Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 23
The Flat Plate in Parallel Flow – Heat Transfer

In the last class we have seen how the Growth of the Hydrodynamic Boundary Layer on a Flat Plate can be analyzed analytically. Though towards the end we had to resort to numerical solution because the, we could convert the p d to an o d by combining the independent variables in by defining a stream function. However, the resulting ordinary differential equation that we obtain was non-linear and analytics solution was not possible and then we have to use numerical solutions.

However, those numerical solutions give us some very interesting results, looking at the value of the stream function, value of the dimensional stream function and its relation with v x and v y and noting that the v x and v y would be 0 at y equal 0. Due to the no slip condition we could obtain the expression for the growth of hydrodynamic boundary layer as a function of operational parameter, which is the velocity with which the fluid approaches the plate. And the length scale which is the axial length in the direction of flow and thermo physical properties which are mu and rho.

So, therefore, the expression for delta, the thickness of the boundary layer was expressed in terms of Reynolds number, when you combine all them it becomes Reynolds number. So, the expression for, expression was obtained as delta to be equal to 5 x by root over Re x.

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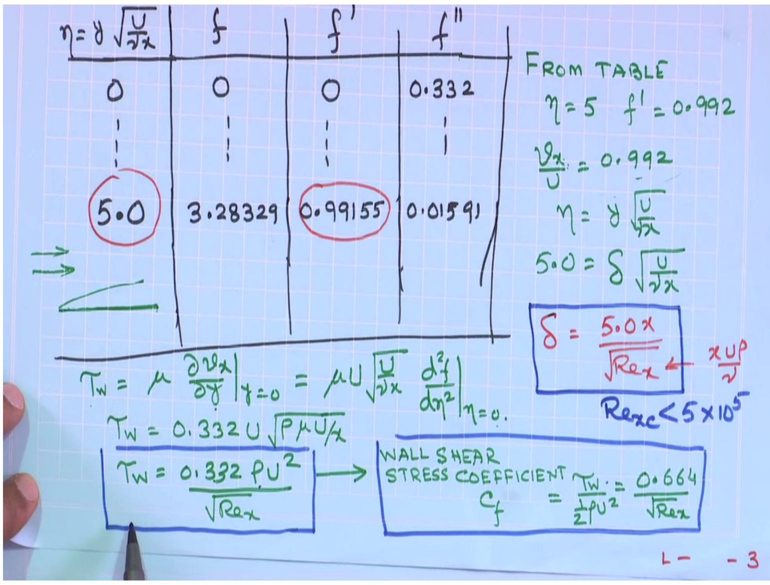

For the case of shear stress, rather the wall shear stress we obtained this expression for the wall shear stress and the another quantity of engineering importance is the wall shear stress coefficient or C f which is, which turned out to be 0.664 by Re x. And these two rows in the numerical solution where important which gives us some idea of eta equal 0 which corresponds to no slip condition.

So, from there we obtained the expression for the wall shear stress sorry, the thickness of the boundary layer. What we would now like to do is use this knowledge to analyze flat plate in parallel flow, but this time it is going to be the heat transfer case.

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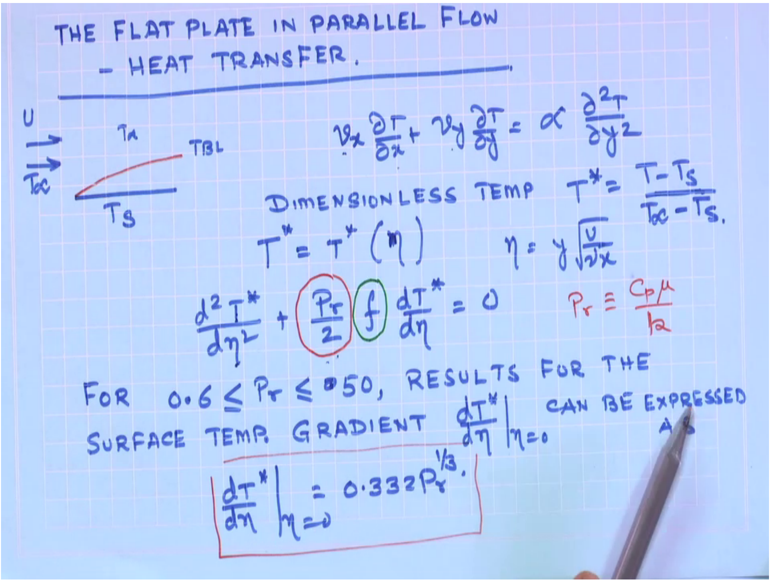

