Heat Transfer

Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 22
The Flat Plate in Parallel Flow – Hydrodynamics and Momentum Transfer

We have already seen what kind of approximations one can make to the equation of motion and equation of energy when these 2 equations are applied for the case, where we have flow over a flat plate. So, which is like flow parallel to a flat plate and we know that the 2 boundary layers hydrodynamic and the thermal boundary layer we will grow on the plate, whose thickness will slowly increase with the actual distance x.

And, inside the boundary layers the flow is going to be 2 dimensional outside of which the flow can be treated as one dimensional. So, we are going to start using the reduced form of these equations for the case of flow over a flat plate, when the plate temperature and the temperature of the fluid which is approaching the plate are different.

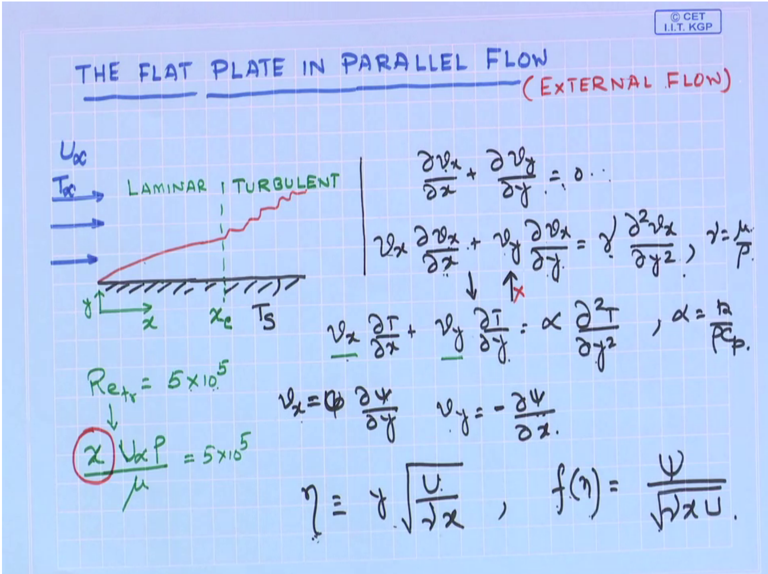
So, there would be there would be heat transfer from the plate to the fluid, as well as formation of a thermal and momentum boundary layers on top of the plate, which within which most of the transport processes are going to be concentrated. Outside of the boundary layer, that is going to be no exchange of either momentum or energy.

So, it is extremely important the concept of boundary layer and the fact that all the transport processes are confined, within a thin layer close to a surface is very important from an engineering stand point. Because, whatever we do we need to let us we need to maximize heat transfer from a surface.

So, our point of interest or other our zone of interest, we will always be the region very close to the surface, within the thermal boundary layer. And, we should make design modifications to ensure that we are going to get the maximum heat transfer from the surface by tweaking by manipulating the flow conditions inside the thermal or the momentum boundary layer.

So, we would we would first see the form of the equation, which we have derived in the previous class. So, if you see what we have over here.

(Refer Slide Time: 02:39)



This is a flat plate, which is placed parallel to the flow where the flow is coming with a velocity equals to U infinity and it is temperature is at T infinity.

So, these fluid when it comes in contact which the surface, which is let us say at a temperature of T S, there is going to be exchange of heat between the surface and the fluid. And, this denotes the thermal boundary layer the edge of the thermal boundary layer. So, temperature changes from T S to T infinity within this thin boundary layer. For clarity this thickness is greatly exhorted. So, there could be real thickness of a boundary layer would be very small of the order of millimeters and so, it will live very close to the solid surface.

But, it is shown like this over here. Initially the flow inside the boundary layer even though it is 2 dimensional it will remain laminar, but as we move more and more into the x direction, after certain point there is going to be turbulence present of the presence of turbulence inside the boundary layer cannot be neglected.

And, this initiation of turbulence inside the boundary layer will change the thickness of the boundary layer in a more drastic fashion and in a more irregular pattern. So, what we have on the right hand side of this dotted line is the existence of turbulent flow, in the boundary layer and the demarcation the artificial demarcation, where it takes place it is denoted as x c or the transition or the transition length.

So, beyond x c all the flow inside the boundary layer is treated as turbulent whereas, before or before the before this x c the flow inside the boundary layer is treated as laminar. As, with the case of laminar and turbulent flow, while flow is taking place through a pipe. There is no distinct number. That is change from laminar to turbulent is not sudden, rather it is a gradual process in which the fluctuations present inside the flow cannot be contained, cannot be damped enough by viscous forces. And therefore, they start to grow and create a turbulent condition.

