Heat Transfer
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Lecture - 46
Tutorial Problems on Epsilon - NTU Methods

We are going to continue on our study of Epsilon NTU method. I have solved one problem on epsilon NTU in the previous class. This class is also going to be tutorial in which I will solve 2 more problems on epsilon NTU method. So, hopefully by the end of today, you will have a fair idea of how to solve problems involving epsilon NTU method. So, the first problem that we are going to deal with is a concentric tube concentric pipe heat exchanger. So, it is a double pipe heat exchanger. So, do not know whether it's operate be it should be operated in counter flow or in parallel flow. Some of the property is are given for example, what is the mass flow rate of the of one fluid, the mass flow rate of the other fluid, the cp heat capacity of one and the heat capacity of the other.

Some the temperatures are also provided, in fact 3 of the temperatures are provided. We need to find out what is the maximum possible heat transfer rate which is possible which is which can be achieved in this heat exchanger also what is the value of epsilon. And secondly, we have to decide whether this should be operated in counter flow or in parallel flow. So, that is in a nutshell is a problem, first find the maximum possible heat transfer rate and secondly, whether it should be done in counter flow orientation or in parallel flow.

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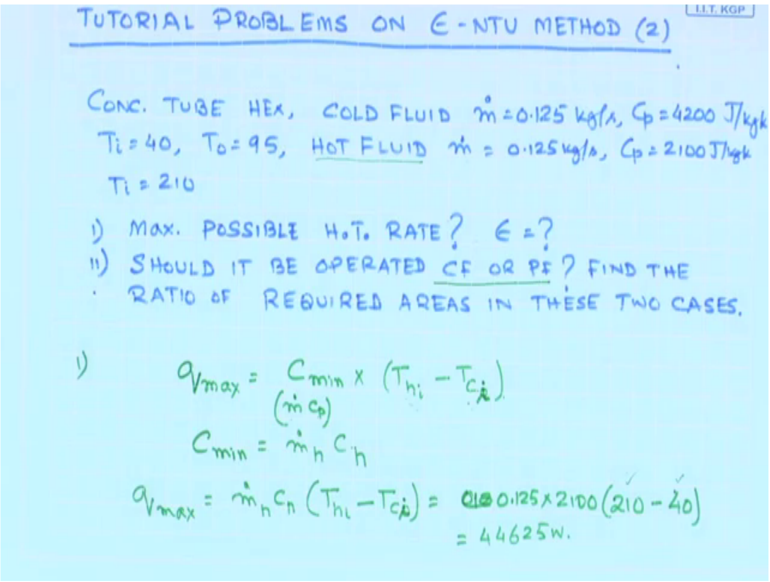

TUTORIAL PROBLEMS ON ϵ-NTU METHOD (2)

CONC. TUBE HEX, COLD FLUID $\dot{m} = 0.125$ kg/s, $C_p = 4200$ J/kgK
$T_i = 40$, $T_o = 95$, HOT FLUID $\dot{m} = 0.125$ kg/s, $C_p = 2100$ J/kgK
$T_i = 210$

i) MAX. POSSIBLE H.T. RATE? $\epsilon = ?$
ii) SHOULD IT BE OPERATED CF OR PF? FIND THE RATIO OF REQUIRED AREAS IN THESE TWO CASES.

i)
$$q_{max} = \underset{(\dot{m} C_p)}{C_{min}} \times (T_{h_i} - T_{c_i})$$
$$C_{min} = \dot{m}_h c_h$$
$$q_{max} = \dot{m}_h c_h (T_{h_i} - T_{c_i}) = 0.125 \times 2100 (210 - 40)$$
$$= 44625 \text{W}.$$

So, if it come now come to the number, so here what you see in this problem is it is a concentric tube heat exchanger, the cold fluid m dot C p, the hot fluid m dot and C p both are given. And temperature of the fluid which is entering at the cold fluid which is entering is 40 degree, the temperature of the outlet of the cold fluid is 95 and for the hot fluid the inlet temperature is mentioned as 210. So, if you see these 3 temperatures are given so; obviously, by simple heat balance the fourth temperature can be obtained. However, in order to find the maximum possible heat transfer rate and finally, to find out the epsilon we need to make our calculations based on epsilon NTU method and secondly, we need to find whether it should be operated in counter flow or in parallel flow. So, I started the at the first part of the problem which is to find out what is the maximum possible heat transfer rate.

