Heat Transfer
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Lecture - 05
One Dimensional Steady State Conduction

Previously, we have derived what is the conduction equation for a Cartesian co-ordinate systems, cylindrical systems and spherical systems. In the development of those equations, we have assumed the temperature can vary in all 3 possible directions and it can vary as a function of time as well. And there may be a situation in which some heat is going to be generated inside the control volume.

So, the equation that we have that is the generalized governing equation for conductive heat transfer in a solid medium. But we are going to see some of the special cases which arise out of this equation which has practical applications; for example, in many of the situations we see that a plane wall has layers of insulation on it, in order to reduce the heat loss during winter or heat gain during the summer.

So, when a plane wall has a number of insulations attached on one side of it and if we know the temperatures of each of the junctions between the insulator and the wall insulator 1 and insulator 2 and so on; then it is possible to predict what is the total heat loss from the wall to the outside at steady state conditions. Because in practical applications we mostly are concerned with what is going to happen at steady state, what is the steady rate of heat loss from a room which is exposed to a very cold environment?

So, it is important therefore, to simplify the heat diffusion equation for special conditions of when you have only a plane wall and at steady state. We would also add we will also add another restrictions that there is no heat generation in the control volume that we are considering. Again going back to the example of heat loss from the walls of a room at steady state with insulations on the outside of the wall; obviously, there is no heat generation in the insulation.

So, how do I express the temperature distribution inside these different layered materials? We would also; we would also need to realize, appreciate the fact that at that at steady state no matter whatever be the order of the insulation, the thickness of the insulation or the material of the insulation; the same heat goes through all the layers that is very important. So, the heat rate is constant the amount of heat which will be lost by a hot room with layers of insulation on the outside is a fixed quantity. So, no matter what you have on the insulation as the insulation; the thickness of it and so, on; at steady state the same amount of heat will flow through all these layers.

And secondly when you think about; that when we talk about plane wall and insulation on top of it, then you do not have any variation in the area any variation in the heat transfer area. So, the same area is exposed to heat transfer from inside the room; to the outside the cross sectional area does not change. So, therefore, the characteristic there are special characteristics of a plane wall is that the area remains constant. And if the area remains constant and we understand from physical principles; that in since there is no heat generation and at steady state, the same amount of heat must travel through all these layers of insulation.

So, the heat rate is constant area is constant and therefore, heat flux which is heat rate by area that is also going to be a constant. So, for planar systems the heat; both the heat rate and the heat flux are constant which may not be the case for cylindrical systems or spherical systems. Because as you grow out the radius increases and since the radius increases the area available for heat transfer will also increase. So, the quantity of heat will remain constant, but the flux will not be constant. So, that is the major difference of conduction in planar systems, at steady state and conduction in radial or spherical systems; cylindrical or spherical systems at steady state where the heat rate will; obviously, has to be constant because there is conservation of energy.

But the heat flux will vary; so, these are the different things and in the fact that heat flux may vary will also lead to interesting observations and they will also affect the temperature profile; what you would expect in a plane wall at steady state with no heat generation and a cylindrical system with no heat generation at steady state. In one case as we are going to see very soon that the heat the temperature distribution is going to be linear; whereas, in the case of cylindrical and spherical systems, they are going to be non-linear.

So, our starting point for analysis of conductive heat transfer in plane walls starts with the heat diffusion equation. And then we are going to simplify the equation based on its one dimensional heat transfer; no generation of heat and at steady state.

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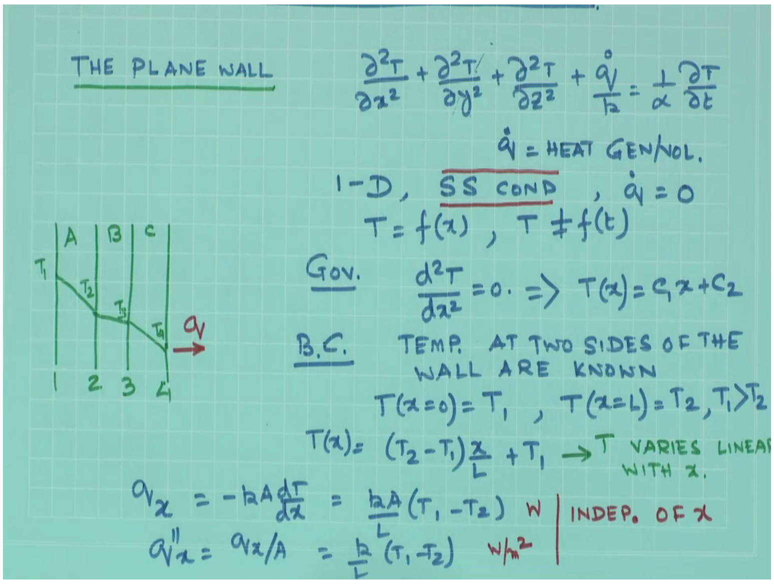