So, there is no magic number beyond which the flow is treated as laminar and before that the flow beyond, which the flow is treated as turbulent, but for convenience as we have done for the case of pipe flow there is 20 one Reynolds number equals 21 100 and beyond is taken to be turbulent flow. Similarly, for flow over a flat plate the flow over a flat plate when the flow is panel to the flat plate the transition Reynolds number, which beyond which the flow is will be turbulent is taken to be as 5 into 10 to the power 5.

So, the Reynolds number transition is taken to be equal to 5 into the 10 to the power 5 in beyond which the beyond which the flow is going to be turbulent, and this is defined as this Reynolds number is defined as. So, this is a velocity which is u infinity rho by mu where mu is the viscosity of the fluid. And I have a length scale over here this length scale is taken as x.

So, this is the length scale should value of Reynolds number will be different and different points, and when the Reynolds number transition exceeds 5 into 10 to the Reynolds number exceeds 5 into 10 to the power 5 this x is going to be equal to x c.

So, this when it is equal to 5 into 10 to the power 5 the flow is taken to be taken to be turbulent. So, the difference with the pipe flow Reynolds number is in the length scale. So, this x keeps on increasing. So, beyond the certain point we are going to get turbulent flow in the system.

So, this is this is this is a universally accepted to be the value Reynolds number value and before which it is laminar beyond which it is turbulent. So, if we look at these are the 3 equations which we have derived using the boundary layer approximation. So, this is the equation of continuity, this is the equation of motion, kinetic equation of motion in the x component this is the kinetic viscosity mu by rho and when you look at the energy equation, it is also using boundary layer approximations and this alpha is the thermal diffusivity which is denoted as k by rho c p.

So, the presence of v x and v y the presence of v x and v y in the energy equation mix this equation coupled with the momentum equation, the x component of momentum equation, whereas the absence of temperature in these 2 equations make them decoupled from the energy equation.

So, there is a one way coupling between the energy between the momentum equation and the energy equation, but the reverse coupling the reverse is not true. This would be there would be a coupling as well if the parameter here the physical property mu and rho start to vary significantly with the change in temperature. If, that does not happen, if the mu and rho can be treated to be a constant within the temperature zone of our operation, then there is going to be only one way coupling between the 2.

So, it is important that before we start even before we even start thinking about the solution. So, thinking about solving these 2 obtain T as a function of x and y. We, need to first solve the these 2 equations, which was which was done in your fluid mechanics class well very quickly briefly go through this without solving it.

First of all these are 2 equation, which will have to be solved simultaneously. So, the one way of reducing the number of equations that we need to solve, we can define a stream function as v x to be equal to del psi by del y. And, we can define the other one has a v y to be equal to del psi by del x. So, when we do that and since psi is an exact differential when we so, that therefore, the order of the differentiation is unimportant. So, when you put this over here and this one over here.

The equation of continuity gets satisfied automatically. So, do we do not need to solve equation of continuity separately, if we define our velocity in terms of a stream function psi and utilizing the definition of v x and v y in terms of psi, we can see the this equation is automatically satisfied. So, we will just have this equation to solve.

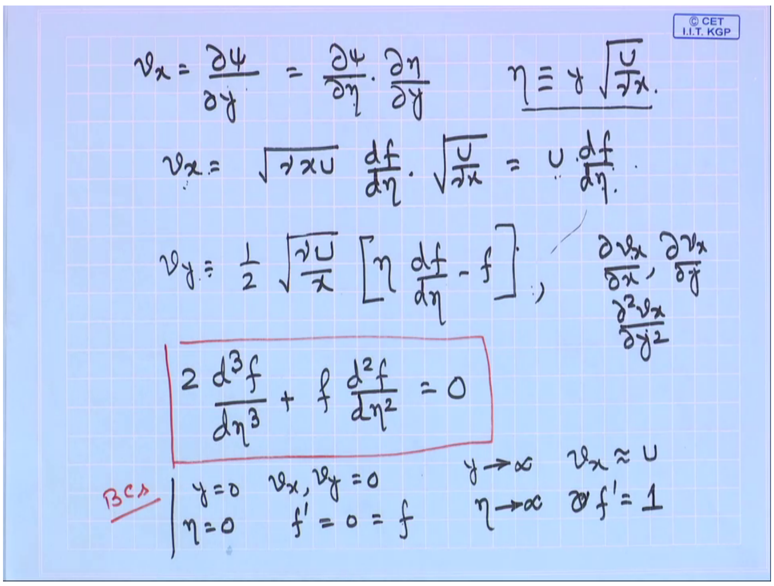
Now, this equation being a partial differential equation, it can be we can try to solve the using a method of combination of variables. So, what we what is done in method of combination of variables is a new independent variable eta is define, which is for this specific case is defined as y times U by nu U is a free stream velocity nu is the kinetic viscosity x and y are the independent variables.