So, we know that from our previous studies the maximum amount of heat transfer which is possible in any heat exchanger is C minimum, where C is the capacity which is nothing, but the product of m dot times C p wherever it is the minimum multiplied by the maximum possible temperature change that can take place. So, the maximum possible temperature change that can operate provided you use you give enough length of it is going to be T h i, which is the temperature of the hot fluid at the inlet and the temperature of the hot fluid at the outlet. So, all this m dot Cp etcetera are given. So, here if you look at the cold fluid and the hot fluid; obviously, the minimum of m dot Cp has to be for the hot fluid.

So, m dot Cp for the hot fluid is less than that of the cold fluid therefore, my C minimum the capacity minimum would simply be m dot of the hot fluid times C of the hot fluid, which is 0.125 times 2100 And therefore, q max can simply be calculated as the C minimum which is in this case is m dot h times C h and the temperature maximum possible temperature drop which is T h i minus T c o. So, when you plug in the numbers what you would get is the amount of heat transfer which is possible for this case times 210, that is the inlet temperature of the hot fluid minus what is the outlet what is the minimum temperature which is possible; this in this case this is going to be 40.

So, when you calculate the numbers it is going to be 44625 watts. So, this is what we have as the maximum heat maximum temperature that can happen in. In fact, this should be Tc i. So, therefore, this is the maximum temperature drop. So, the maximum temperature drop multiplied by C minimum, which is the minimum of m dot and C p and when you put this number, this is the maximum amount of heat which can be transferred in the heat exchanger. And of course, this is an idealized situation in order for the temperature of the hot fluid this temperature difference to be achieved in any heat exchanger, you have to provide infinite length of the heat exchanger heat exchanger pipes.

So, when you as we have discussed, the epsilon the maximum possible heat transfer that is achieved will take place, when the temperature of the hot the maximum temperature difference in existing in the system is achieved. So, therefore, the in counter flow that is only possible in counter flow when the temperature of the hot fluid at one end, and the temperature of the cold fluid at the other end at the inlet condition both will they both will change. And, the temperature of the cold fluid and the temperature of the hot fluid at the inlet this difference if, you can exchange all the heat between these two fluids that the maximum possible heat transfer which is given by the formula that I have just written it is m dot C minimum multiplied by the temperature difference maximum temperature difference that is possible in the system.

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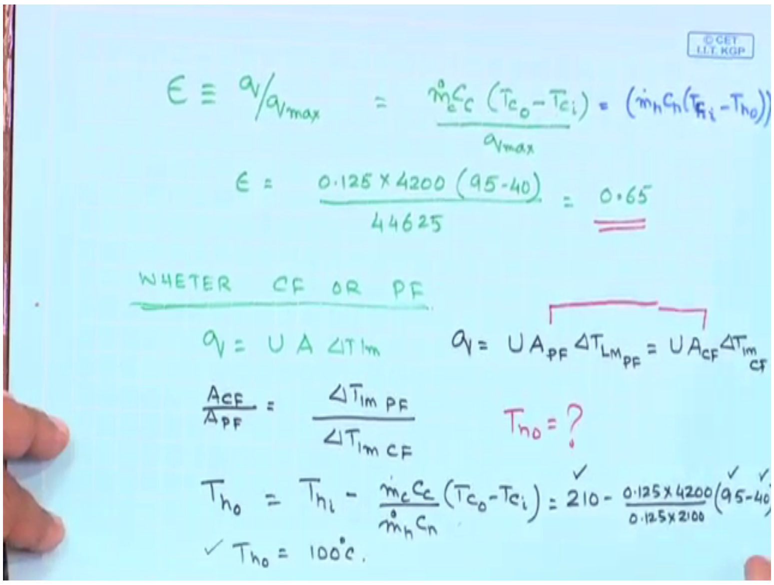

$\epsilon \equiv q/q_{max} = \frac{\dot{m}_c C_c (T_{c_o} - T_{c_i})}{q_{max}} = (\dot{m}_h C_h (T_{h_i} - T_{h_o}))$

$\epsilon = \frac{0.125 \times 4200 (95-40)}{44625} = \underline{\underline{0.65}}$

