Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 25
Turbulent External Flow

Previous class we were discussing about the concept of eddies and, how eddies transport additional momentum and heat in turbulent flow? So, whenever there is turbulent flow the small packets of fluids, otherwise known as eddies they automatically form in the moving fluid.

These eddies transport from one point to the other carrying with them additional momentum and energy. So, they are the region why we expect we encounter more heat transfer, higher values of heat transfer, and momentum transfer in turbulent flow. We have also discussed that how the growth of the boundary layer, which was really continuous and slow the rate of growth of boundary layer was slow for the case of laminar flow, it suddenly starts to increase at a rapid rate.

So, there is no clear demarcation about where laminar flow ends and turbulent flow begins, but just for convenience we assume we assign a specific value of actual Reynolds number to the flow. So, when the Reynolds number is 5 into 10 to the power 5 and we define this Reynolds number by through the use of x that is the axial length in the length scale of Reynolds number.

So, the Reynolds number defined for flow over a flat plate is x v rho by mu where x is the actual length. So, of course, as I move into move along the plate the value of x increases, the value of Reynolds number also increases. And therefore, there would be a transition number assigned as 5 into 10 to the power 5 beyond, which it is assumed that the flow is entirely turbulent.

So, based on this in based on the concept of eddies there is 2 terms which we have discussed in the last class, eddy diffusivity and eddy thermal conductivity. Now both these terms are to be evaluated experimentally, because there is there is lot of there are lot of theory, which have been proposed to estimate the value of these eddies the eddy diffusivities.

However, a clear cut expression for the for these eddies is yet to be universary accepted. So, people are using different ways different of different techniques to decide about what would be the momentum transfer in laminar flow? As well as in turbulent flow and what would be the value of heat transfer? And the parameter of interest in convective heat transfer as I have discussed before as well is the heat transfer coefficient, denoted by H or the dimensionless form of this dimensionless number involving H, which is known as the Nusselt number.

So, the entire study of convective heat transfer is to is to obtain the write expression for the Nusselt number for a variety of flow conditions. So, similarly the for the case of momentum transfer the parameter engineering parameter of interest is c f or the friction coefficient. So, this c f the expression for c f in laminar flow is known to us we have already derive that, what would be the value of c what would be the expression for c f in laminar flow.

But, the question still remains is how do we extend this expression for c f in turbulent flow and is there a way through which once I know the expression for c f, the expression for Nusselt number can be automatically reduced, automatically obtained from the known expression of c f. In other words we are trying to see that if there exists a similarity between momentum transfer or c f and heat transfer there is Nusselt number.

So, the relation between c f and Nusselt number and under what condition this relation of equivalence between c f and Nusselt number can be used is going to be the topic of this class and 2 or 3 subsequent classes. Because, what I what I have told you earlier is at the heat transfer is coupled to momentum transfer, but momentum transfer is not couple to heat transfer provided the thermo physical properties do not vary do not change.

So, we can solve the heat the fluid mechanics part of it fluid mechanics part of it transport phenomena independent of heat transfer.

So, but the heat transfer part cannot be solved without solving the momentum transfer part. So, our energy is therefore, directed to solve the problem which is simpler of these 2, that is the momentum transfer and momentum transfer in turbulent flow. I will not discuss about the technique or the method that that was used to obtain an expression for c f. You probably have read it in your fluid mechanics class, if you have not there is an excellent treatment of turbulent flow expression for c f, in the text book fluid mechanics by fox and McDonald.

In that textbook in the textbook of fox and McDonald, what they have shown is that they have introduced something called, which is called the momentum integral equation. So, using this where the integral equation means it is going to look at the phenomena over a certain wavelength and therefore, any quantities that are obtained out of the solution of that equation the momentum the integral the integral equations are not point values, but they are length averaged values. So, momentum integral equation and the Newton's law or other the equivalent form of Navier stokes in integral form provides a powerful tool to examine what is happening for flow inside a boundary layer?

This momentum integral equation is not only where is there is no restriction that it is to be used for laminar flow only. Whereas, for the case of Navier stokes equation, that you are aware of it is valid only for laminar flow, whenever there is turbulence present in the system the additional terms would appear for to account for the enhanced momentum transfer. And those additional terms collectively are known as Reynolds stresses.

So, the Navier stokes equation as such is not valid for turbulent flow. And these Reynolds stresses as I have as I have told you in previous classes their extremely difficult to evaluate extremely difficult to get a specific value. So, therefore, the integral momentum integral equation, since it is not limited to laminar flow only gives us a way to address turbulent flow inside the boundary layer.