So, we start first with this equation which we have derived previously in the class. So, this is temperature as a function of x y z as well as time.

So, there is a transient component also attached to this and this q dot if you remember this is q dot is the heat generated per unit volume of per unit volume. So, therefore if it is 1 dimensional at steady state condition then I can say that T is a function only of x; let us assume that x is the direction in which the heat flow takes place and T is not a function of time.

So, when I talk about steady state what I mean is that the temperature can be a function of position, but at a given position the temperature is not a function of time. So, for the 1 dimensional case temperature is a function of x, but temperature is not a function of time and we will also assume that there is no heat generation in the system therefore, q dot is 0. So, if we apply these 3 conditions to this equations what I understand what I see is that T is not a function of y; so this term would disappear, T is not a function of z; so this term would also not be present, q dot is 0 and temperature does not vary with time.

Therefore the governing equation reduces from the heat diffusion equation as d 2 T dx square is equal to 0. So, this when you integrate this it would simply give you temperature as a function of x as C 1 x plus C 2 where C 1 and C 2 are constants of integration. So, now in order to solve this I need boundary conditions; so 2 boundary conditions are required and what you would assume that the temperatures at 2 sides of the wall are known if the temperatures at 2 sides of the wall across which heat transfer heat transfer is taking place.

So, what I am going to say then is T at x equals 0; that means, at one edge of the wall the temperature let us say it is a T 1 and T at the other edge of the wall which is at x equal to L E is equal to T 2. And we will assume that T 1 is greater than T 2 though the derivation the conceptually it will work fine; if you if you do it in the other way that T 2 is greater than T 1; only the direction of heat transfer will be different.

So, when you use these 2 boundary conditions with this profile what you get? I am only writing the solution that T x is T 2 minus T 1 x by L plus T 1. So, this tells me that temperature varies linearly with x. So, this clearly again shows that the temperature at any position is a function of x only and it depends on the imposed temperature difference which is T 1 minus T 2; it also depends on the geometric parameter, this L is the thickness of the wall and the imposed temperatures.

From Fourier's law we know that q in the direction this is this q is the heat flow in the x direction is equal to minus k times A d T d x where k is the thermal conductivity. So, one can write looking at this expression and finding out what is d T; d x is k A by L T 1 minus T 2 and the heat flux is simply q x by area which would be k by L; T 1 minus T 2.

So, the important point to note here is that the heat flow; the total rate of heat flow with units of watt and heat flux with units of watt per meter square both are independent of x; no x appears in any of these 2 equations. So, as I told you before that the not only the heat rate at steady state without heat generation is constant in a planar system, the heat flux is also a constant.

So, our derivations simply shows what we have started with the basic premise which we have started with that both the heat rate and heat flux are constant in planar systems; which we would say subsequently would not be the case for a spherical or radial systems, where the cross sectional area keeps on changing as you move in in the plus x plus R direction.

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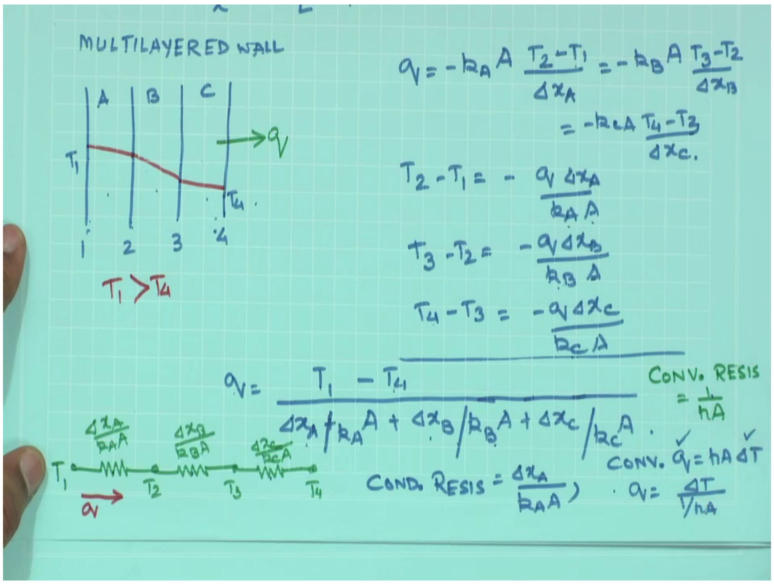