WHETER CF OR PF

$q = U A \Delta T_{lm}$ $\quad$ $q = U A_{PF} \Delta T_{LM_{PF}} = U A_{CF} \Delta T_{lm_{CF}}$

$\frac{A_{CF}}{A_{PF}} = \frac{\Delta T_{lm\,PF}}{\Delta T_{lm\,CF}}$ $\quad$ $T_{h_o} = ?$

$T_{h_o} = T_{h_i} - \frac{\dot{m}_c C_c}{\dot{m}_h C_h}(T_{c_o} - T_{c_i}) = 210 - \frac{0.125 \times 4200}{0.125 \times 2100}(95-40$

✓ $T_{h_o} = 100^\circ C$.

So, therefore, the next part is rather straight forward. The epsilon by definition is whatever, be the heat that is transferred divided by whatever be the heat the maximum amount of heat which could be transferred in this condition. So, the heat that is transferred is m dot of the cold fluid times C of the cold fluid and Tc temperature of the cold fluid at the outlet minus temperature of the cold fluid at the inlet which is the value of q divided by q max. You could write this also as equal to this is also equal to m dot h C h times T c at the T hot at the inlet minus T hot at the outlet.

But since, the tea hot at the outlet is not known to me at this point. So, therefore, I am simply using these values since they are known to me. So, what I do next is I will simply put in the numbers over here. So, the epsilon of simply be equal to 0.125 that is the mass flow rate times 4200, this is the capacity of the cold fluid times the temperature drop, which is 95 minus forty divided by the value of q max which we have already obtained as 44 625. I will go back to the previous slide, where the where in the description of the problem it is mentioned that the for the cold fluid the temperature change is be from 40 to 95.

Therefore, this temperature difference is provided and therefore, this value can be calculated as 0.65. So, that is the value of epsilon, which can which is obtained in the present scenario. In order to do the second part that is whether it should be run in counter flow or in parallel flow. In order to decide that; obviously, a heat exchanger is going to be would prefer a heat exchanger which would require the least amount of heat exchange area. So, the requirement of heat exchange area is the prime factor in deciding whether, to go for counter flow or to parallel flow. And of course, as an engineer in order to reduce the cost you would go for that type of flow, which would result in the minimum heat transfer area for a specific operation.

So, our next job is to find out what is the ratio of the 2 areas, required for counter flow and required for parallel flow and see which one is the smaller one. The one which would give the smaller area of heat transfer, smaller heat transfer area requirement would be the preferred mode of operation. So, you would calculate: what is the heat exchange area requirement for parallel flow, as well as for counter flow. So that is what we would do next and to decide which one it should be. So, let us start with this area requirement calculation. We understand that q is equal to U which is the overall heat transfer coefficient A and A times delta T l m dot

So, this is the log mean temperature difference, this is the heat transfer area which is required. So, therefore, the q would should be equal to U times area for the case of parallel flow, delta T for the case delta T L M for the case of parallel flow and this because of heat balance must be equal to U which remains unchanged; whether you have parallel flow or counter flow, area required in counter flow delta T log mean based on counter flow. So, these are this is a relation which we are going to use and therefore, I need to see which one of these two is going to be the smaller one. In order to do that, so I find out what is A counter flow divided by A parallel flow and what I see is that this U will cancel. And therefore, it is going to be the inverse ratio of the log mean temperature difference for these two cases. Therefore, this is simply going to be delta T log mean in the case of parallel flow by delta T log mean in the case of counter flow.

So, this area if I could evaluate it would give us the idea of which one is going to be the more efficient one. Now the unknown in here is I still do not know what is the value of T h o is unless I calculate what is T h o, I will not be able to evaluate these log mean temperature differences. And the T h o the heat transfer the temperature of the hot fluid at the outlet would simply be T h i minus m dot c C c by m dot h C h and T c o minus T c i. So, what we have done is I have simply used this relation where m dot Cc T co minus T c i is m dot C h T h i minus T h o; that is the simple heat balance equation; that means, the heat gained by the cold fluid must be equal to the heat lost by the hot fluid. And therefore, T h o is simply going to be this relation.

So, you put these values in here the T h i is 210 and this is 0.125 into 4200 and the denominator is 0.125 into 2100. And the temperature jump, temperature jump for the cold fluid is 95 minus 40, which would give you a value of T h o to be equal to 100 degree centigrade.

