Heat Transfer
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Lecture – 29
Tutorial Problem in External Flow and Convection

We are going to solve another very interesting problem on convective heat transfer where the flow, air flow is taking is taking place over a flat plate. Now, this flat plate is slightly interesting, because this flat plate consists of several flat plates one after the other. So, there of the same size, so, one flat plate a slice of a flat plate is put next to slice of another flat plate and another one another one and so on. So, all my fingers essentially are they consist deform the flat plate with each section is thermally isolated from the next.

So, for all practical purposes they are plates like this where they are thermally separated from each other, but together they form a flat plate of uniform width with uniform width, but sections like this that are thermally insulated from each other.

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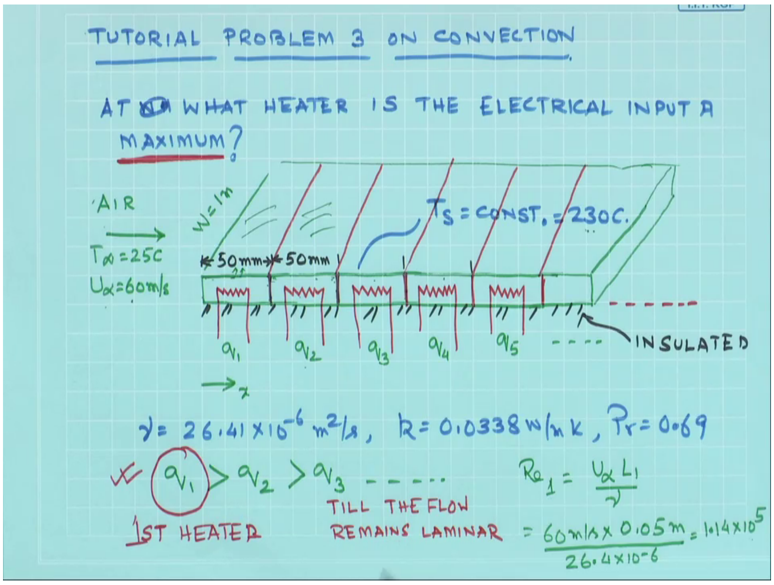

So, I have drawn the figure over here, where you can see that this is one strip of 50 millimeter in length next strip same length, same and so on. So, it continuous like this and the width of the plate is 1 meter. Now, each if you if you now concentrate only on this is part each small part is insulated isolated from the next small part and so on. Each of these small parts will have a heater connected to it which is going to provide some heat which is equal to q 1 for strip one, q 2 for strip two, q 3, q 4 and so and so on.

The purpose of these strip heaters the purpose of these heaters is to maintain the temperature of this entire plate at a constant value of 230 degree centigrade. So, each heater would probably have to provide different amounts different quantities of heat to these sections of the plate. Since, the value of heat transfer from over here is going to be different from the value of heat transfer over here and so on. So, each of these heaters will then have to provide different amounts of heat to these small sections to maintain the temperature of all such sections at a constant value of 230 degree centigrade and air at a temperature of 25 degree centigrade and with a velocity of 60 meter per second flows over this.

So, that is going to with convective heat transfer and since the temperature of the plate is more than the temperature of the air each of them will lose some amount of heat to the flowing air stream. But the amount of heat lost to the flowing air stream would definitely depend on the value of the local convective heat on the value of the convective heat transfer resistance. So, the problem then is identifying that the heat transfer resistance for each of these strips will be different from each other.

So, the amount of heat to be provided by each of these heaters would have to be regulated in such a way that all these strips will be maintained at a constant temperature of 230 degree centigrade. The question that is asked is at what heater is the electrical input a maximum? That is the question which has been put to us. We understand that each of the heaters are providing different quantities different values of heat q 1, q 2, q 3, q 4 and so on.

So, the question and these strips are not limited to two to six strips. There are six or seven strips, they go on to maybe hundred such strips. There are hundreds of heaters connected to and there are hundreds of heaters then. So, we need to find out which heater among all these would have to be provided the maximum would have to be would have to deliver the maximum amount of heat in order to maintain the temperature at a constant of 230 degree centigrade.

