Heat Transfer
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Lecture - 21
Momentum and Thermal Boundary Layers

So, we have seen the use of energy equation. And, now we will go back to our original aim to obtain an expression for H, the convective Heat Transfer coefficient. And, we are going to choose the simplest possible flow geometry, which is flow over a flat plate. So, flat plate is kept stationary in a flowing air stream, which whose temperature is different from the temperature of the flat plate.

So, we see that every day in around us where a hot plate is getting cooled or heated, when it is exposed to a flow of air over heat. And, let us assume that the flow is laminar and at steady state and you would like to see what would be the expression for Nusselt number, which contains the convective heat transfer coefficient H for such case. But, before we do that, I would recall once again that there would be the formation of a hydrodynamic boundary layer, and the formation of a thermal boundary layer on the surface on the solid surface, inside and let us say that the air is approaching it as with a constant velocity.

So, the velocity of the air, which is coming is constant and is not a function of is not a function of x is not a function of y or of z. So, it is an 1 dimensional flow of air, which is approaching a flat plate. So, when this air comes in contact with the solid due to viscosity and due to no slip condition, the flow near the plate very close to the plate it slows down ok. And, the velocity of air which is flowing over the plate will increase from 0 on the plate due to no slip to the velocity of the air, which is approaching it beyond a certain thickness of air layer, which is known as the boundary layer.

So, a hydrodynamic boundary layer is that thin layer of air in contact with the stationary plate inside which, you would expect a variation in velocity with the a y direction ok. So, the velocity varies with the y direction and it asymptotically reaches the value of the approach velocity beyond that layer. So, inside the layer since the flow varies with y, the equation of continuity looking at the equation of continuity you would realize, that the flow inside the boundary layer is 2 dimensional ok. We will for the time being assume that the depth of the field is very large compared to the thickness of the plate.

So, therefore, the velocity or the temperature remains fixed it is not a function of z, it is a function only of x and it is a function of y, but it is not a function of z. So, if you assume a 2 dimensional flow, then inside the boundary layer the flow the hydrodynamic flow of air is going to be 2 dimensional. There is going to be a variation in velocity with x and variation in velocity with y. Similarly, when we think of the temperature boundary layer or the thermal boundary layer, the temperature of the fluid which is approaching the solid plate at a different temperature this is constant, this is invariant this does not depend on either x y or z.

But, the moment it comes in contact with let us heated plate. The temperature on the plate of the immovable air molecules due to no slip condition, the temperature is going to be equal to the temperature of the solid plate. As, we move up the temperature gradually changes till it asymptotically becomes equal to the temperature of the incoming fluid.

So, the thickness of the layer in which the temperature varies from the temperature of the base plate, from the temperature of the substrate to the temperature of the free stream that free stream air, which is coming is known as the thermal boundary layer. And, as again the temperature inside the thermal boundary layer is going to be a function of the actual position; that means, it is going to be a function of x and it is going to be a function of y.

So, outside of the boundary layer be it a hydrodynamic boundary layer or a thermal boundary layer the flow is 2 dimensional. So, the velocity is a constant, velocity is equal to the approach velocity, velocity is equal to the free stream velocity, for a flat plate this approach velocity in the free stream velocity; that means, the velocity outside the boundary layer they are equal. And therefore, the flow inside the boundary layer both hydrodynamic and thermal would be 2 dimensional.

So, we would have to see how to solve the equation? For laminar flow, inside a boundary layer, inside the thermal boundary layer, but as I have told you before since the momentum transfer and the heat transfer equations are coupled, we probably first need to solve what is the expression for velocity inside the boundary layer? Inside the hydrodynamic boundary layer and then use that knowledge for the solution of the thermal boundary layer to obtain an expression for Nusselt number, that is our objective our objective is to obtain an expression for Nusselt number.

But, before we go up to that objective, we need to modify, we need to simplify, the equation of continuity, equation of motion and equation of energy, that dictate these 3 equations dictate the, the momentum equation, the energy equation and obviously, the continuity equation which is nothing, but a statement of conservation of mass. So, as statement of conservation of mass equation of continuity, the momentum which is the equation of motion and energy which is the energy equation that we have use can we simplified that.

So, it seems that there is a way to simplify these equations to make it more compact such that we can try we can attempt and analytics solution, if not analytic a semi analytics solution, but the goal is to obtain the heat transfer coefficient. Now, in order to obtain the heat transfer coefficient, we first need to see what those approximations are which are collectively known as the boundary layer approximation.