Therefore, I have I now know what is the temperature of the hot fluid at the inlet, temperature of the hot the cold fluid at the outlet, temperature of the cold fluid at the inlet and temperature of the hot fluid at the outlet. Since four temperatures are known to me, I would be able to calculate what is the log mean temperature difference when it is in parallel flow and when it is in counter flow.

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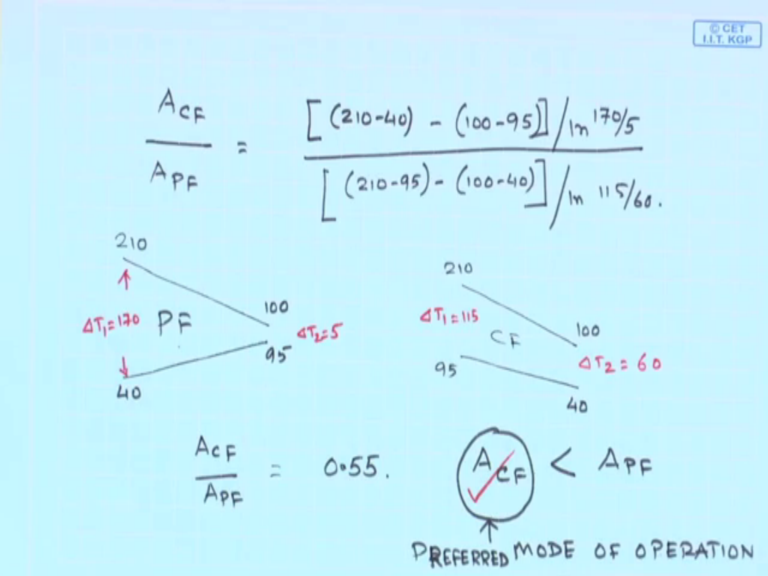

$$\frac{A_{CF}}{A_{PF}} = \frac{\left[(210-40) - (100-95)\right] / \ln 170/5}{\left[(210-95) - (100-40)\right] / \ln 115/60}.$$

$$\frac{A_{CF}}{A_{PF}} = 0.55.$$

$$A_{CF} < A_{PF}$$

PREFERRED MODE OF OPERATION

So, therefore, to write this the area required for counter flow divided by the area required for parallel flow would simply be the log mean temperature difference for the case of parallel flow which is, this is the delta T 2 minus delta T 1 divided by ln 170 by 50. And, for the case of count for the case of counter flow the l m T d for the counter flow would be 210 minus 95 minus of 100 minus 40, this divided by ln 115 by 60.

I would draw the profile once again, this is a hot fluid. This is the cold fluid when they are in parallel flow. So, the temperature changes from 210, which is the inlet temperature of the hot fluid, the outlet temperature is 100. So, I will put 100 over here and for the case of this one it is changing from 40 to 95. So, this is 40 to 95. So, therefore, the delta T 1 in this case, delta T 1 is going to be equal to 170 and the delta T2 would simply be equal to 5, so that is for parallel flow. Where is 4, the where when the flow is in counter flow what we have is the same 210, but here this is 95 and from 210 it reduces to 100 and this starts at 40.

So, therefore, the delta T1 for the case of counter flow is 210 minus 95 that is the , so this is equal to 115 that is the delta T1 and delta T2 would simply be equal to 60. So, that is my logic of evaluating the log mean temperature difference based on parallel flow and based on counter floor. So, when you do this when you evaluate this ratio, this A C F by A P F counter flow by parallel flow would turn out to be 0.55. So, which simply shows, shows that the area required in counter flow is less than the area required for parallel flow. So this is the one which should which is the which is the mode of the mode of operation, the preferred mode of operation is so, the preferred mode of operation is therefore, going to be the counter flow.

So, this simple problem would give you the, would clarify some of the idea about the maximum heat transfer rate which can be achieved, when the temperature of the cold fluid outlet which is the temperature of the hot fluid at the outlet. And then, what is the what is so that is the that is the maximum possible heat transfer. Then you find out what is the actual heat transfer, the ratio of these 2 would give you the idea of the effectiveness epsilon of the exchanger. Secondly, you are also finding out what is the area requirement in two cases, one in counter flow the other in parallel flow. And you would see that they are; obviously, that they are going to be in the inverse ratios of the log mean temperatures for these two log mean temperature differences for these two cases.