Now, first of all which you tells I mean how is heat delivered by the heater related to heat transfer coefficient. We understand that higher the value of heat transfer coefficient, higher would be the value of value of heat which would be dissipated from the hot plate to the following air and higher is the heat dissipation from the plate to the air more and more heat would have to be provided by the heater which is connected to that specific strip of 50 millimeter length.

So, the question then boils down to how do we calculate the variation of heat transfer coefficient on width length on each of these heaters and find out which is the maximum, where the value of h is maximum more heat would be dissipated and more heat would have to be supplied by the electrical heater. But as I am I have mentioned before there are hundreds of heater hundreds of strips like that. Are you going to calculate each one of them separately to find out where your heat transfer coefficient where your q is the most or is there any shortcut to find out logically based on your heat transfer studies so far?

You would be able to say that, no, I do not have to calculate the heat to be provided to each of this heaters. I only have to take a look at first, second or maybe third, three or four possibilities if I take into account if I calculate the heat to be provided to those three situations and find out which is the maximum, then definitely I have the max over all maximum of heat to be provided by the heater at that specific location.

So, how do I find out the possibilities that one must examine to arrive at the solution for this problem that is at which heater the heat input has to be the maximum? So, when we think about this problem I think the problem statement is clear to you there are each individual segments which form the entire plate. Each segment has a heater attached to it. So, in air flows over air at constant temperature flows over all the segments, which heater has to provide the maximum heat that is the question. So, in order to do that, I will again go back and show you the variation of heat transfer coefficient with actual distance that I have plotted in the last class.

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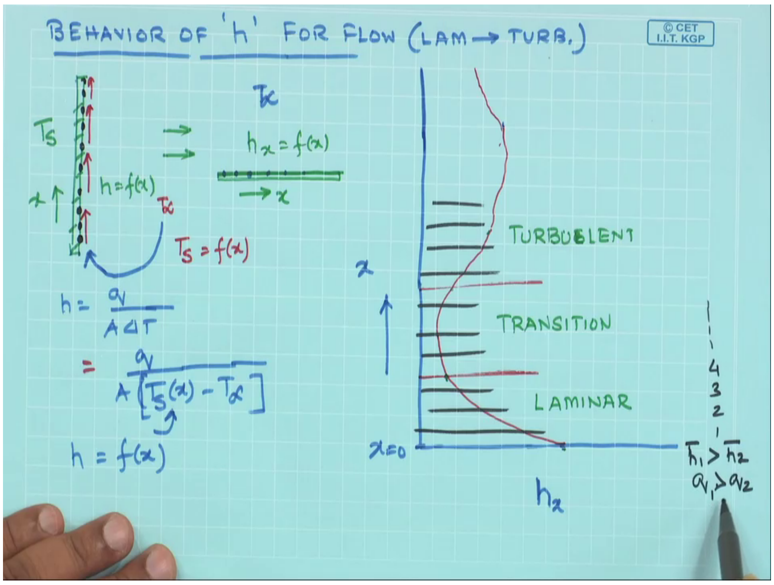

So, let us take a look at this once again the page that the figure that have drawn over here. The value of heat transfer coefficient is very high in the beginning it reduces, it goes to transition, its turbulent and keeps on increasing, keeps on increasing and then at some point of time it will start again decreasing as the value of the as the value of the thickness of the turbulent boundary layer will keep on changing. So, why do I think the heat transfer coefficient and you have the so called strips like this of equal thickness.

So, let us call them is 1 2 3 4 and so on. So, which is which are going to be the ideal candidates for which I have to find out what is the heat transfer. Let us see between 1 and 2. When I look at the amount of heat transfer between 1 and 2, I can clearly see that the heat transfer coefficient over the first region is going to be more than that of the second region and more than that of the third region and so on.

So, I can safely say that q 1 is going to be greater than q 2, since h 1 is greater than is greater than that of h 2 or since we are talking about the average value of heat transfer coefficient over the entire region I should write h 1 bar which is the length average value of heat transfer coefficient over region 1 and this is a length average heat transfer coefficient over length 2 and since h 1 is greater than h 2 more heat will have to be supplied by the heater to strip 1 as compared to strip 2. So, therefore, in this based on our understanding so far we understand that q 1 is going to be greater than q 2 and going to be greater than q 3 and so on, but to up to which extend? Is it going to be q 4, q 5, q 6 and it continuous? It does not continue like that if you again if you look at the behavior of heat transfer coefficient how it changes when it goes from laminar to turbulent?