So, this eta contains both y and psi and when you evaluate this v x v x v y del v x del x del v y del y and del square v x by del y square, in terms of eta then something interesting would at would automatically you can see. And, the dimensionless stream function which is f eta f as a function of eta is defined as psi by root over nu x U. So, these 2 are the definitions one is about a variable, which is a combination variable of the independent 2 independent variables that appear in this equation and this is simply non dimensionalizing the stream function by this.

So, therefore, utilizing these 2 our aim would be to express this equation not in terms of x y and v x or v y, but everything should be express in terms of eta, where eta would become the new independent variable and f would become the new dependent variable. So, if we can do that if we can show that f is a function of eta only, which is which is from this equation then the partial differential equation can be converted to an ordinary differential equation.

So, that is the whole beauty of this approach when you combine to independent variables to a new independent variable. And, all these v x v y v x v y del v x del x del v y del y etcetera are expressed in terms f or it is derivative. I will just show you one example and then will present the solution.

(Refer Slide Time: 12:50)



So, what is v x v x is by definition of a stream function del psi by del y which would be del psi by del eta times del eta by del y. And, eta as I have seen mentioned in the previous slide is y root over U by nu x.

So, therefore, you can you can see that v x would be equal to nu x U d f d eta times root over U by nu x, which is nothing, but U times d f d eta. The point to note here is that f is a function of eta only. So, starting with del psi del eta del eta del psi I am going to express this v x in terms of this. In a similar fashion that I am not going to right derive you one can write v y to be equal to half nu U by x eta times d f d eta minus f.

So, this is the expression for v y, in a similar fashion one can find out what is the expression for del v x del x del v x del y and del square v x by del y square, all the other terms present in the x component of the momentum equation. When, you substitute all of this and expressions for this in the equation what you get is this equation?.

So, this becomes the new governing equation for flow inside the hydrodynamic boundary layer. What is the advantage that we see that this is now an ordinary differential and this is now an ordinary differential equation which is non-linear, but it can it can it we can possibly try to solve it using the methods, using the new using the methods which are available to us and the best method to solve such an equation would probably be the numerical method.

So, the all these v x v y etcetera now expressed in terms of the dimensionless stream function f and the independent variables x and y are clubbed together in f. So, now, let us see in order to solve this equation, if you look at the solution we need to have 3 conditions 3 boundary conditions, if we need to solve the equation.

So, let us try to find out what are those boundary conditions. So, I have a plate and the flow is taking flow is taking place over the plate. So, what are the conditions that you can see about v x and v y on the plate. So, we on the plate means that is y equal 0 for any values of x, for y equal to 0 and if you look at the expression for eta y equal to 0 stands for eta equal 0. So, what happens at y equal 0 on the plate both v x and v y would be equal to 0 due to the no slip condition.

So, the no slip condition dictates that both v x and v y would be equal to 0. So, that is those are the 2 conditions what one can write based on the location at y equal 0. Let us see what is going to happen on the outer edge of the other edge of the boundary layer, which is the edge of the boundary layer beyond which the flow is one dimensional. What happens to the velocity of the velocity of the fluid the x component of the velocity of the fluid, the velocity of the fluid x component is 0 on the plate as we move away from the plate progressively the velocity in the x direction increases. And, from the edge of the boundary layer and beyond the velocity or in the x direction would be equal to the free stream velocity of the fluid which is coming towards the plate.

So, the other condition could be that a as y tends to infinity the mathematical speaking, the velocity in the x direction approaches that of the free stream velocity. So, those are the 3 conditions. What happens at y equal 0 and what happens as y tends to infinity? At, y equal 0 no slip condition dictates that the velocity both components of velocity would be 0 and at as T tends to 0, at a point far from the solid plate the velocity, if of the fluid stream would be equal to the free stream velocity. That means, v x is simply going to be equal to U these 3 are the boundary conditions, physical boundary conditions, which we need to express in terms of f and eta and try to solve the non-linear ordinary differential equation that we have that we have obtained over here.

So, I am going to write those boundary those equations over here first. So, for y equal 0 both v x and v y would be equal to 0. So, what is y equal 0. So, y equal 0 corresponds to if you see the definition of y definition of eta y equal 0 corresponds to eta equal to 0.

