Heat Transfer
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Lecture - 27
Mixed Boundary Layers

In the previous class, we spoke about Turbulent Boundary layer, Turbulent thermal boundary layer and Hydrodynamic boundary layer. We have also introduced the concept of similarity, analogy that can be obtained between Heat transfer and Momentum transfer. With the analogy that means, writing the governing equations in dimensionless form, identifying the similarity parameter and also identifying that after a specific value of one of the similarity parameters that is Prandtl number being equal to 1.

And also noting that the dimensionless boundary conditions are same for flow over a flat plate in laminar or in turbulent flow on the surface of the plate and in the free stream, we could make the two governing equations identical such that two systems; one undergoing momentum transfer and the other undergoing heat transfer can be made dynamically similar. So, when that happens? A solution of one can be used as a solution of the other.

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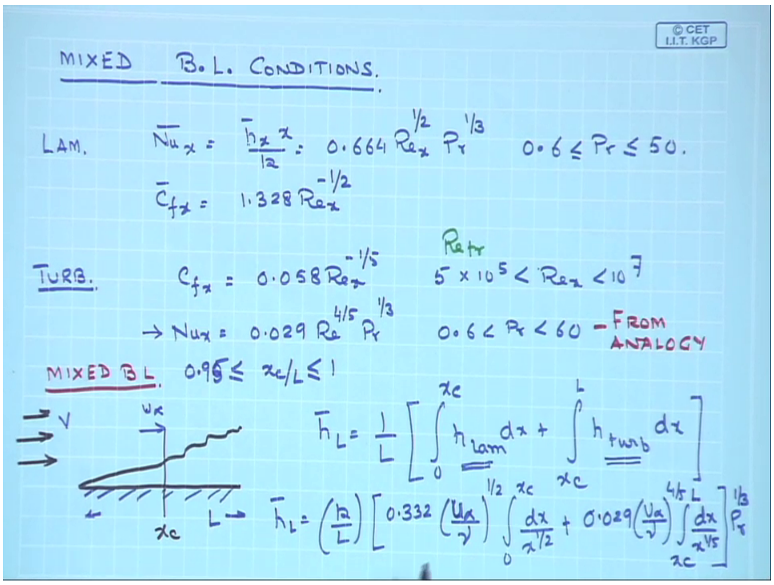

So, let us look at what we have what we know so far and we are going to look at what is boundary layer mix boundary layer; that means, we would understand that in real situations, boundary layer should always start as laminar and after you cross a certain value of Reynolds number which we have decided to be equal to 5 into 10 to the power 5, it changes from laminar to turbulent. So, we have got these relations based on our analysis as presented before that the coefficient of friction, it is the average value is 1.328 Reynolds the power half and Nusselt number would be of this form and it is valid for a Prandtl number range between 0.6 and 50.

In yesterdays class, we have seen that the turbulent flow can be expressed in the C f the friction factor for turbulent flow can be expressed in this form; where the Reynolds number lies between 5 into 10 to the power 5 which is the Reynolds number at transition, Retr and it should be between 5 into 10 to the power 5 to 10 to the power 7. And from analogy modified Reynolds analogy or Chilton Colburn analogy what we saw is that the Nusselt number is going to be a function, Reynolds and Prandtl and the functionality relationship can be expressed in this way for a Prandtl number range between 0.6 and 60 which you have obtained from analogy.

But we understand that the situation is not going to be purely turbulent from the beginning. It starts to become for turbulent from some point which is denoted by x c in the figure. So, if I have a flow over a flat plate and if I have simultaneous heat and momentum transfer, when we talk about heat transfer then it is going to be the thermal boundary layer is going to be laminar and then, it becomes turbulent. So, this expression for turbulent flow heat transfer coefficient is valid only when the flow is turbulent from the very beginning.