So, as long as the flow is laminar the value of h will continually decrease. So, for the entire range of laminar for the entire laminar range you only have to find out what is the value of q 1, since you know that q 2, q 3, q 4 etcetera are all going to be smaller than q 1. So, of all these then I have written. So, far I need to calculate: what is the heat that is to be dissipated to be supplied on the first heater. So, this first heater is definitely going to be one of the contenders of my one of the contenders of my calculation. So, I must calculate the value of heat to be supplied by heater 1.

On the side I can find, I can calculate: what is the value of Reynolds number based on one and just to ensure that I have laminar flow in this case. So, Reynolds number for the case of 1 would be U 1 L 1 by nu and this would be 60 since the flow velocity 60 meter per second times 50 millimeter, 0.05 meter divided by nu which is provided as 26.4 into 10 to the power minus 6. So, this turns out be 1.14 into 10 to the power 5. So, the entire first heater first strip is under laminar flow. This way it will continue if I find out what is R e 2, what is R e 3, R e 4 and so on till the point when I heat 5 into 10 to the power 5 because I understand that Reynolds number of 5 into 10 to the power 5 the flow becomes turbulent.

So, whatever we were discussing so far would be valid this would be valid till the flow remains laminar I think that is clear to clear to all of you that till the flow remains laminar the value the value of the value heat to be provided to the first heater is going to be the maximum as compared to any of the subsequent heaters well while all of them are in the laminar zone. So, if I have to calculate one heat input in the laminar zone it has to be in the first heater. Now, let us see what is going to happen to this when I have the changeover from laminar to turbulent.

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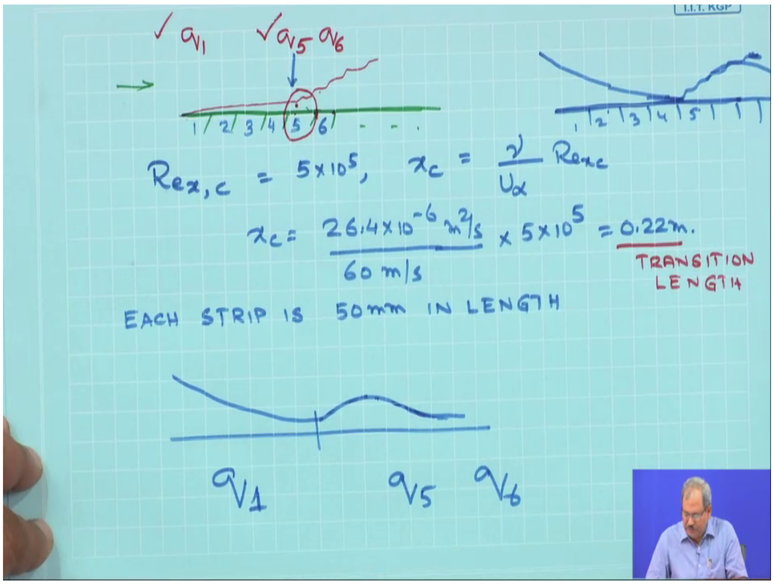

So, this is my plate and I have 1 2 3 4 5 6 and so on the flow takes place over here I need to find out what is at which point the flow becomes from laminar to turbulent. So, in order to do that I find out I understand that Reynolds number based on length at the value of transition is 5 into 10 to the power 5. So, therefore, the value of x c the length at which the transition takes place can be simply written as the kinetic viscosity by U infinity and R e x c.

So, when you put the values over here and the value of Reynolds number at transition is taken as 5 into 10 to the 5 which is 0.22 meter. So, if it is 0.22 meter an each strip is 50 millimeter in length. So, therefore, the transition takes place on 1 2 3 4 5th heater. So, fifth heater is the one over which the flow is going to start become turbulent this is clear from the value of x c the length of transition. So, this is the transition length.

So, since turbulence takes place the changeover from laminar to turbulent takes place on fifth heater the value of heat transfer coefficient in this region is going to significantly jump from the value of the heat transfer coefficient which existed prior to the point where it changes from laminar to turbulent. In other words value of the heat transfer coefficient from this point onwards is going to be very high as compared to the heat transfer coefficient that existed before this.