So, we will go back to the we are we are going to do something else with this equation, which simply tells me that q double prime x q double prime x is equal to k by L T 1 minus T 2 ok.

Now let us assume that we have a system of a plane wall with 3 materials which are A, B and C; these are denoted by 1, 2, 3 and 4 these are the junctions of this and let us assume that the temperature profile is something like this ok. And as you can see that as you move from 1 towards 4 the temperature decreases and since heat always flows in the direction of decreasing temperature, the direction of heat flow is simply going to be this q and at a steady state as we have seen before the heat flux as well as the heat will remain constant.

So, at steady state the same q will flow through material A, B and C. So, we are looking at what would be the result for this; so, this is a multi layered wall. So, in a multi layered wall what one I can write since my q is constant in; so, this material A, B and C; this is minus k A times area of the material A; T 2 minus T 1 by delta x A which would be the same as minus k B, area remains the same T 3 minus T 2 by delta x B. And similarly for the third one which is minus k C A; T 4 minus T 2 by delta x C.

So, this is nothing, but the Fourier's law. So, now, what one can do is you can express T 2 minus T 1 in terms of all these quantities that is T 2 minus T 1 is equal to minus q times delta x A by k A A. T 3 minus T 2 is minus q delta x B by k B A; T 4 minus T 3; this 3 T 4, T 3 is minus q delta x C by k C A. So, this k A, k B and k C are the conductive thermal conductivities of material A material B and material C and delta x A, delta x B and delta x C are the thicknesses of each of these materials.

So, you sum them all together what you are left with is this T 1 and T 4; they are the one who are which are going to remain in this and on this side if you take q common. So, you have delta x A by k A A, delta x B by k B B and delta x C by k C. So, these are the ones which would remain; so when you rearrange what you are going to get is T 1 minus T 4 divided by delta x A plus by k A area plus delta x B by k B area plus delta x C by k C area.

So, this is what you are going to get. So, what is T 1 minus T 4? T 1 is the temperature at this point and T 4 is a temperature of this point. So, the total flow of heat between these this multilayered wall is given by the overall temperature difference T 1 minus T 4 divided by the sum of these 3, which is for material A, for material B and for material C. So, I can; so, this we are we are familiar with this type of equations from other branches of engineering.

So, what I can think of this as T 1 is one of my potential and let us say I have a resistance, then I have T 2 another resistance and I have T 3 and another resistance and I have T 4.

So, temperatures can be viewed as potentials and the heat transfer resistance is a nothing, but delta x A by k A A; this one is delta x B by k B A and this is delta x C by k C A. So, what you have then is the system is the potentials T 1, T 2, T 3 and T 4 the resistances are like these and the entire thing is in series. And as a result of since T 1 is greater than T 4, since T 1 is greater than T 4, what I have then is flow of heat in this direction and as with the as with the series condition with the Ohms law where the resistances are in the series what you would see is that the same heat will flow through all these walls.

So, what you have then is the overall potential difference divided by the sum of resistance. So, only thing is the potential difference to be replaced in Ohms law to arrive at the heat transfer law is overall temperature difference by sum of all conductive resistance. So, what are these conductive resistances? They are delta x A by k A into A; delta x A is the thickness of medium A; of material A, k A is the thermal conductivity of the material A and A is the cross sectional area.

So, you have 3 or 4 materials sandwiched one after the other and if you maintain temperature at one point the; the if you do if you do know what are the temperatures at the 2 end points, then you can find out what is the flow of heat through this combination materials; through this multilayered walls. And the result that you would get is something similar to what you have obtained in the case of Ohms law ok.