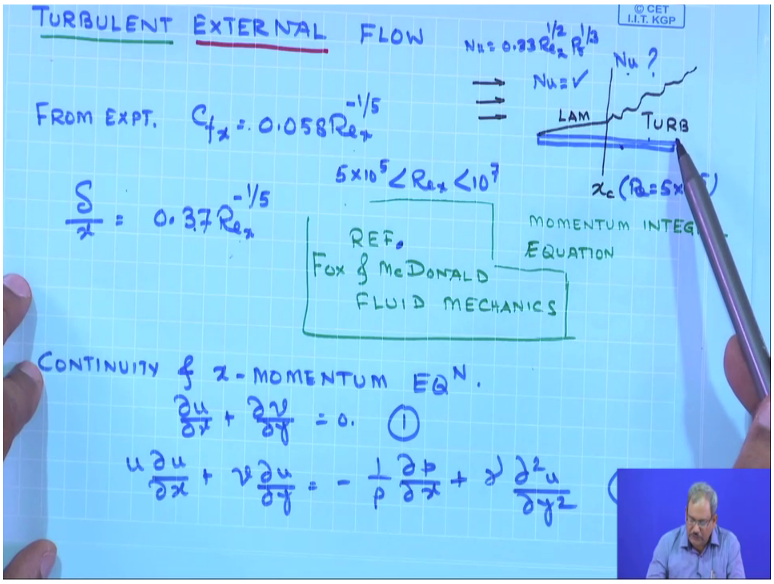
This integral equation and an another advantage of this is the solution of is this momentum integral equation for specially for Newtonian fluids would give rise to an ordinary differential equation, which can simply be integrated to obtain the parameter of interest, which is delta the thickness of the boundary layer as a function of Reynolds number. The same way it was done for the case of laminar flow.

And, once you have delta then it is also possible to obtain the expression of c f, but in there we have to invoke the blazes friction factor or in other words the f that you see in Moody's diagram of friction factor versus Reynolds number. So, with the knowledge of frictions f versus Reynolds number as is given in moody diagram and through the use of momentum integral equation it is possible to obtain and expression for c f in turbulent flow. So, I would derive that expression since it is something which probably has been covered in fluid mechanics.

So, I leave that part and it will directly provide you with the expression for c f in the case of turbulent flow or the expression for delta how does delta change with x or with Reynolds number for the case of turbulent flow. The important thing that I would like to stress I would like to show you in this class is once this knowledge of c f and delta they are known to us, how do we get the expression for Nusselt number, when we have heat transfer as well inside the thermal boundary layer?.

So, our analysis for starts with the a priory knowledge of the expression of c f that was obtained from the fluid mechanics, momentum integral equation, Moody's diagram and so, on.

(Refer Slide Time: 10:14)



So, if we come to see over here the from the this is what is the flow that is and you have initially laminar at x e the transition point it changes from laminar to turbulent. And as you can see the growth of the turbulent layer is much more rapid as compare to the growth of the laminar boundary layer.

So, what a I have written down the expression for C f x. So, this is from experiment and also from theory the friction coefficient as a function of actual location can be expressed as 0.058 into Reynolds to the power minus 1 1 minus 1 by 5. And, any relation is valid when you have to specify the limits up to which the expression is valid.

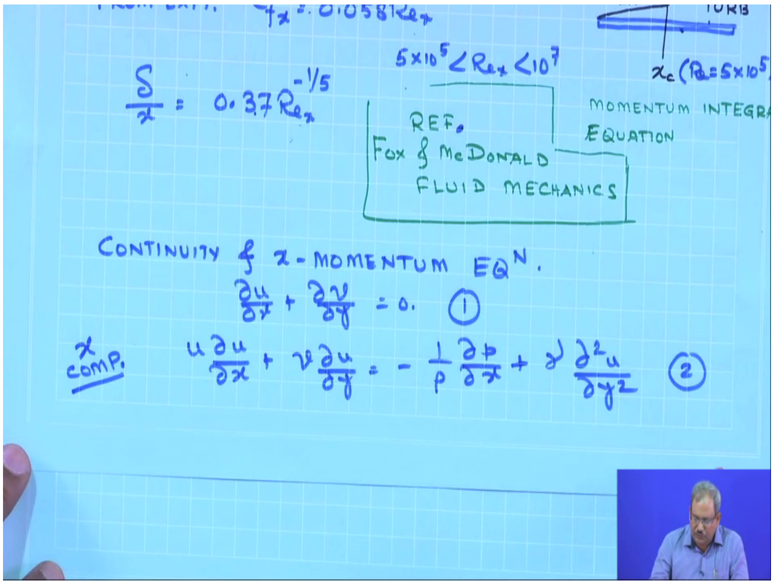
So, for a Reynolds number, which is greater than 5 into 10 to the power of 5; that means, this x e corresponds to Reynolds equals 5 into 10 to the power 5. So, when the flow starts to become turbulent from this point up to a Reynolds number of 10 to the power 7 this is a expression which provides us with which gives us the value of the friction coefficient. And it can also be shown it has been shown that delta the hydrodynamic boundary layer thickness by x where x is the actual location is 0.37 R e x to the power minus 1 by 5.

