes to a value equal to 0, almost close to 0 and 2 different incident radiations are falling on the surface. So, the 2 radiations are q lambda 1 incident radiation and q lambda 2 incident radiation. One is concentrated in the range 0 to lambda 1, the other is concentrated in the region beyond lambda 1.

So, since in this region, it is absorptivity is equal to 1, almost all of the incident radiation is going to be absorbed by the surface. Whereas, over here since the spectral distribution of the incident radiation is such that since the value of alpha 2 here is about 0, none of the absorption, none of the incident radiation is going to be absorbed on the surface ok. So, the values of alpha, they are drastically different in these 2 cases ok. So, the based on the irradiation field, you your values of alpha lambda are drastically different and the value of epsilon on the other hand is independent of irradiation.

So, what I mean here that the value of lambda if you calculate based on an irradiation like this and the value of alpha when you calculate your value of alpha based on irradiation like this this two are drastically different. On the other hand, the value of epsilon is independent of irradiation therefore, there is no basis to say that alpha will always be equal to alpha is always going to be equal to epsilon.

So, absorptivity and emissivity; hemispherical absorptivity and hemispherical emissivity may not be equal. So, we have to be careful while using emissivity equal to absorptivity, this relation to know what is the irradiation field? has it been mentioned that the surfaces that are we are talking about the surfaces and the surfaces are gray.

So, they do not have any directional, they do not have any spectral dependence of the properties. So, using Kirchhoff's Law, one has to be very careful to use it with the most restrictive condition that is alpha is equal to epsilon or the with a restrictive condition in which epsilon lambda is equal to alpha lambda. So, for these two cases one has to be careful.

But, this spectral dimensional emissivity will always be equal to spectral directional absorptivity. So, if you look at the development that I have done in today's class, starting with epsilon lambda theta equal to alpha lambda theta; how I have obtained alpha equals epsilon and what are the approximations or assumptions that I had to make in order to get alpha equals epsilon.

So, this should be very clear in your mind, I am going to solve few problems on this Kirchhoff's Law concept. So, that it is going to be very clear to all of you that is when and how you can use Kirchhoff's Law and what are the simplifications that we can obtain once if we can assume that it is their gray surfaces that is overall emissivity overall hemispherical emissivity is equal to overall hemispherical absorptivity. Once you solve the problems, it will be more clear to you which we would take up in the next class.