So, quickly draw the profile for parallel flow and counter flow as I have shown you. Find out what is delta T1, what is delta T2, calculate what is the ratio of the lm Td and that ratio would tell you: what is the area which one would require the smaller heat transfer area weather in parallel flow or in counter flow. And you would see that, the counter flow for this specific case is going to require the lesser amount of heat transfer area. So, that is going to be the preferred heat a preferred mode of operation for this heat exchanger. So, I guess this concept is clear to you now, I will quickly solve one more problem and then we will move on to the next topic. So, therefore, the next problem which we are going to deal with is again it is a double pipe heat exchanger and it is it uses hot and cold water.

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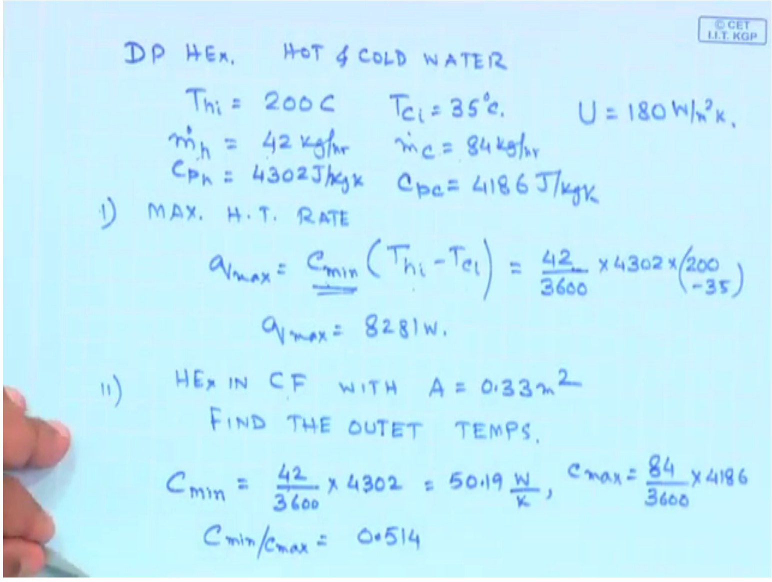

DP HEx. HOT & COLD WATER

$T_{hi} = 200 C$ $T_{ci} = 35°c.$ $U = 180 W/m^2K.$

$\dot{m}_h = 42 kg/hr$ $\dot{m}_c = 84 kg/hr$

$C_{ph} = 4302 J/kgK$ $C_{pc} = 4186 J/kgK$

1) MAX. H.T. RATE

$$q_{max} = \underline{\underline{C_{min}}} (T_{hi} - T_{ci}) = \frac{42}{3600} \times 4302 \times (200-35)$$

$$q_{max} = 8281 W.$$

11) HEx IN CF WITH $A = 0.33 m^2$

FIND THE OUTET TEMPS.

$$C_{min} = \frac{42}{3600} \times 4302 = 50.19 \frac{W}{K}, \quad C_{max} = \frac{84}{3600} \times 4186$$

$$C_{min}/C_{max} = 0.514$$

The T h i the inlet temperature of the hot fluid is 200 degree centigrade and the inlet temperature of the cold fluid is 35 degree centigrade, the flow rates of the hot fluid, so therefore, m dot h for the hot fluid is 42 kg per hour, m dot c is going to be equal to 84 kg per hour and the value of overall heat transfer coefficient is provided as 180 watt per meter square per Kelvin. So, the first one is find out what is the maximum heat transfer rate. Now if you look at the values of the m dot h, this one is going to be this is going to be less half of whatever be the cold water flow rate. So, therefore, the maximum heat transfer rate q max, as we understand is going to be C minimum multiplied by the temperature drop maximum allowable temp[rapture] maximum possible maximum achievable temperature drop in the system which is T h i minus T c i.

So, these two temperatures are known to me and note here that both the outlet temperatures unknown. So, this problem is a prime candidate for solution using epsilon NTU method. So, and this C minimum must be equal to m dot h since, it is flow rate is half of that and the value of c p is going to be roughly the same. In fact, the value of C p for the for the hot fluid is equal to 4302 joule per kg Kelvin and C p for the cold fluid is 4186 joule per kg Kelvin. So, therefore, the product of these two is definitely going to be less than the product of these two. So, C minimum is the hot one and therefore, if I put this in there, so 42 kg per hour, this makes it 42 kg per second multiplied by 4302, which is the C p of the hot fluid multiplied by delta T maximum delta T which is 200 minus 35.