So, these 2 expressions the derivation of these 2 expressions how these 2 expressions were obtained? You would see you can see if you if you wish you can see it in the reference, fluid mechanics by fox and McDonald there they have shown how these 2 expressions are obtained through the use of momentum integral equation?.

So, we will not solve this will take these 2 and proceed to see, how our knowledge of momentum transfer in turbulent flow can be extended expanded to take into account the heat transfer in turbulent flow. Or in other words the expression for Nusselt number, when heat transfer is taking place from this plate to the fluid. What would be the expression for Nusselt number in turbulent flow, because you already know, what is the expression for Nusselt number in laminar part?. So, that part is that is that is known to us, which we you if you recall it simply 0.332 R e x to the power half and Prandtl to the power one-third.

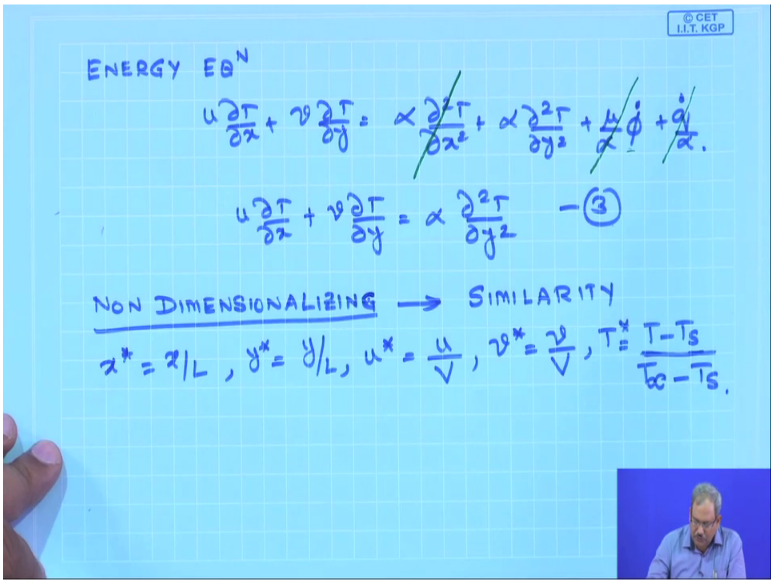
So, this Nusselt number is simply going to be 0.33 Reynolds to the power half into Prandtl to the power one-third. So, we would like to see what would be the value of the expression for the Nusselt number in turbulent flow, because in practice most of the flows are in the turbulent region. So, would like to see the expression for that. In order to do that, I start with writing the continuity equation which is a which is a statement of the conservation of mass. And the momentum equation for flow inside the boundary layer I do not I neglect the diffusive transport of x momentum in the x direction.

(Refer Slide Time: 13:44)



So, this is the x component of the momentum equation in through the use of the standard boundary layer approximations, that I have introduced previously.

(Refer Slide Time: 13:56)



So, when you add to this the energy equation, what you get is this and we understand from our previous discussion, that the momentum that the thermal transport in the x direction by diffusion by conduction is small as compared to the transport of heat in the y direction, because of the small length scale that is the boundary layer thickness, which is associated in the y direction. This is a viscous heat dissipation and for most of the practical cases most of the cases that are that we encounter the viscous dissipation is small and can be said equal to 0. This is a heat which is generated by joule heating or by nuclear sources and so on.