So, what is v x v x is u times f prime d f d eta d eta. So, it should give f prime to be equal to 0, which comes from v x to be equal to 0. And, then comes v y to be equal to 0, we understand that from here if prime is 0. So, in order for v y to be 0 f must be 0 as well. So, the other condition is f prime and both f prime and f will be 0, since my velocity would be 0 at y equal 0.

So, y equal 0 corresponds to eta equal 0, v x is 0 corresponds to f prime is equal to 0, v y to be equal to 0 then it must be f has to be equal to 0.

So, these are 2 boundary conditions from which one can write. And, the second condition is as y tends to infinity v x would be equal to v x would be equal to the free stream velocity. So, what is y equal tends to infinity. So, as y tends to infinity eta tends to infinity my v x is U v x is going to be equal to U which tells me that my f prime will be equal to 1.

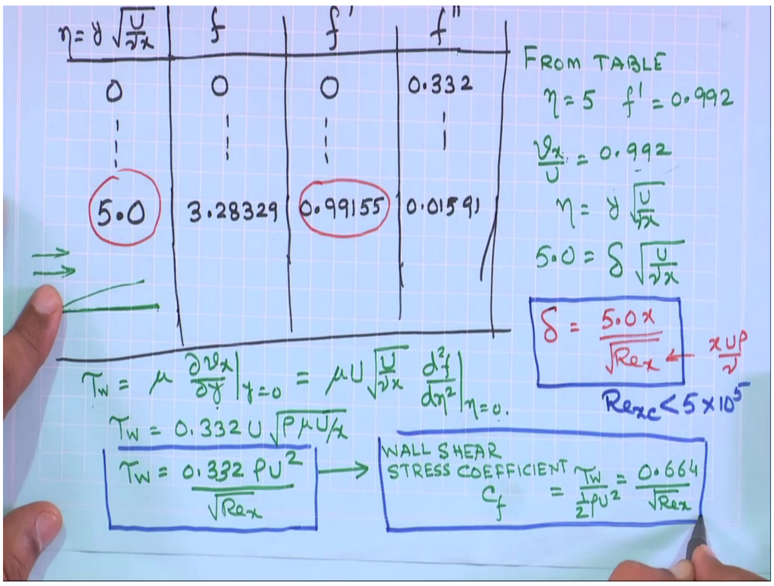
So, v x tends to U. So, therefore, f prime must be equal to one as eta tends to infinity. So, these 3 are the boundary conditions that one needs to have in order to solve for this. So, I think the physical nature of the equation and the boundary conditions are very clear to you.

Now, the next thing is even with these simplifications, even with the identification of the boundary conditions and analytical solution for this equation is not possible. One has to result to numerical solution of this equation and see what those results tell us to explain the flow and momentum transfer inside the hydrodynamic boundary layer. I am spending time on this, because as we have seen without knowledge of the hydrodynamics inside the boundary layer, one would not be able to solve the thermal boundary layer.

So, let us quickly see how would a numerical solution? What I am going to give you just the table containing the values of eta and the values of f prime and so on. And, some of the values I will least. Those values would be sufficient to tell as give us some idea of what is going to be the important parameters? For example, the thickness of the boundary layer or the shear stress coefficient for flow over a flat plate. If, we can get these 2 informations they would be should be sufficient to correlate the results with those of the thermal boundary layer.

So, let us look at the how would the result look like in the case of hydrodynamic boundary layer.

(Refer Slide Time: 21:54).



So, the table that we have over here, this is from the numerical results. So, the first one is eta which we know is defined as y root over U by nu x. The second one quantities f, the third one is f prime and the fourth one is f double prime.

So, at eta equal 0 at eta equal 0, we know that v x is equal to 0. So, f prime is 0 and f is also 0 from v y. So, both f prime f and f prime would be 0 and the numerical solution of the double derivative of f which is d to f d eta square it is 0.3 3 2.

So, there are several numbers which are listed over here, I will I not write everything what you would a interesting thing is when this eta is 5 the value of f is about 3.28329, the value of f double prime is 0.9 9155 in f f single prime and f double prime in 0.01591. There are other numbers as well which I will not I will not do not need right now.

So, what is the significance of this table? This table which is obtained from the numerical solution of the governing equation using the boundary conditions that I have already discussed. So, from the table it is clear that for eta equals 5, for eta equals 5 f prime is 0.9 9, f prime is equals 0.992. And, what is f prime, if you again look at f prime this f prime is v x by U, which is f prime. So, v x by U is 0.992, what they what the tells us is the interesting; that means, for a value of eta equals 5 the value of v x by U or v x reaches 99 percent of the free stream velocity.