So, if my if I need to calculate the value of q 1 to ascertain which is going to give me the highest value of heat be applied I should also calculate and find out what is q 5 because q

1 is where the flow is entirely laminar, the film thickness is the list and therefore, there is a possibility that this q 1 is going to be not only the maximum amount of heat transfer that is maximum amount of heat transfer that is taking place in the laminar region, but it also going to be a global maximum when we consider all the heaters subsequently.

So, of course, between 1 to 4, q 1 is going to be the maximum, but I think that there is a possibility that q 1 is going to be the highest among all. So, there is I definitely need to calculate the value of q 1. Similarly, q 5 is the one in which the flow is going to be partially laminar and then it is going to become turbulent and since we know that heat transfer in turbulent flow is much more as heat transfer in turbulent. So, q 5 is the one in which the first turbulent flow appears. So, the chances of q 5 being the global maximum cannot be discounted so, I must find out what is the value of q 5 as well. So, in between the first and the fifth heater I need to calculate the value on the first heater and the value on the fifth heater.

Now, let us see what is going to happen on the sixth heater. Sixth heater is the first heater where the flow is turbulent entire flow is turbulent, fifth is partly laminar then turbulent, sixth is entirely turbulent. So, sixth is entirely turbulent and we understand that turbulence increases the value of the heat transfer coefficient then q 6 is also a potential contender for being the maximum in terms of the heat to be supplied to the heater

So, what is the difference between q 5 and q 6? q 5 gives you the heat which is to be supplied when part of the flow is in laminar zone the rest is in turbulent zone. So, I need to calculate the heat transfer heat to be supplied to q 5 and compare that with q 1 and now, I am going to q 6; q 6 is the first heater where the flow is turbulent at the beginning and turbulent at the end. And since turbulent promotes heat transfer so, of course, q 6 has to be a contender and the value of heat to be supplied to the sixth heater must be compared with the value to be supplied to the fifth heater and the value to be supplied to the 6 heater true to the first heater.

What happens to seventh heater and on? As we move into seventh heater eighth heater ninth and tenth the value of the turbulent boundary layer thickness thermal boundary layer thickness will keep on increasing. So, as it starts to increase the value of the heat transfer coefficient will be will be decreasing. So, the highest value of heat transfer coefficient in turbulent flow will remain on fifth partially and on sixth. Any point beyond 6 the value of the heat transfer coefficient will be less than that of 6.

So, with increase in x the value of the heat transfer coefficient will start to reduce and therefore, q 7 will definitely will less than q 6, q 8 would be less than q 7, q 9 would be less than q 8 and so on. So, therefore, of all the conditions that we can think of for flow over a flat plate where the flow changes from laminar to turbulent there are only three heaters. Where we have to calculate the amount of heat to be supplied by each heater in order to ascertain which is going to be the highest one which are the first heater where the flow is entirely laminar.

The fifth heater where the transition from laminar to turbulent takes place and the next which is the 6 heater where the flow is entirely turbulent; So, q 1, q 5 and q 6 calculate these three and find out which is which is which has the highest value. So, that is what we are going to do next and I think that you understood then logic of this. If you if you plot the value of heat transfer coefficient for 1 2 3 4 5 6 7 and so on it is going to be high, it will progressively come to 1 2 3 4 5 and then suddenly it starts to increase and it would be like this. So, the behavior of this would be something like this.

So, this is where the transition take place, it increases initially, but then as the heat transfer as the thermal boundary layer thickness increases it is starts to decrease again. So, it is the q 1, q 5 and q 6 which these are the three which we need to calculate.