That can happen if there is if there is some sort of turbulence promoter at the edge of the plate which makes the flow turbulent which this disturbs the flow in such a way that it becomes turbulent even though the value of Reynolds number at that point is very small. So, in the absence of turbulence promoter you are always going to have a laminar part first and then, it becomes turbulent. So, the equation that we have here can only be used if this is the total length; let us say the total length of this is L.

So, the total length if this ratio there is x c by L, if it is if it if it varies within 0.95 and 1; that means, for most. So, x c by L; where L is the total length of the plate, if x c by L is greater than equal to 0.95; that means, if the transition occurs at the very beginning, then one can use this equation for the entire length even though we understand that there has to be some sort of a some sort of a of a laminar flow to start way.

However, in order to remain more in tune with us with what happens in the real situation the expression for h L should take into account the value of the laminar heat transfer coefficient, the value of the turbulent heat transfer coefficient. And if I am making an average of the heat transfer coefficient, the length average heat or convective heat transfer coefficient, should need to integrate this from 0 to x c; where x c is the distance at which the flow takes from laminar to turbulent and substitute for h laminar, this expression and from x c to L the flow is turbulent.

So, the expression for h turbulent will have to be substituted from here. So, from 0 to x c and x c to, if we leafy integrate with the appropriate expression of the convective heat transfer coefficient that is applicable in the flow domain, then and dividing the whole thing by L I should be able to obtain what is the average value of the heat transfer coefficient.

So, this can then we expanded this can after you put the values over here this h bar L would simply be equal to k by L which would be taken outside. And it is going to be 0.32 U infinity by nu. U infinity is this is U infinity and this is V and we have discussed previously that V would be equal to U infinity for flow over a flat plate which is also known as the Zero pressure gradient flow.

But in order to remain a keep the generality of the solution intact, we are using u infinity over here. Infinity to the power of half integration from 0 to x c d x by x to the power half, you would get this expression simply by substituting the expression of h x from here to this and then, for the case of turbulent this I am going to use this expression. So, it should give me 0.029 U infinity by nu to the power 4 by 5 and the integration here would be from x c to the total length d x by x to the power 1 by 5 and the Prandtl number dependence 1 by 3 would come over here.

So, I have done anything additional here, I just identified that up to x c it is remains laminar beyond x c it becomes turbulent. So, in order to obtain an average value of the heat transfer coefficient which is something that you would like to have while making any engineering calculation; you are not interested only with the laminar flow heat transfer coefficient or the turbulent flow. You rather like to know, what is the overall heat transfer coefficient? What is the length averaged heat transfer coefficient over the entire plate over which the flow is taking place?

So, in order to obtain let us see the heat that is lost from a hot surface when it is in contact with a flowing fluid, you simply use the average value of a heat transfer coefficient; average over the entire length. So, the heat loss from the plate would be h bar L a of the plate area of the plate multiplied by temperature of the surface minus temperature of the fluid at a point far from the plate or in other words it would be h bar L a times T s minus T infinity.

So, your if you want to tell if want to find out the heat loss from a slice on the plate, you have to use the local value of heat transfer coefficient. At that point if you want to find out what is the total heat loss from the substrate, then you have to use the average value of heat transfer coefficient. And what we are doing right now is finding an expression for the average value of the heat transfer coefficient when both laminar and turbulent flow exist on the same plate.

So, up to x c, it is laminar and beyond x c, it becomes turbulent. So, let us the whole idea the aim a objective of this exercise. So, now let us look at would this become. So, this is this is what the complete expression is and once you integrate this what you are going to get here is Nusselt number based on the length.