So, that is Fourier law Fourier's law of conduction being applied to multilayered walls ok. So, the next what we would do then is a system in which I have one more thing; I would like to quantify here is that is the conduction resistance as I said is delta x A by k A A if you think about convection, the convection is Newton's law of cooling where q is h A times delta T.

So, if we if this is the this is the effect which is the heat flow and this is the cos; then I can I can write that exactly like this equation my if I look at this then my resistance must be equal to I can write this as q as delta T by 1 by h A. So, your conduction resistance is delta x A by k A A, where as my convection resistance is equal to 1 by h A. So, if this wall is connected with the convection environment then I am going to have 3 conduction resistances in series and a convection resistance being added to this where the convection resistance is nothing, but 1 by h A.

So, the conduction what is conduction resistance and what is convection resistance? For the case of planar systems must now be clear to all of you; then we would go to a situation which is more practical, where these walls through which conduction takes place they are exposed to an environment outside air. So, the room that I am sitting in it is an air conditioned room; it has multiple layers of insulation on the outside and then it is exposed to the outside ambient temperature outside ambient.

So, the amount of heat loss from this wall which anybody would like to minimize is simply sorry in this case the amount of heat gained by this room; since this room is cooler, than the outside ambient, this is going to be combination of all the conduction resistances due to the insulation and the wall that we have for this room; as well as what is the convection resistance on the outside of the building which connects it to the ambient temperature.

So, if we think of the driving force for heat transfer from the outside of the room to the inside; is what is the temperature outside the ambient temperature and what is the temperature inside the room; the difference between these two is the cause of heat transfer. So, it is a system when I have the insulation in the outside the ambient; both taken together, it is a system of conduction and convection both present in the system.

So, we will quickly see how the result would look like for a system in which I have both conduction and convection; however, it is a planar system such that both the heat rate and the heat flux are going to be constant. So, let us see what form this equation would take.

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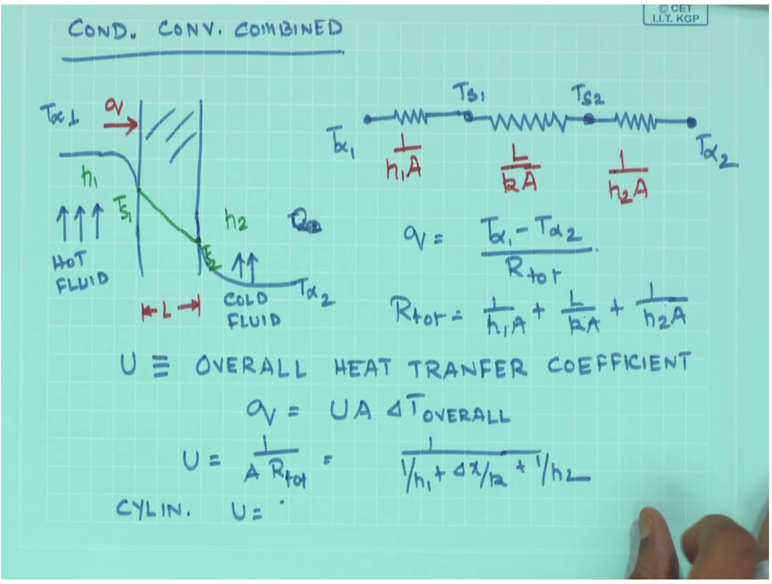

So, I have conduction, convection both are combined in the diagram probably looks something like this, where the this side is T infinity 1 and this side let us say the temperature is T infinity 2.

So, I have a hot fluid which is moving in up in this direction on this side of the wall and I have a cold fluid which is moving on the outside of this; so, this my wall. And the temperature here is T infinity 1, the temperature here is T infinity 2; T infinity 2 and if you recollect what we have done before is that it is going to sharply fall because of the thermal boundary layer that we have discussed before.

So, let us say this is the surface temperature of 1 and then I have a T S 2 which is the surface temperature of 2 and then it falls to T infinity 2. So, over here the fall is also going to be like this and in between T S 1 and T S 2; since it is a plane wall, at steady state without any heat generation; the heat the temperature profile must be linear. So, this is what the and the temperature T S; T infinity 1 is more than T infinity 2 and therefore, the flow of heat is going to be from the left to the right.