So, in the heat transfer when there is going to be a difference in temperature between the plate and the fluid which is approaching. So, this is let us say is a T S and the fluid which is approaching is at a temperature T infinity with its velocity equals to U and then that is going to be the growth of the thermal boundary layer. It may or may not coincide, may lie above or below the hydrodynamic boundary layer.

But this is the thermal boundary layer, let us call it as TBL and the energy equation corresponding energy equation would be v x del T del x plus v y del T del y is the alpha times del 2 T by del y square. So, this is the equation which now would, we would have to solve, but we already have an idea of the value of v x and v y at different values of x. So, our hydrodynamic solution has already given us the, from this table it has already given us what would be the value of f and f prime which are required to calculate the v x and v y.

So, we would now go for again define a dimensionless temperature, temperature which is defined as T star to be equals T minus T S where, T S is the temperature of the solid surface divided by T infinity; Where T infinity is the temperature of the fluid outside of the thermal boundary layer, T infinity minus T S. And we also assume the T is going to be a function, the T star, this one is a function of, we understand it is function of both x and y. The value of T star would depend where we are in terms of the axial location as well as in terms of the vertical location

So, therefore, this T star is a function of eta, the eta that we have defined before. So, this is what we have assumed and when we make the substitutions, in this equation the governing equation turns out to be d 2 T star by d eta square plus P r by 2 f dT star by d eta equals 0. This is a very interesting, very interesting result, I skip the steps where you substitute v x and v y, del T del x and del T del y and so on by assuming this T as a function of eta and we understand that eta is equal to root y times root over U by nu x. When we do that the ordinary, the partial differential equation gets transformed into an ordinary differential equation. The point to note here is first of all the appearance of f prime in the energy equation mix it, coupled with the with the hydrodynamic part of the solution.

So, the presence of f denotes that the energy equation and the momentum equation are coupled, you would not be able to solve the equation, you would not be able to solve the energy equation unless you solve the momentum equation. So, you need to know the value of f priori before even start solving this equation; the second one to note is the appearance of P r Prandtl number in here. So, this Prandtl number is defined as you know as C p mu by k where, C p is the thermal capacity heat capacity, mu is the viscosity and k is the thermal conductivity. So, if you look at this expression and you already have, you already know from this table that: what is the value of f for different values of eta.

So, the value of eta and f are already known to you from the solution of the hydrodynamic part of the momentum transfer equation. So therefore, this f as a function of eta is known to you. So, if this is known then I should be able to numerically solved for T star as a function of eta. Since, my f verses eta is known, I can solve for T star as a function of eta provided the value of Prandtl number is known to me.

So, in order to numerically solve this equation, the only thing I need to do is specify the value of Prandtl number, the moment I specify the Prandtl number to be let us say 0.7. Then I should be able to numerically solve it because, if dependence of f on eat is already known to me and therefore, I should be able to obtain T star as a function of eta for a specific value of Prandtl number. So, that is the only thing we have need to realize in here that, T star as a function of eta can be obtained at a specific value of Prandtl number.

So, that is the only thing which is which is which is remaining here, which is to be clarified here and it has been found that for this range of Prandtl number 0.6 to 50, 5 0. results for the surface temperature gradient which is dT star by d eta at eta equal 0, can be expressed as dT star by d eta at eta equal 0 to be equals 0.332 Prandtl to the power one third. This is very interesting very important result, as we have seen we need to have, we need to know the numerical value of Prandtl number to solve it.