And this q max is equal to 8281 watt. So, that is straight forward, the second one is if the heat exchanger is the heat exchanger is being of is operated in counter flow, so, the heat exchanger is in counter flow with area heat transfer area to be is equal to 0.33 meter square, find the outlet temperatures. So, if you if you once again the problem takes that the heat exchanger, the above heat exchanger is operated in counter flow with an area of 0.33 meter square, the value of U has already been provided find out the outlet temperatures. Now we can we can clearly see that, this is again a method the epsilon NTU method should be the preferred one because, both the outlet temperatures are unknown to us, I do not know what is T h o and also what is T c o.

So, we have to use epsilon NTU method for this case. So, in order to find out the epsilon NTU method, first we have to find out: what is the value of C minimum and what is the value of c maximum. Thus, the C minimum is going to be 42 by 3600 into 4302. This is the m dot h times C p h, the same thing we have done over here we should to be equal to 50.19 watt per Kelvin and similarly C maximum is going to be 84 is the flow rate in kg per hour, so divided by 3600 would be in kg per second multiplied by 4186, which is the which is the Cp of the cold fluid and when you take the ratio of C minimum by C maximum, this should turn out to be 0.514.

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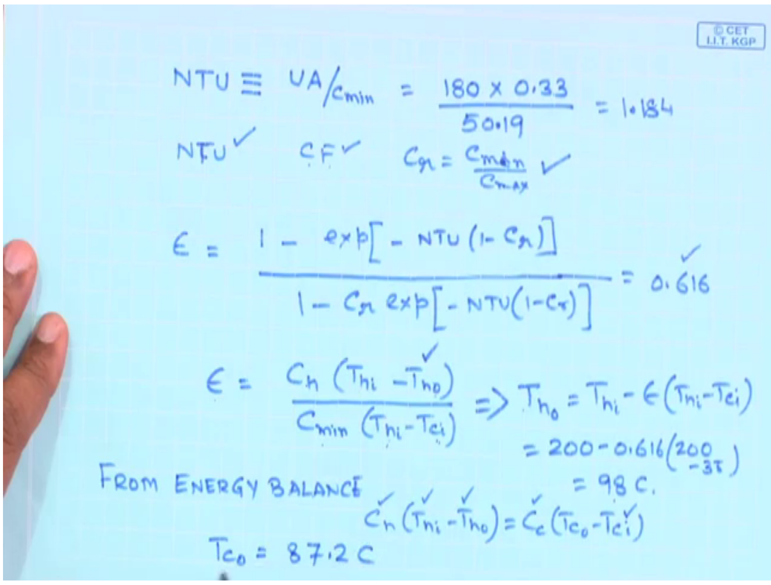

$$NTU \equiv UA/C_{min} = \frac{180 \times 0.33}{50.19} = 1.184$$

NTU ✓ CF ✓ $C_r = \frac{C_{min}}{C_{max}}$ ✓

$$\epsilon = \frac{1 - \exp[-NTU(1-C_r)]}{1 - C_r \exp[-NTU(1-C_r)]} = \overset{\checkmark}{0.616}$$

$$\epsilon = \frac{C_h(T_{hi} - \overset{\checkmark}{T_{ho}})}{C_{min}(T_{hi} - T_{ci})} \Rightarrow T_{ho} = T_{hi} - \epsilon(T_{hi} - T_{ci})$$

$$= 200 - 0.616(200 - 35)$$

$$= 98\ C.$$

FROM ENERGY BALANCE

$$\overset{\checkmark}{C_h}(\overset{\checkmark}{T_{hi}} - \overset{\checkmark}{T_{ho}}) = \overset{\checkmark}{C_c}(T_{co} - \overset{\checkmark}{T_{ci}})$$

$T_{co} = 87.2\ C$

So, that is a straight forward substitution of the values. The next is, we also know that from definition the NTU the number of transfer unit is due is due defined as UA divided by C minimum. So, the U provided is 180 watt per meter square Kelvin the area which is provided is 0.33 and the C min that we have just now calculated is 50.19. So, therefore, this is equal to 1.184. Therefore, I know my values of NTU, I know that this is be operated in counter flow and my C r which is C max C sorry C minimum by C max this value is also known to me. So, when all these are known to me, either you can read from the graph or you can use the relation which is provided for counter flow operation in a double pipe heat exchanger.