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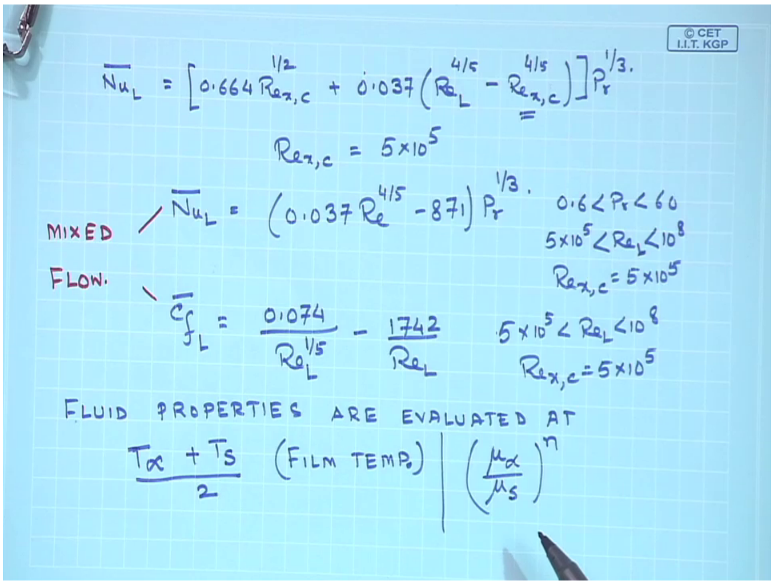

The average value of Nusselt number would be 0.664 Reynolds number local value at transition to the power half plus 0.037 Reynolds number based on the total length of the plate to the power 4 by 5 minus Reynolds number x c at the transition 4 by 5 and Pr to the power 1 by 3. So, this over here I can put Re x c that is a Reynolds number at transition to be equal to 5 into 10 to the power 5 as per our assumption; as per a what are we have discussed before.

So, when you do that the critical Reynolds number value when you take this to be equal to 5 into 10 to the power 5, the Nusselt number average value of Nusselt number then length entire length average value of Nusselt number when you have both laminar. And turbulent flow would simply be 0.037 Reynolds power 4 by 5 minus 871 to the power into Prandtl to the power one-third. So, this is of course, valid for conditions at 0.6 Prandtl 60 and the Reynolds number is lying at the end of the plate that is Re L. It is greater than 5 into 10 to the power 5.

So, it is laminar; but it is within 10 to the power 8 that is the upper limit of the relation correlation that we are using and the Reynolds number transition is taken to be 5 into 10 to the power 5. And the similarly, the friction coefficient C f L when you have both laminar and turbulent can be obtained. I am not write I am not doing it over here again. It is Re L to the power 1 by 5 minus 1742 by Re L and these numbers the 1742 or 871 comes from the choice of our Reynolds number at transition to be 5 into 10 to the power 5.

So, the corresponding ranges of the parameters which need to be checked before you use this expression; one is Reynolds number is in the range of 5 into 10 to the power 5 to 10 to the power 8 and the transition is taken to be 5 into 10 to the power 5. So, these 2 together are for mixed flow. So, if you have mixed flow on a plate, where heat transfer or momentum transfer is taking place; then the corresponding relations to be used are these two. Whereas, if it is only laminar or if it is only turbulent, then you use is this either this or this based on whether you have laminar flow or turbulent flow and the transition is always taken as 5 into 10 to the power 5.

So, I think what you what we have a and one more 0.1, I should mention here is that all the fluid properties for example, in Reynolds number the rho, the mu; in Nusselt number the thermal conductivity k and so on. The fluid properties in order to calculate the Reynolds number or for the Nusselt number are evaluated at the arithmetic average of the temperature at the infinite position temperature at a point far from the plate plus temperature of the surface divided by 2.

This is also called the Film Temperature. This average is known as the Film Temperature. And the property which gets mostly which is most affected by a change in temperature is the viscosity mu. The thermal conductivity and other properties are not that strong a function of temperature. So, sometimes in some cases all the properties are evaluated at film temperature and if there is a substantial variation between T infinity and T s; then, a correction factor mu infinity by mu s to the power n.