So, as I different values of Prandtl number and start solving this equation 0.5, 0.75, 1, 1.25, 10, 20 so and so on. What is interesting, the observation that was obtained is that dT star by d eta at eta equal 0, which you can obtain from the solution of this, provided you assume a value of Prandtl number, can be expressed as a function of Prandtl number over a wide range of Prandtl number.

So, the observation once again is that for, when Prandtl number lies in this range the surface temperature gradient; that means, at eta equal 0 dT dT star by d eta can be expressed as a function of Prandtl number in this functional form. Now, why is it important, why I am at all interested in dT star d eta at eta equal 0. The same reason I was interested in trying to find out what is the velocity gradient at y equal 0; that means, at eta equal 0 because velocity gradient at y equal 0. That means, on the surface, at this at the at the solid surface gives me what is the force exhorted by the moving fluid on the solid, in other words I am, I was defining an important transport coefficient or defining transport phenomena related to force exhorted by a moving fluid on a, on a stationary solid.

Similarly, dT star, d eta at eta equal 0 that means, the velocity gradient at y equal 0 should give me some idea of the heat transfer process that is taking place in the situation, where as cold fluid flows over a hot plate. So, the temperature gradient at the interface should give us an idea of what is the convective heat transfer coefficient in this process and we understand the entire study of conviction is try to find out what is the convective heat transfer coefficient. What is the relation or correlation that can be used to obtain the value of the heat transfer coefficient, convective heat transfer coefficient and in order to do that, the knowledge of dT star d eta at y at eta equal 0 will play, does play a very significant role.

So, next what we are going to see is how this dT star d eta at eta equal 0 can be related to the convective heat transfer coefficient, but to do that I need to have an idea of what is the Prandtl number. So, what we have seen is that for a large range of Prandtl number, the temperature gradient at the interface can be expressed as a function of, can be fitted to the variation in Prandtl number. So, dT star d eta at eta equal 0 can be correlated to Prandtl number to the power something with a constant in front of it.

So, I will start writing it and then I will show you how it would give us an expression for the heat transfer coefficient. So, let us start with that, so this dT star, d eta at eta equal 0 is equal to 0.332 Prandtl to the power one third this is our starting point, the heat transfer coefficient h suffix s x.

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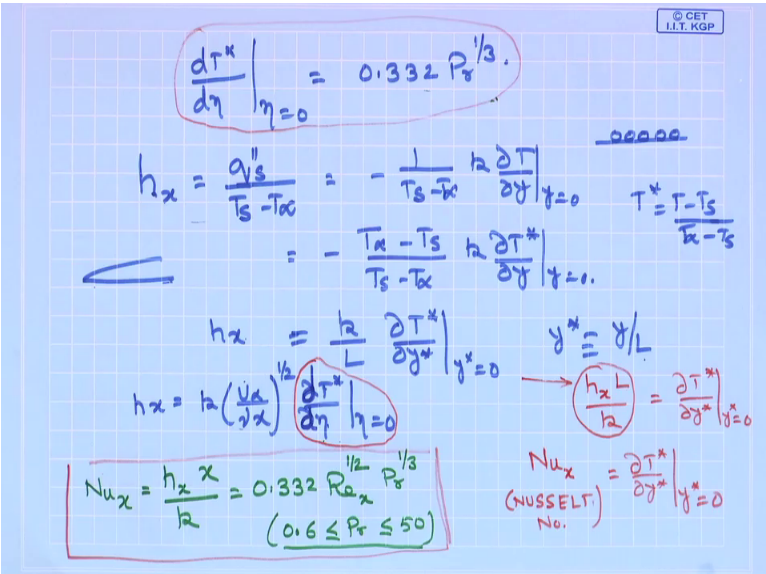

That means, it is a local value of the heat transfer coefficient can be defined as, from Newton's law of cooling S cube double prime S by T S minus T infinity. T S is the temperature of the solid surface, t infinity is the temperature of the liquid. So, if this is the flux the Newton's law of cooling connects the convective heat transfer coefficient with q double prime is in this way. So, I can write it as T S minus T infinity times k, del T by del y at y equal 0.