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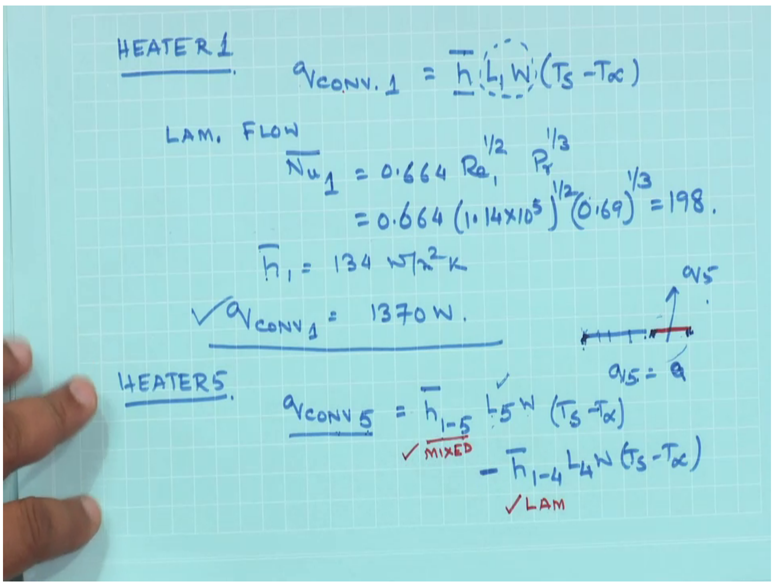

So, let us start with the heater 1, I need to find out what is q convection for one which would obviously, the h bar L 1 times W, T of S minus T infinity. So, the area available for heat transfer is this is the heat transfer coefficient average since averaged over the entire length of the first one T S minus T infinity and this is definitely as we have calculated it is definitely under laminar flow.

So, the average value of Nusselt number for 1 from our relation 0.664 R e to the power half Prandtl to the power one-third and when you put the values in here, this is the value of the Reynolds number the value of Prandtl number is provided as 0.69 to the power one-third which when you calculated it would be 198 and h bar 1 would simply be 134 watt per meter square Kelvin. And therefore, q convection 1 would be 1370 watt when you put in the values of h in here L 1 is 50, W is 1 meter T S and T infinity are known. So, you calculate what is q convection 1 so, this part is done.

Now, I have to find out what happens on heater 5. The fifth heater the q convection 5, the fifth heater let us say this is the fifth heater, ok. I need to find out what is q 5 and q 5 is in order to find q 5 you remember everywhere I am finding out the average value of the heat transfer coefficient. So, q 5 is simply the would not the amount of heat which is lost from the fifth heater q convection 5 should be h bar the average value of heat transfer coefficient between 1 to 5 between 1 to 5 times the total area which is L 5 times W. So, it is 5 times the length times T S minus T infinity minus of h bar 1 times 4 L 4 W T S minus T infinity. If you if you understand this then we can move on then the rest is simple.

I need to find out exclusively what is the heat to be supplied to the fifth to the fifth stream. Now, in order to do that I need to find out what is the heat transfer coefficient over here; that means, the value of the average heat transfer coefficient from 0.1 from this point to this point, but we do not have any such correlation whatever relation that we have would give me the value of the heat transfer coefficient from the very beginning. So, I can find out what is the heat average value of heat transfer coefficient from this point to the beginning of the first fifth heater and from this point to the end of the fifth heater.

So, I do not have any way to find out what is the heat transfer coefficient where the starting point is this and the endpoint is this. In order to circumvent that problem what I am going to do is I am going to find out the total amount of heat lost from this point to the end which is h bar average heat transfer coefficient considering the length from the very beginning up to the fifth heater the length the area which encompasses the total area of five heater, so, it is L 5 times W times delta T and subtracting from this total heat the total heat which is lost by heat from by strip 1 to 4. So, from the total heat between 0 to 5 and I subtract that from the total heat between 0 to 4 this, the result would give me the heat which is lost from the fifth strip only.

So, that is why I have written it in this way and I understand that this is going to be 5 times the length of each strip this is going to be the 4 times that of each strip and this allows me to find out what is q convection on 5. Now, we also understand that 1 to 4 is under laminar condition, but one to 5 when I go to the 5 one this is under mixed condition. So, this 1 to 4 is to be evaluated using laminar correlation and 1 to 5 is to be is to be evaluated is in the mixed condition mixed relation.