This correction factor is sometimes added to these expressions to take into account the strong change the substantial change of viscosity between the surface temperature and the film temperature and the temperature at a point far from the plate which is the T temperature at y take y infinity or which is denoted by T infinity. And so for special cases with very large variation in this temperature from T s to T infinity of viscosity correction factor is sometimes added to the expression that we have we have derived so far.

So, this concludes our discussion on what is going to happen when you have laminar flow in terms of heat transfer; when you are going to have laminar flow over a flat plate; when you have turbulent flow over a flat plate and most likely scenario when you have mixed flow on the flat plate. So, the first thing that you do while solving a problem is first find out based on the entire length of the plate calculate Reynolds number; that means, you calculate Re L and see if this is greater than 5 into 10 to the power 5. If it is greater than 5 into 10 to the power 5, then you are going to have laminar flow at the beginning and turbulent flow thereafter.

So, your relation to be used would be the relation for mixed flow. On the other hand, if you do not reach the transition Reynolds number that would mean that the entire plate is going to be under laminar flow and you choose your equation, choose your relation appropriately. In some problems, it would be mentioned that the flow is disturbed from the very beginning in such a way to induce turbulence in the system.

So, if that is mentioned if the flow is disturbed the beginning from the beginning itself, then you can safely use the turbulent flow equation; the turbulent heat transfer equation without taking into account the presence of a laminar region at the beginning because this system is set up in such a way, then it becomes turbulent from the very beginning.

So, the first job is to find Reynolds number, choose the relation; find out what is the value of the heat transfer coefficient and proceed like that. So, in our study of external flow in convection, the main aim was to obtain an expression for h that we have obtained through our through the through whatever we have studied in the past 3 or 4 classes.

Now, comes the application. How these relations are going to be applied which would clarify which would clarify conceptual mode? So, in the coming few classes, I am going to solve problems of different type which are all external flow which are when the heat transfer is taking place over a flat plate or over some other geometry. So, all these are external flows. So, will start with the which one problem today and will have will solve other problems of different types all involving external flow and when that is done, then only we will start our study of internal flow.

When the flow instead of taking place on the outside of a surface is going to have inside of a can do it. So, if have a tube through which you have a hot fluid flowing, then what would be the value of the heat transfer coefficient based on the inside of the tube ok. So right now, we have studied the external flow and we will study internal flow; but before that let us solve the few problems. So, the first problem that we are going to look at is about the Analogy.

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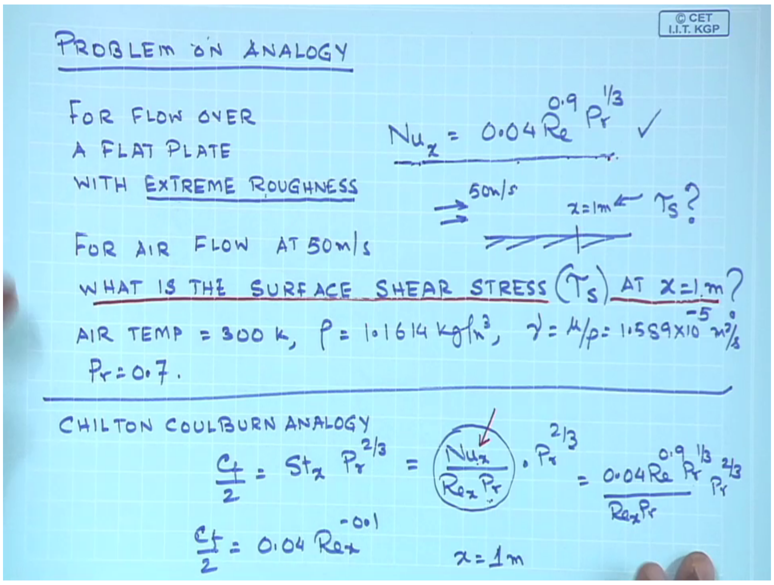

The Analogy that we have obtained is the first problem is about the modified Reynolds analogy or Chilton Colburn analogy. What it mentioned is that for flow over a flat plate with extreme roughness when I mention extreme roughness, it simply tells me that the relations that we obtained for the case of laminar flow. For example, this relation, it cannot be used for the problem that we are discussing because these Nusselt number relations are more or less for a smooth surface.