So, this is Newton's law of cooling and this is substituting Fourier's law of conduction for the surface heat flux because, as we have discussed many times before on the surface the fluid molecules are immovable. And therefore, the transport of heat through this immovable layer of molecules of the fluid can take place, does do take dose take place only by conduction and that is why I am substituting the Newton's law of conduction in here. So, this now can be modified little bit, it is T infinity minus T S by T S minus T infinity, I am trying to use the definition of T star. Where the definition of T star is T minus T S by T infinity minus T S this times k del T star by del y at y equal 0.

So, this to cancel, the negative sign will disappear so, my h x is k by L thermal conductivity by L because, I am converting this by into dimensionless one as well, del T star by del y star at y star equal to 0. So, y star is simply defined as y by L, where L is the length of the entire length of the plate over which this heat transfer is taking place. Look at this equation carefully, what you having here is, a if I bring this L on the other side and k over here and of shoot of this equation is h x local value of heat transfer coefficient.; x denotes the location L by k is equal to del T star by del y star, at y star equal to 0. What is this, h L by k this h L by k is dimensionless and it is denoted by Nusselt number, Nu local value that is why I put the substitute x.

So, this is Nusselt number which denotes conviction is equal to del T star by del y star at y star equal 0. This is an unique definition of Nusselt number, which simply tells you that the Nusselt number is nothing, but the dimensionless temperature gradient at the interface, Nusselt number is del T star del y star at y star equal to 0. So, the proper definition of Nusselt number is not simply h L by k, the correct way to express Nusselt number is it is physically the dimensionless temperature gradient at the solid liquid, in solid fluid interface.

So, that is one way to look at the genesis or the physical significance of Nusselt number. So, now, let us proceed with this and see that with our knowledge of del dT star d eta at eta equal 0, can somehow be plugged into this expression to obtain a more compact, more useful expression for Nusselt number in laminar flow for flow over a flat plate.

So, I continue with this once again and what I do is this h x is k U infinity by nu x to the power half, del T star by del eta at eta equal 0. So, I convert this y to eta and that is why these terms do appear in front of it and since T star is a function only of eta. So, therefore, the partial sign can be replaced by ordinary differential, ordinary differential signs. So, t star is a function only of eta and this I have already seen; what is the expression for this from our previous study.

So, now I am going to write this, bring this over here take the x to you other side and write the final expression for Nusselt number, the local value of Nusselt number as h x local value of the convective heat transfer coefficient; x by k to be equals 0.332 Reynolds number based on local Reynolds number to the power half into Prandtl to the power one third. The limitations is this, the Prandtl number has to lie between 0.6 and 50 which was, which was used to obtain this expression.

So, what I see here then, that a compact expression for Nusselt number can be obtained by simply invoking the result over this, and realizing the T star is a function of eta. And this expression is going to be extremely useful for solving or for solving or for analyzing situations in which we have flow over a flat plate where the temperature of the plate and the temperature of the fluid which is flowing over it is, they are different.

So, we are going to have convective heat transfer taking place and this is the first time that you see an expression for h is obtained in terms of Reynolds number which characterizes the flow, which brings in the which brings in the hydrodynamic part of it into the expression. And a function of Prandtl number which is C p mu by k and it gives some idea of the convection and conduction the relative importance of convection conduction, momentum diffusivity, thermal diffusivity into our discussion.

This gives me the average, this gives me the local value of the heat transfer coefficient, like before we have obtain C f x that is the friction coefficient at a specific x. The expression that I have obtain for heat transfer it is N u x or h x the convective heat transfer coefficient at a specific x. But as an engineer, you are probably more interested in finding out; what is the average value of the heat transfer coefficient or what is the average value of the friction coefficient over the entire plate surface. You do not want to know most likely what is the value of the heat transfer coefficient at a specific location, you rather try to find out what is the average value of heat transfer coefficient.