So, this is a system in which I have conduction in the plane wall, convection on the inside as well as on the outside of the plane wall. So, if I draw the resistance diagram for this where the outside potential is T infinity 1 and the potential over here is T infinity 2; these are the 2 temperatures then I have some sort of a convection resistance which brings me to T S 1 and then the conduction resistance which brings me to T S 2 as per the diagram and then I have a convection resistance which then results in asymptotically and smoothly merges with T infinity 2.

So, this is my convection resistance which must be equal to and let us assume that this hot fluid maintains a convection resist; convection coefficient the h 1 over here and a convection coefficient of h 2 at this point. So, therefore, the resistance is simply going to be 1 by h 1 A; as we have discussed before. This is going to be L by k times A; where L is the thickness of the wall, A is the area, k is the thermal conductivity and over here it is going to be h 2 A.

So, that is what we would get in this case therefore, the total flow of heat q is simply going to be T infinity 1 minus T infinity 2 divided by R total; where R total refers to the algebraic sum of these 3 resistances; since they are in series. So, this is R total would simply be equal to 1 by h 1 A plus L by k A plus 1 by h 2 A that is the total resistance to heat transfer in this system.

Now, these 3 are it is sometimes advisable to express this 3 resistance is in terms of an overall heat transfer coefficient. And we would see this the reference to overall heat transfer coefficient coming many times in convective heat transfer not only in convective heat transfer, but more commonly in the case of designing of heat exchange equipments, where instead of each of these individual resistances we express them in terms of an overall resistance and this is generally denoted by the symbol U and therefore, the q the heat flow is going to be U A, delta T overall as I mentioned the U appears mostly in the case of conduct convective heat transfer.

So, the total amount of heat transfer is simply going to be according or similar to Newton's law of cooling as q equals U A delta T, where U is the overall coefficient which may contain both conduction and convection as we have seen in the example that we are discussing. The A is the cross sectional area through which the heat transfer takes place and delta T is the overall temperature difference ok.

And it; obviously, does not assume that it is a linear or anything, but we understand that it is going to be linear in the solid, but it is going to have a shape which is where the temperature from the ambient comes to the temperature of the surface in a; in a fashion which is consistent with our explanation based on thermal boundary layers.

So, it changes sharply from its value at the; at the free stream at the ambient to the temperature on the surface of the wall. So, this heat overall heat transfer coefficient has significance specially when we are going to design heat exchanging equipments. So, the looking at the governing or defining equation of U; the overall heat transfer coefficients and looking at the equation which we have just derived, which gives us the conductive flow of heat through a wall, which is experiencing convection on both sides; I can also find out what is the expression for the overall heat transfer coefficient.

So, that is what am going to do next. So, if you compare between these 2 what you would get is that U is the overall heat transfer coefficient 1 by R, total R. So, which is 1 by 1 by h 1 plus delta x by k plus 1 by h 2. And we would see that for a hollow cylinder what would be the form of U that we would see in the next class.

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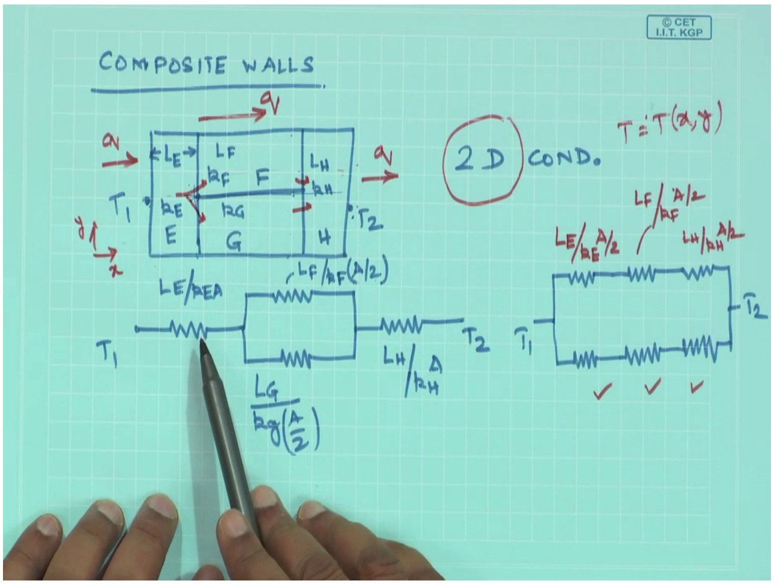

But before I conclude this part of the class let us just briefly for 2 minutes; we look at composite walls which are slightly different. Let us say you have a system in which up to this point is mid is made of some material and from this point onwards it is of another material, but in between you have 2 different materials.