So, which is unique because this tells us that at a value of eta equals 5 the velocity inside the boundary layer is almost equal to 99 and the definition the working definition of boundary layer thickness is that point, that point in y where the velocity reaches 99 percent of the free stream velocity. So, this point then refers to the edge of the boundary layer. Now, if this is edge of the boundary layer and if I since I know eta equals y root over U by x, then when I put the value of eta to be equal to 5 from here this y must be equal to the delta, which is the which is the local thickness of the boundary layer.

So, this delta is a function of x as we move along the plate the value of delta will increase, but for a value of eta equals 5 then y must be equal to delta. So, when you reorganize this equation what you are going to get is a very important relation, the delta the film thickness is going to be equal to 5 x by root over r e x where R e x is the local value of the Reynolds number defined as x v rho. So, this r e x is x U rho by mu local value the Reynolds number. And therefore, delta is going to be equal to 5 x by root over R e x.

So, this gives you a compact expression for the thickness of the boundary layer at different axial locations. Now, this equation since we are assume it is only laminar flow this is valid for a Reynolds number transition Reynolds number of 5 into 10 to the power 5. So, as long as your Reynolds number is less than 5 into 10 to the power 5 the thickness of the boundary layer at any location thickness of the hydrodynamic boundary layer at any location, can be expressed by this formula.

On a similar note one can write the wall shear stress that is the shear stress felt by the wall by the plate over which this flow is taking place. So, this is the shear stress felt by the solid surface as mu times del v x del y at y equal 0.

So, this is the stress at y equal 0. So, that is why we have this subscript w signifying wall which tau in this after a bit of simplification by converting this y to eta and so on. This is can be written as I am not writing all the steps in here, as mu times U root over U by nu x d 2 f by d eta square at eta equal 0.

So, the wall shear stress this is Newton's law of viscosity and then after converting the v x to f and y to eta this is the expression that you should get. So, the wall shear stress that is knowledge of wall shear stress therefore, requires the value of f double prime at eta equal 0 and from the table over here you know that what is the value of f double prime at eta equal 0. So, therefore, one can write tau w as 0.332 U by root over rho mu U by x or the expression of tau w to be equals 0.332 rho U square by root over R e x.

So, this is another important result as it gives you the value of the wall shear stress for flow over for flow over a flat plate and a corollary of this is the wall shear stress coefficient, which is expressed as C f, which is defined as tau w by half rho U square, which would be equal to 0.664 by root over R e x.

Let us when some more time on this wall shear stress coefficient. So, first of all why we would we would that like to have the concept of wall shear? The wall shear essentially is the engineering parameter of interest, because whenever you are trying to design something whenever you are trying to evaluate, the force exhorted by a moving fluid on a stationary platform. You need to find out what is the wall shear stress.

The wall shear stress then you will have to be multiplied by the area in order to calculate the total force.

So, therefore, shear stress plays a very important role the wall shear stress plays a very important role, in the design of the of a surface evaluation of the force on a surface and so on. And, they contain they are function of the local value of the Reynolds number, the properties of the liquid rho and mu and the impose condition which is the velocity of the fluid, which is flowing over the plate.

And, another engineering parameter of interest is wall shear stress coefficient, which is expressed as wall shear divided by half rho U square, which is also known as the dynamic pressure. So, the wall shear stress coefficient is a function is the ratio of the wall shear by the dynamic pressure and which is x which can be obtained directly from this as 0.664 by root over R e x.

I think this much of fluid mechanics should be sufficient for us to progress to find out what is a find out how to handle the heat transfer thermal boundary layer the heat transfer part of it? Since thermal and the fluid mechanics is linked to heat transfer by the appearance of velocities in the expressions one first need to know, what is the velocity? What are the components of velocity? What is the thickness of the boundary layer? What is the shear stress coefficient and then try to get into the thermal boundary layer in order to obtain the solution.

So, what would be the equivalent to one for the case for the case of for the case of heat transfer, the C f in heat transfer is related is similar in concept to the heat transfer coefficient or the Nusselt number. So, our aim is to obtain Nusselt number. So, what will you do is based on the knowledge of the flow inside the hydrodynamic boundary layer, we would convert this knowledge to our thermal boundary layer and try to derive the quantities which are of relevance in heat transfer. Namely the temperature and the heat transfer coefficient. So, we are spending so, much time on the fluid mechanics just to make sure that we understand the next step, when we have to use these results in order to obtain the Nusselt number which we do in the next class.