Because, that would give you some idea of what is the total amount of heat transfer if there is a way to evaluate the total the average value of heat transfer coefficient, then the total heat transfer coefficient can simply you obtained as h a delta T ok, you do not have to use h x. So, our next small item which is remaining is, how to obtain the average value of heat transfer coefficient from the expression of the local value of heat transfer coefficient. So, let us do that and see how, what would what would happen in this case. So, the average value of a of this the average value.

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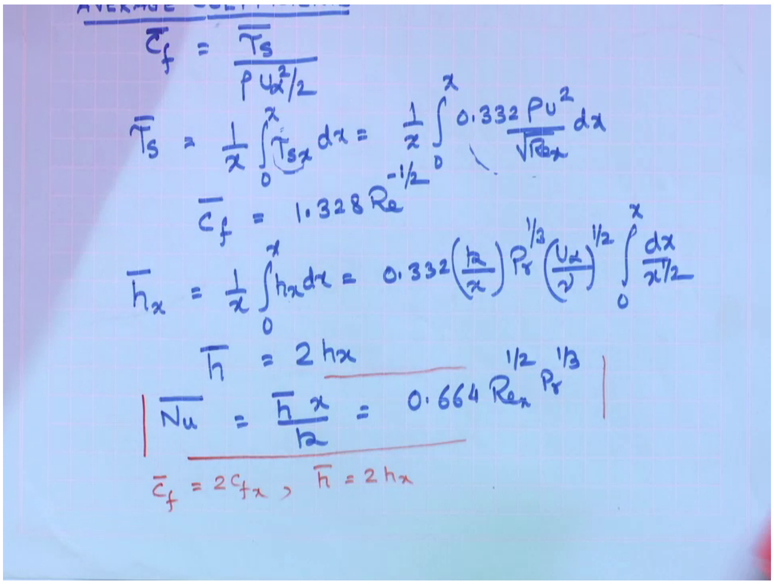

So, this is about average coefficients. So, C f and I put a bar over it to say that this is the average value, must be tau average, tau surface average by rho U infinity square by 2that is the definition of C f. But here I am using the average one so, what is tau this thing then, the average value of the shear stress is from 0 to x tau s x d x. So, this can be written as 1 by x, 0 to x 0.332 rho u square by root over Re x d x. This tau x is simply I am using the expression of tau x which I have obtained before and once you do this integration, you would see that C f, the average value of C f is going to be 1.328 Reynolds to the power minus half ok.

And in a similar to the, momentum transfer the value of, the average value of h x would simply be 1 by x 0 to x, that is the definition of lengthwise average h x d x, which would be putting the expression over here k by x. Prandtl number is a constant so it goes out of the integration sign, U infinity by nu which is to the power half out constant, out of the integration sign 0 to x d x by x to the power half.

So, once you do this your h, the average heat transfer coefficient is going to be equal to twice of the local heat transfer coefficient. Therefore, your average value of Nusselt number, the expression for Nusselt number would be average h times x by k and this would turn out to be 0.664 Re x to the power half Prandtl to the power one third. So, this gives you the average value which is more important in realistically, in engineering situations then the local value of the heat transfer coefficient. So, what we say then is C f average is equals twice C f local and h average is twice local.

So, we have two equations then, one equation is for the local value of the Nusselt number or local value of the heat transfer coefficient and simply finding out the average. And the average in is done by integrating it over a certain length and dividing it by the length, which is the standard definition for average based on the length. What you would see is this the average, the local value and the average value differs only by a factor of, only by a factor of 2 and that is why the, this is the relation between the average and the local.