So, if you have extreme roughness present in it, there it was mentioned that experimentally one would obtain, one would have obtain the local value of Nusselt number related to the Reynolds number and the Prandtl number in the following way. So, the laminar so, the convective heat transfer coefficient is now given by this empirical formula. You have air flow. So, this is this is completely new; this is something new which has been experimental evaluated for surfaces with very large value of roughness for air flow at 50 meter per second which is taking place over the over the plate.

So, the air is coming at 50 meter per second and what is required is: what is the surface shear stress? The surface shear stress let us denote it as tau and since it is on the surface, I would say tau surface to x at x equals to 1 meter. So, that is the question. So, this is x equals to 1 meter, what is the value of tau s at this point? The only information that you have is the heat transfer, the Nusselt number local value of Nusselt number for the convective heat transfer can be correlated to Reynolds and Prandtl number by this relation.

Nothing has been said or mentioned about what would be the, what would be the what would be the expression for friction factor for the hydrodynamic transport case? Only the heat transfer correlation is provided to you. It is mentioned that the air temperature which is coming onto the plate is 300 Kelvin. The rho density of air is 1.1614 k g per meter cube. The kinetic viscosity which is nu which is mu by rho is 1.589 into 10 to the power minus 5 meter square per second and for air the Prandtl number is 0.7. So, this is the problem, what you have to find out is: what is the surface shear stress at a specific location ok?

So, the first thing we realize that this problem deals with heat transfer. Heat transfer relation is provided to you; but the question that was asked to you is about momentum transfer. So, the Nusselt number relation is provided. However, the friction the shear stress is the 6ne that you would have to provide. So, whenever that happens the only way and it is a surface which is extremely rough.

So, the relation that you have for laminar or turbulent flow or mixed flow for C f would not be valid because all those relations are for flat plates. The moment the roughness is introduced, significant roughness is introduced, none of the relations or correlations that we have used to drive to obtained so, far would be valid in this case.

Now, the only way it this problem can tackled is if you go for the analogy between heat transfer and momentum transfer. So, what is the analogy; but this in this analogy, we have to we cannot say that Prandtl number is equal to 1. So, Reynolds analogy is out of question. The value of Prandtl number for this problem has been provided to be equal to 0.7.

So, the Reynolds analogy which assumes that Prandtl number is equal to 1 cannot be used here. What we have to do? We have to use the modified Reynolds or the Chilton Colburn analogy. With that and with the knowledge of the heat transfer relation, I should be able to obtain what is the expression for C f or what is the numerical value of C f? The moment I know the value of C f, I understand by definition C f is shear stress by dynamic pressure; shear stress tau divided by half rho v square.

So, since the velocity is known the free stream velocity is known to me, the only unknown tau can be evaluated by through our use of the of the modified Reynolds analogy; that is the whole problem. So, let us quickly work it out and see, what is the value of shear stress that one can expect for this kind of a situation of flow over an extremely rough surface where the heat transfer relation is provided to you?

So, we start with our Chilton Colburn analogy, it simply tells us that C f by 2 is Stanton number based on Stanton number times Prandtl the 2 by 3 and Stanton is Nusselt number local value of Nusselt number divided by Reynolds number into Prandtl number times Prandtl to the power 2 by 3. So, this is what Stanton number is; that is the definition of Stanton number and the value of the expression for Nusselt number is we can we can obtain, we can we can provide the expression of Nusselt number in here in the relation.