So, the expression that I am writing from the text and I am again mentioning that, if such a problem comes, the expression connecting epsilon and an NTU would be provided. You do not have to memorize this and you only have to identify that this is a counter flow situation and this request you need to find out what you have the value of NTU, you know what is the value of C r, that is the C ratio of C minimum and C maximum. So, choose the relation which represents this situation and try to and an use it to find either the unknown epsilon or the unknown NTU. In this case the unknown is epsilon, so I am using a relation epsilon equal to some function of NTU, C minimum etcetera. Had this been the case of unknown NTU then I choose the reverse relation where NTU is related to epsilon and other parameters.

So, the expression which is to be used in this case is 1 minus epsilon of minus NTU into 1 minus C r, where it is just the ratio divided by 1 minus C r exponential minus NTU times 1 minus Cr. Please look at these relations in your text and be sure to use the right relation for the system at hand. So, when you put these values in here I am not doing all the calculations, you would you would see that this epsilon would turn out to be 0.616. So, that is the value of epsilon which you should get for such for after you put in the values in there. We also know that, epsilon is the actual heat transfer which could be C h multiplied by T h i minus T h o, as well as the maximum heat transfer which is possible that is T minimum T h i minus T c i.

I will repeat once again, I have now the value of epsilon evaluated. I also know the definition of epsilon, which is the heat transfer that is taking place which could be T h i T which could be T h i minus T h o multiplied by C h divided by the maximum allowable heat transfer possible in such a situation which is T C minimum multiplied by the maximum drop into maximum change in temperature that is possible in the heat exchanger. So, this in here, you would see that the unknown is T h T h o. So, therefore, from here you can calculate what is T h o to be T h i minus epsilon of T h i minus T c i. So, when you plug in the values in here this should be 200 minus 0.6 616, which is 616 multiplied by 200 minus 35.

So, this should be equal to 98 degree centigrade. So, the temperature of the hot fluid can be evaluated as simply 98 degree centigrade. And from energy balance they are other unknown is what is T c o, the temperature of the cold fluid at the outlet, so from energy balance you can write that C h times T h i minus T h o is C c times T c o minus T c i. So, this two unknown T h i is known, T h o is, T c i is known, so the only remaining unknown quantity T c o which can be evaluated as 87.2 degree centigrade. So, this again shows you the utility of epsilon NTU. In this specific case you do not know either of the outlet temperatures, what you know are the mass flow rates, the specific heat capacity, the overall heat transfer coefficient and the heat transfer area for heat transfer area available for such operations.

So, using an LNTT approach to solve for this would require a long a iterative procedure. So, epsilon NTU give provides an alternate elegant and shortcut method to find out what are these two unknown temperatures. So, the all the problems that I have solved in the last class and in this class and this course the utility of epsilon NTU method and the concept of epsilon is very important which is just the ratio of the actual heat transfer divided by the maximum heat transfer possible in an exchanger provided you give enough length; mathematical speaking ideally infinite length such that the maximum achievable temperature drop is possible. Once you have the value of epsilon known to you and the other parameters are known to you then you should be able to calculate: what are the outlet temperatures for the hot fluid, as well as for the cold fluids.

So, I hope that with these solved problems you are in a position now to solve problems on your own and I would request you to look at your text book or any other text book in heat transfer, see some of the problems which have been solved in the book, as well as do exercise problems on your own. And, also do they are the tutorial sheets which would be provided and the assignments which would be provided. And, if there are any questions the TA and I would be available to answer them online and if there are any confusion please do not hesitate to ask us and we will try to help you to master this specific technique which is commonly used when both outlet temperatures are not known to us.

So, in the next class we will discuss some other modes of some other types of heat exchange equipments which are also very important in industry. For example, evaporators, the forward feed evaporators, the backward feed evaporators, their advantages and disadvantages and types and so on. And after that is over we will then move on from conviction to the last part of this course, which is going to be the radiative heat exchange between surfaces.