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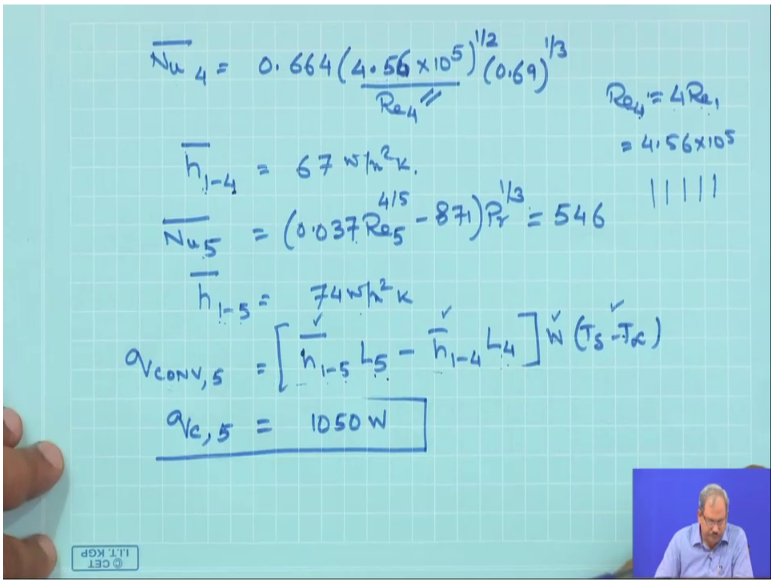

So, what is the average Nusselt number for 4 since it is laminar and simply going to find out the value of the Reynolds number. So, Reynolds number based on basically 4 to the power half, Prandtl number remains unchanged. So, I have calculated Reynolds number up to 4. So, Reynolds number up to 4 is simply 4 times Reynolds number at one since the length is the base is 4.56 into 10 to the power 5. From this I calculate this h bar average value of heat transfer coefficient considering the length up to the fourth heater to be equal to sixty 7 watt per meter square Kelvin. In contrast the Nusselt number average length average value of Nusselt number constrain the fifth heater, since it is in mixed flow I am going to use the mix flow relation which we have obtained before.

So, this is Reynolds number based on the 5 heaters to be 5 strip together minus 871 Prandtl to the power 1 by 3 which when you put the values would come to be 546 and this would give you h bar 1 to 5 to be equals 74 watt per meter square Kelvin. So, just look at this once again the when I finding out the average heat transfer coefficient up to the fourth heater everything is under laminar flow. So, I use the laminar relation accept the Reynolds number here is going to be Reynolds number up to 4 which is 4 times Reynolds number up to one since these strips are of the identical length.

So, I can calculate the Reynolds number and then I can calculate: what is the heat transfer coefficient, when I consider the entire region from 1 to 4. When I consider the entire region from one to 5 it is in it is in a mixed region and so, I use the mix correlation to find out what is the value of this, what is the value of this one. So, the q convection 5 q conversion for fifth heater would simply be equal would simply be equal to h bar 1 to 5 L 5 minus h bar 1 to 4 L 4 and this entire thing is to be multiplied by W times T S minus T infinity.

So, when you see this first part would give you the total amount of heat to be supplied between 1 to 5 total amount of heat to be supplied between 1 to 4, you know the values of this, you know what is L 5, what is L 4, W is known, this part is known so, q convection can be calculated and it would turn off to be 1050 watt. So, this is your q convection 5. So, that is another one.

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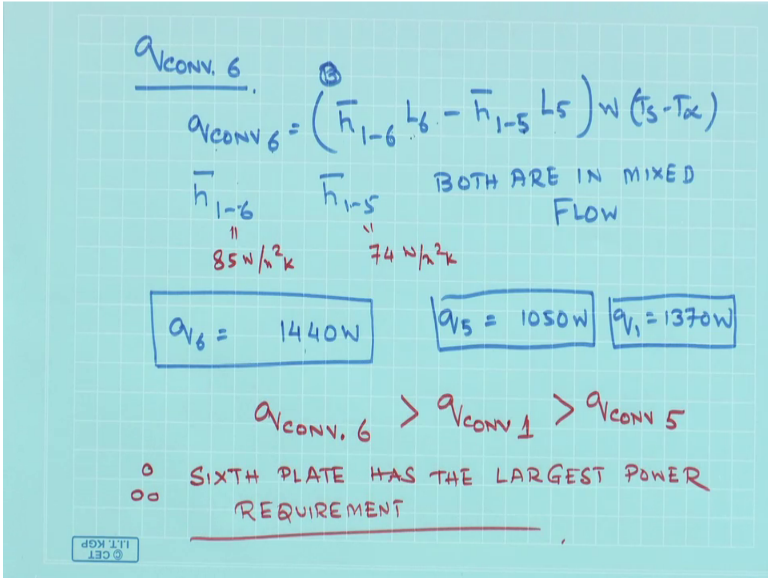