So, it would be 0.04 Reynolds to the power 0.9 Prandtl to the power 1 by 3 and I have the Reynolds times Prandtl and over here, I have Prandtl to the power 2 by 3. So, what this is provided then is C f by 2 is 0.04 Re x to the power minus 0.1 which it should be. But the point note here is that from our knowledge of Nusselt number the which is this expression, I have obtain an expression for C f. C f since it deals with momentum transfer alone, it should not contain any Prandtl number.

Because Prandtl number only appears in heat transfer and not in momentum transfer and here it what we would see that the expression for C f that we have obtained for an extremely rough surface is this and it does not contain any Prandtl number. So, now what is remaining is find out what is the value of Reynolds number at location? So, this is you have to do it at x equals to 1. So, at x equals to 1 meter; what is the value of Reynolds number? Plug the value in here and find out what is C f?

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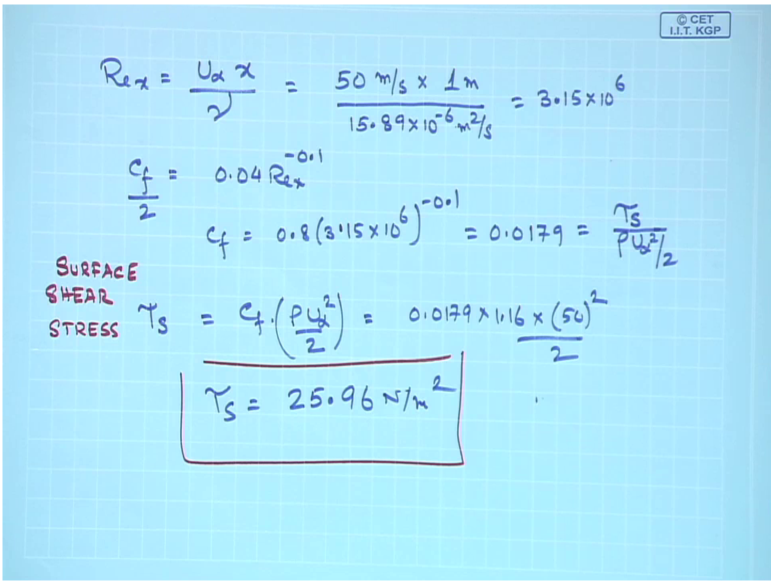

So, the Re x the Reynolds number is U infinity x by nu; where nu is the kinematic viscosity. The value of U is 50 meter per second and x is 1 meter; the nu has been provided as 15.89 into 10 to the power minus 6 meter square per second. So, this is 3.15 into 10 to the power 6. That is the value of Reynolds number. So, the C f would be equal to C f by 2 is 0.04 Re x to the power minus 0.1.

So, your C f would be 0.8 3.15 into 10 to the power 6 to the power minus 0.1 and the numerical value would be 0.179 which by definition of C f is the wall shear stress by half rho v square, in this case U infinity square. I know the value of U infinity I know the value of rho.

So, I should be able to find out the surface share stress. So, this tau s is the surface this could be C f times rho U infinity square by 2 and this would be 0.0179 into 1.16 is the density; 50 is the value of U infinity by 2 and tau s is 25.96 Newton per meter square. So, this is a perfect example of using the analogy to obtain the heat transfer coefficient.

We need to solve a few more problems and to see what are the interesting it is an extreme interesting part of the course, where you can do several calculations with real life real life applications.

What I am going to do in the from the next class is I pick problems which are relevant, which you can visualize with which you can identify and try to see what would be the value of heat transfer, heat transfer coefficient, how much cooling you can obtain when you have flow of cold air over surface when heat is being generated? Which is the common occurrence in many of the computer chips, many of the circuit boards where each transits studies going to generate some amount of energy or dissipate some amount of energy?

How you are going to take that heat away from the transistor such that it remains at working temperature well below the maximum or the limiting temperature is essentially a problem of convective heat transfer? So, how much heat we can extract over a circuit with a flow on top of it? Would look at different problems like that and try to see how convection plays a critical role in the designing of many of the equipments that we use in our everyday life.