So, let us assume that this is also not present in this particular system. So, the energy equation for flow in flow and heat transferred inside a boundary layer can simply be written as alpha times del to T by del y square and I call this as equation 3. So, these 3 equations we are going to work with these 3 equations, the continuity equation, the x component equation and the energy equation in this one is for the x component of the any different. These 3 equations we would like to use and to see, if anything can be can be seen I can be done to use this expressions for C f in order to obtain the Nusselt number expression for turbulent flow, that is the job we have in at our hands.

So, we start with the Non-dimensionalizing the equations. So, if I non- dimensionalize this equations it would give us something like the similarity of the transport processes. And, these similarity of the transport processes are going to be valid for certain conditions. So, we have to identify that what are the conditions, that one needs to check before they can use this similarity between momentum transfer and heat transfer.

Now, whenever you try to do something and exercise like that, the first thing one should do is to non-dimensionalize the equations. The advantage of non- dimensionalizing any governing equation is that at times you would see the automatic emergence of certain dimensionless groups, which would give you a clearer picture of the transport, which is taking place inside the system. So, a governing equation governing equation differential equation being when you non dimensionalize you would should see the important dimensionless parameters automatically coming out of the equation, which would let you compare 2 systems to completely different systems as long as their dimensionless equations are identical.

And, they become identical when the similarity parameter that you get out of these equations they are equal. In simple terms if you have Reynolds number to be equal to 5000, you know what is the flow condition that is that exists in inside the boundary layer? So, it does not matter whether you have flow of air or flow of water or flow of glycerin and over a flat plate, which is giving rise to this Reynolds number this is to the specific value of Reynolds number.

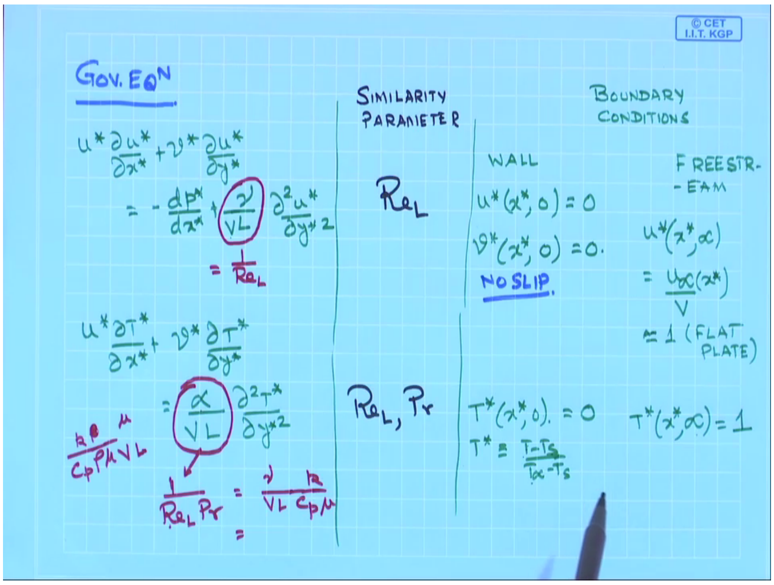
If this specific value of Reynolds number is identical, then you can get a dimensionless solution, you can get a solution of the dimensionless film thickness or the friction coefficient, which would can be which can be compared for flow between 3 dissimilar fluids. So, the behavior of the flow or behavior of heat transfer inside the boundary layer will be identical if the Reynolds number is the same, if the Reynolds number is the same the behavior of the flow would be the same.

But, would I need to specify some other dimensionless groups or some other conditions such that I can say it is not only the flow the heat transfer would also be the same. So, that is what we are trying to do. So, the first step of doing that is non-dimensional is non-dimensionalizing the governing equations and see, what or which groups come out of that automatically?.

So, the parameters that we use for non dimensionalization are x star these are all dimensionless quantities x star is x by L where L is the length of the plate over, which the flow takes place y star is y by L u star is u by V, this is the x component of velocity, the y component of velocity is similarly this is the approach velocity, the T star the dimensionless temperature is defined as T minus T S where T S is the temperature of the solid surface the plate, and T infinity which is the temperature at a point far from the plate in the y direction in the upward direction minus T S.

So, these are the parameters, which are which are defined in this. So, when you do that something interesting happens and I am going to write what in this is.

(Refer Slide Time: 20:09)



First, I would write what is the going to be the governing equation? So, the governing equation for the case of flow is going to be u star del u star by del x star plus v star del u start by del y star is equal to minus the pressure gradient the dimensionless form of it plus nu by L nu by v L, this is a kinematic viscosity, this is approach velocity, this is the length scale del square u star by del y star square. And this is for the momentum equation x component of the momentum equation.