Now, how about q convection 6? For the six one where it is both so, q convection 6 can be as before can be calculated as h bar 1 to 6 L 6 minus h bar 1 to 5 L 5 times W T S minus T infinity, exactly the same logic as before. All the heat to be supplied up to the 6 heater, all the heat that is to be supplied up to the fifth heater, subtract one from the other and you find out exclusively what is the heat to be supplied on the sixth heater.

Now, in order to h bar 1 to 6 and h bar 1 to 5 both are in mixed flow condition, since the transition has taken over transition has taken over here. So, you can find out what is the value of h bar 1 to 6 this would turn out to be 85 watt per meter square per Kelvin and the same way then q 6 would be calculated and this we have previously evaluated to be equal to 74 watt per meter square Kelvin. So, when you plug in these values over here and find out and use the correct value of L 6 and L 5 what you would get is 1440 watt. So, this is a value of q 6 that you are going to get.

So, what you would then see as the final result when you and the q 5 you have evaluated to be 1050 watt and q 1 you have evaluated to be equal to 1370 watt. So, when you compare between these three you see that q convection 6 is going to be greater than q convection 1, which is greater than q convection 5 and therefore, sixth plate has the largest power requirement. So, I guess you understood the problem that we have just now solved. It requires not only an idea of which correlation to be used for which situations for laminar a relation, for mixed another relation.

Now, we have also discussed and in depth how would the heat transfer coefficient vary based on the flow region. So, when flow starts it starts as laminar flow in most of the situations and the value of the heat transfer coefficient is the largest. Since the thickness of the boundary layer is the least. As you move along the plate the thickness of the boundary layer thermal boundary layer keeps on increasing, as the thickness increases the value of the heat transfer coefficient start to decrease. So, whatever be the average value of heat transfer coefficient on one first segment is definitely going to be more what you have on the second segment, third segment and so on. So, h the average h is a monotonically decreasing function of distance as we move along the plate provided we have laminar flow.

Then comes the transition so, we have we assume that the transition is a is a sharp point at before which the flow is entirely laminar, beyond which the flow is turbulent the moment transition takes place there is going to be sudden spurt, sudden increase in the value of the heat transfer coefficient because of the onset of turbulence. But this sudden increase as the turbulent boundary layer keeps on increasing the effect of turbulence will be more than offset by the by the rapid increase in the thickness of the turbulent thermal boundary layer.

So, it is starts with high value, but immediately stocks to decrease, though the value of the heat transfer coefficient will be higher than that of the laminar part of the thermal boundary layer. So, initially laminar high, sharp decrease, sudden increase and then again gradual decrease this is how the heat transfer coefficient behaves for convection, for in external flow and this concept is fundamental I hope you have understood it correctly. And once, you must once you understand this concept then the tutorial problem that we have just now solved will tell you this concept, will tell you that only three possibilities are to be considered to find out where you are going to get the maximum value of the length average heat transfer coefficient.

We do not have to do it for all, do it for the first one laminar flow where the heat transfer coefficient is very high. Also do it for that for that plate over which the transition takes place because you have a sudden increase in the value of the heat transfer coefficient. Do it for the subsequent one as well because that is the first plate which is going to be under turbulent flow from the very beginning till the end. So, turbulent first plate, transition in between and the next to that, but the flow is turbulent; these three would be the only possible choices that you need to examine in order to find out which is going to give you the which is which plate is going to required largest amount of electrical energy to maintain it is temperature at a constant value.

So, this is a direct example of how the relations and the concept that we have developed in the first few classes for external flow heat transfer can be utilized for a practical problem; in finding out the amount of heat to be provided in order to maintain a surface at a constant temperature, which is requirement in many practical applications. We will solve few more problems in the coming classes, before we move on from external flow to internal flow.