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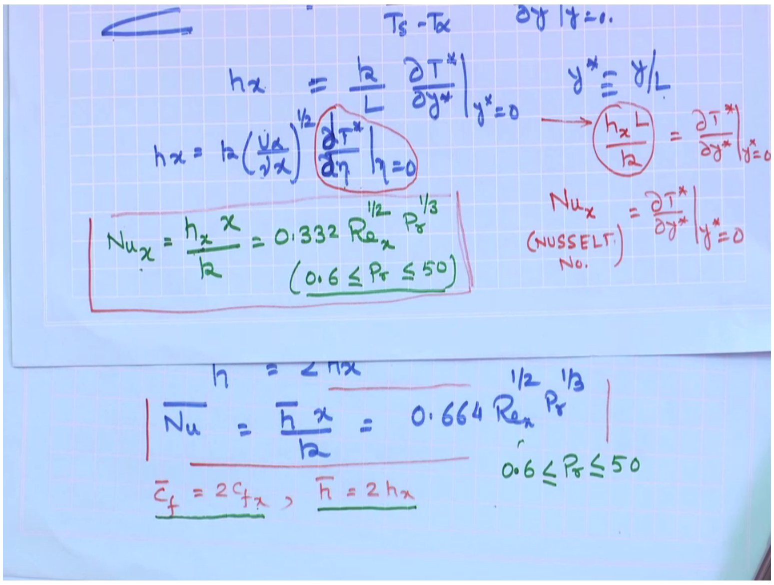

Which is the same as that between the shear stress coefficient the local at the average, but we also need to know, that need to remind you once again that this expression is also valid for a Prandtl number range: between 0.6 to 50. So, what we have done in this class, in this exercise and the previous class is to show you that to why it is important to study what is happening inside the boundary layer, inside the thermal boundary layer and inside the momentum boundary layer. Because all the phenomena is concentrated in a region very close to the solid surface, in one case its momentum transfer and in the other case it is heat transfer. The momentum transfer case can be handled by defining a combination variable eta and the dimensionless stream function f.

When I do that what I would have, what I would obtained for the case of hydrodynamic boundary layer is an ordinary differential equation, non-linear higher order differential equation containing f being the dependent variable and eta being independent variable. That equation was solved numerically to obtain a table containing what will be the value of f for different values of eta, in using the values and the concepts we obtained compact relations for the growth of the boundary layer that is delta as a function of x. And you have also obtained what is going to be the shear stress coefficient as a function of x properties and the imposed flow condition.

The equation for energy transfer when we non-dimensionalized it, everything that take the temperature dimensionless temperature T star was a function of eta, which is the combination variable. But a term f appears their denoting or underscoring the coupling between the fluid flow and heat transfer, Prandtl number also automatically appears in the governing equation. This equation can be numerically solved because, I already have the numerical solution of f versus eta, but in order to solve it numerically I first need to need to assign a value to the Prandtl number, any number any realistic number to Prandtl number and solve for this, assign another value and solve it again.

When that was done for a large range of Prandtl number it was found that d T star d eta at eta equal 0; that means, the temperature gradient at the liquid solid interface can be expressed as a function of Prandtl number. Purely a fitting between the temperature gradient and Prandtl number, with that we have seen how to express the heat transfer coefficient using Newton's law of cooling and then Fourier's law. And obtain fundamental definition of Nusselt number to be the dimensionless temperature gradient at the interface. We proceed with that utilizing the values that we have obtained and we got an expression for local Nusselt number, as a function of Reynolds number and as a function of Prandtl number, That 0.332 Reynolds to the power something, Prandtl to the power something valid within the range of Prandtl number.

Then we realize that it would probably be easier to work with the average quantities, the average value of heat transfer coefficient or the average value of the shear stress coefficient. So, what we have done, we have done length average in I have finding out the average of these quantities h or C f for the entire length of the plate over which the flow and heat transfer list taking place. So, what we got is that the average values are always twice the local value. So, we have now obtained and expression for Nusselt number, average Nusselt number when flow and heat transfer takes place over a flat plate.

But remember all these are limited to laminar flow, laminar flow conditions prevailing over the solid plate. What happens in turbulent is something very interesting, something entirely different and you would expect that more heat transfer is, more heat transfer is possible when you create turbulent conditions inside the thermal boundary layer. Which we will discuss in the next class, but today's class is all about laminar flow and heat transfer when flow takes place over a parallel plate.