And, if I write the energy equation it would be u star del T star by del x star plus v star del T star by del y star is equal to alpha by V times L this alpha is the thermal diffusivity, which is k by rho C p del 2 T star by del y star square. You can do this non dimensionalization yourself and see this is what you are getting?

The important part of this the all these are dimensionless. So, if all these u star x star del y star del y star d p p star x star everything is dimensionless, then this also must be a dimensionless parameter. And, if you look closely this is kinematic viscosity. So, this is nu by rho. So, when rho comes over here we have rho V L by mu. So, this is nothing, but 1 by Reynolds number right. So, this has to be 1 by Reynolds number Reynolds number based on the entire length of the plate.

So, non dimensionalizing the equation automatically projects are up similarity parameter, which that we know from our experience from our knowledge a priory knowledge that it has to be Reynolds number. Now, let us look at this combination of physical properties, geometrical properties, and operational parameter. This one this is k by rho C p. So, you what you have is in k by rho k by rho c p and here you have V and L.

So, what you do is you take mu over here you take mu over here and you what you then get is or rather what you get here is if you combined these together, this is simply going to be 1 by R e L times P r. Because, 1 by R e L as I know from here is going to be V L by nu and Prandtl number is C p mu by k.

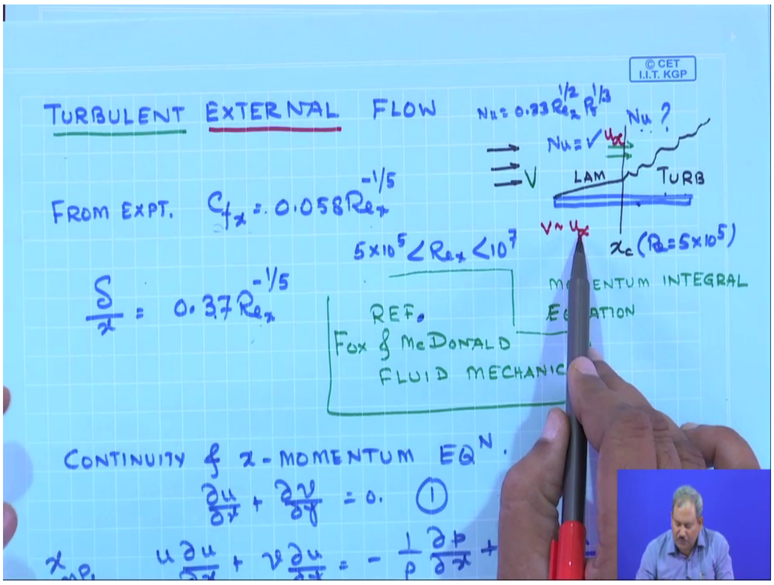
So, this nu is simply mu by k. So, mu and mu will cancel and what you have there is in alpha by V L. So, this is the similarity parameter for heat transfer is Reynolds number as well as Prandtl number, that what it should be because Prandtl number is C p nu by k. So, the this is heat transfer. So, heat transfer should always contain C p k etcetera. And, since this is heat transfer and we also have convective fluid flow present in the system. So, this has to be governed by Reynolds number as well.

So, if I write the similarity parameter over here, this is simply going to be R e L and this one is going to be Reynolds number based on the entire length and Prandtl number. So, these 2 are the similarity parameters 1 for momentum transfer, the other for heat transfer. Now, let us look at what is going to be the boundary conditions.

So, the first thing the first boundary condition that I am going to the first boundary condition is at the wall. So, what happens at the wall at the wall we have no slip condition; that means, both the x component velocity, which is u star at any value of x star, but since it is at the walls y is equal to 0 or y star is equal to 0 would be equal to 0. Similarly, the v component of velocity at any value of x star, but at 0 y should also be equal to 0. So, these 2 are nothing, but the no slip condition, that we are aware of what is going to happen at the free stream? At the free stream; that means, the value of u star for any value of x star, but at y equal to infinity, which is simply going to be u infinity by V u infinity at this value of x star.

Now, if you see this the velocity with which it approaches the solid plate is V and the constant velocity outside of the boundary layer is denoted by u infinity ok.