So, you have a material E with a thermal conductivity k E of length L E; for this one its length is going to be L F, the thermal conductivity is going to be the k F and this is material F. And I have k G in here, this is material G and this is material H where it is going to be L H and k H and the temperature here on this side is going to be the T 1 on this side it is going to be T 2.

So, as you can see that the heat is going to flow; if T 1 is greater than T 2 heat is going to flow from 1 to 2, but in here after it reaches this surface; part of the heat is going to move through this, part of the heat is going to move through this and they are going to emerge in material edge and flow. So, this is the overall direction of q, but at this location at this plane, it is going to get divided and at this plane it going to combine; so that you get the same q and here you also going to get q.

So, there are 2 ways by which ah; so this is definitely a 2 D condition where the where near this the temperature can be there are can be 2 dimensional effects. So, temperature is not a going to be a function of x alone, it could also be a function of y. So, there are 2 ways by which you can represent this in terms of when you talk about thermal I mean the electric circuits. So, you have T 1 on one side T 2 on the other side and here the resistance is simply going to be L E by k E A and here it is going to be L F by k F A by 2; A by 2 because if I assume that half of the area is taken up by the material F and half of the area is taken by material G.

So, that is why I am putting it A by 2; it can be any fraction and it does not matter one third of the area can be made of F and 2 third will be made of G. So, appropriately the fraction of area is to be is to be replaced over here and in this case it is going to be L G by k G times A by 2; as I said a by 2 is just an just for this specific case. And over here it is going to be L H by k H and it is a total area or you can assume that this entire thing is divided into 2 strips and they are parallel to each other.

So, you have 3 resistances in series for the top part and so, this is my T 2 and this is my T 1 and heat is going to flow through this and through this. So, q by 2 q by q by 2 is flowing through this sorry q 1 is flowing through this and q 2 is flowing through this where the sum of q 1 plus q 2 is going to be equal to q.

So, what are the resistance in this case? This resistance for the first part is going to be L E by k E instead of A, it is going to be A by 2 now; instead of A; it is going to be A by 2, this resistance remains unchanged. So, it is going to be L F by k F A by 2 and this is going to be L H by k H; A by 2 and similarly the 3 resistance here are going to be the same as what you see over here.

So, whenever you come across a composite walls; it is customary to express them either like this or in either like this or in this fashion, but in both cases we have to appreciate that near the junctions there would be 2 dimensional effects, 2 dimensional conduction where T is going to can be a function of both x and y where y is this direction and x is at this direction.

So, the explaining these complicated flow of heat through composite walls by a simple resistance in series or resistance in parallel mechanism would only give you an approximate value of the temperature, the junction temperatures and the temperatures the total flow of heat. So, this is an approximation and higher the difference between the thermal conductivities between the adjacent layers the error is going to be error is going to be enhanced.

So, one should be careful about expressing the results in this specific form and most of the time, it is going to be dictated by how first you want a result; whether or not you can tolerate approximations and what is the physical condition of difference in thermal conductivities such that the difference in temperature across a fixed x is not that significant. So, that has to be taken into account before you start solving it.

So, in summary what we have done in this class is that we looked at the heat efficient equation. Reduced or simplified the equation in the case of one dimensional conduction under steady state with no heat generation. And what we saw is that in a plane wall that give rise to a linear temperature distribution, when it is a linear temperature distribution we have expressed the heat flow through a plane wall under those conditions in something similar to that of Ohms law; where the potential difference is replaced by temperature difference and the current is replaced by the heat flow. And the resistance what you get for these kind of systems is simply the thickness of the plane divided by the thermal conductivity of the plane and the area cross sectional area which is perpendicular to the direction of the flow.

And we have also seen what would be the effect when we have convection and conduction combined in such a system in multilayered walls as well as in composite walls attached. So, take a take home message from this part is that the heat flow and the heat flux both are constant. The next class what we are going to do is I will show you that in a cylindrical or a spherical system, the heat rate would remain the same, but the flux will keep on changing and that will give rise to some interesting results which would analyze in the next class.