(Refer Slide Time: 26:43)



So, if it is a curved plate then the then v infinity and u infinity will differ in values, but for flow over a flat plate this V is equal to U infinity. This is true only when you have flow over a flat plate had this been a curve plate this would not be true. Therefore, in order to keep the general generality of the solution I have written it as u star, which is defined as u star at infinite value of y; that means, u star when y is somewhere over here it simply going to be u infinity by V, but we understand that for a flat plate this u infinity by V is going to be equal to 1.

So, this is equal to 1 for the case of flat plate, but the general boundary condition at the free stream is u star at y star is equal to infinity is this. Similarly, let us try to see what is going to be the condition T star at x star and 0 ok? Remember, that T star is defined as T minus T S by T infinity minus T S. So, when y is equal to 0 this T would simply be equal to T S. So, this is going to be equal to 0 and what happens at the free stream?

So, it value of T star at any x star, but at infinite distance from the wall from the plate then at that point T simply becomes equal to T infinity. So, the temperature of the fluid flowing over a flat plate at a point far from the plate; that means, y equal mathematical speaking y equal infinity, this T is simply going to be equal to T infinity and I am going to have this to be equal to 1. So, if you if you realize if you if you understand these cases then the governing equation, the similarity parameter and the boundary conditions.

So, this would give us the starting point to ensure or to drive, when and how the momentum transfer part of it is going to be similar to that of the heat transfer. So, let us talk something about the governing equations, the dimensionless forms and that the boundary conditions. Now, these 2 equations one governing momentum transfer and one governing heat transfer, they would be identical provided these 2 are equal.

So, if your Reynolds number are same and Prandtl number is equal to 1, then in dimensionless form there is no difference between equation 1 and equation 2. I repeat once again. If the Reynolds number values are same in for the case of heat for the case of momentum transfer and for the case of heat transfer. And, if in addition Prandtl number is equal to 1, then these 2 equations become identical. In one case we are talking about velocity x component of velocity, in the other case we are talking about the temperature the dimensionless temperature.

So, the dimensionless form of u and the dimensionless u star and the dimensionless form of e star would be identical provided these similarity parameters these 2 similar parameters are equal and the value of Prandtl number is equal to 1. And, if in addition when you look at the boundary conditions, if in addition the flow is taking place over a flat plate such that u star at x star and y star equals infinity, if this is on a flat plate, then this boundary condition and this boundary condition would be identical.

These 2 are identical anyway so, far a flat plate the second boundary conditions would also be identical. So, therefore, it clearly gives us an idea of what is going to be the similarity? The equivalents between heat transfer and momentum transfer. So, with dimensionless form of the equation simply provides simply tells me that, if the Reynolds number numbers their kept constant and if Prandtl number is equal to 1, and if the flow takes place over a flat plate then the 2 governing equations are identical the boundary conditions are also identical.

So, in that happens when the governing equations and the boundary conditions are the same, the 2 systems become dynamically similar. And, for a dynamically similar for 2 dynamically similar systems the expression of the dependent variable can simply be replaced by the expression of another dimensionless variable. Or in other words what I try what I am trying to say is that? If the dynamic similarity between heat transfer and fluid mechanics is achieved, then any expression of u star can be modified as an expression of T star by simply replacing the quantity element quantities with the relevant dimensionless quantities that is there is therefore, the second flow second transport case.

So, 2 dynamically similar systems I know what is what is u star going to be, if I know that then any derivative of u star any derivative expression utilizing the expression of u star can also be substituted to obtain what would be the expression for T star and let us say del T star del y star the temperature gradient. So, the if I know the velocity expression and if I know the velocity gradient expression, then through dynamic similarity I should be able to obtain what is the temperature expression? And what is the temperature gradient expression?.

And these approach when the 2 systems are made dynamically similar through the choice of equal Reynolds number and through the peculiar at this point choice of Prandtl number equal to 1, because Prandtl number may not be equal to 1 for many of the fluids, but if it is equal to 1 then complete equivalents between momentum transfer and heat transfer will exist. And, the analogy between these 2 for a special condition of equality of Reynolds number and unit value of Prandtl number this special analogy is known as Reynolds analogy.

So, in the next class we would use this Reynolds analogy to see if I can switch from a known expression of c f to an expression for Nusselt number. And, that would underscore the importance of this analogy the Reynolds analogy and somewhat extended are modified form of Reynolds analogy, which would be valid not only for Prandtl number equal to 1, which is unrealistic to expect for many of the fluids, but for over a wide range of Prandtl numbers. So, our next class the topic would be the Reynolds analogy and the modified Reynolds analogy.