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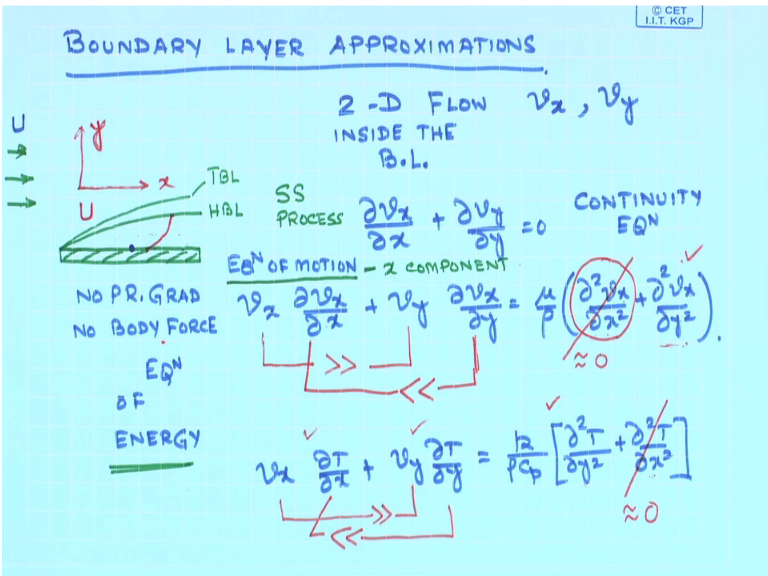

So, I start with boundary layer approximation in this class and I have drawn this substrate which is stationary. But, let us say there is a flow of air towards this plate the velocity here is let us say the velocity here is U, which is a constant and the temperature of this is also a constant. And, it is 2 dimensional flow, which simply tells you that the non-zero component of velocity are v x and v y and you have inside 2-D flow inside the boundary layer. So, we have v x and v y, but v z is equal to 0. The concept of boundary layer and the associated analysis they play, they together play a very important role in evaluating any transport process taking place near a surface when it interacts with a moving fluid.

So, it is important in momentum transfer, it is important in heat transfer and it is important in mass transfer. Not, only that it is also going to be the first step to obtain the similarity between momentum heat and mass transfer process, which you would read in your subsequent airs. So, let us first write what are the equations. So, the equation of continuity would simply give you del v x del x plus del v y del y to be equal to 0.

So, this is your continuity equation. In the momentum equation is v x del v x del x plus v y del v x del y is kinematic viscosity, or mu by rho times del square v x by del x square plus del square v x by del y square ok. So, here I have assumed that the there is no pressure gradient, no imposed pressure gradient, no body force and so on. So, this is this is what the and this so, this is what is the equation of motion? And, in your all cases I am assuming, that it is a steady state process.

So, this is going to be the other up equation and the equation of energy is v x del T del x plus v y del T del y is equal to k by rho c p del 2 T by del y square plus del 2 T by del x square. This is equation of motion I should also say that this is the x component of the equation of motion. So, this 3 are our governing equations, which we need to which we need to solve, but before we solve that let us see if you can reduce this equation. If, you can get rid of some of the terms based on the physical understanding is this term small compared to this or is it small as compared to compared to the second term and so on.

So, let us first heuristically try to see what is happening? This is my x direction and this is the y direction. So, as you can see most of the flow is in the x direction, there is very little flow in the y direction. So, which tells you that this v x must be very large in comparison to v y. So, v x is going to be large in comparison to v y. So, can we neglect the neglect the second term with respect to the first we cannot do that, because v y maybe small, but the v x varies drastically with y the variation of v x with y is going to be very large in comparison to variation of v x with x.

So, this term this part is going to be very large in comparison to this term. So, we need to need to keep in mind that the thickness of the boundary layer is very small. And, the velocity varies from varies from 0 at this point to the free stream velocity capital U, over here. So, since the d y over which the velocity changes from 0 to U is very small this term the gradient with respect to y is going to be very large compared to the gradient with respect to x, which tells to us that none of the terms on the left hand side of the governing equation of the x component of equation of motion can be neglected.

We need to keep both the terms on one both the terms of the left hand side in our equation. So, let us that is our the first observation. What is the second observation? The second observation this is convective flow of momentum, because you have v x and v y in their native form present in the terms. So, both the terms on the left hand side refers to convective transport of x momentum inside the boundary layer. What is over here? I have mu in this case wherever I has mu the same way as in k this presence of mu the significant makes the significance of the two terms on the right hand side, they collectively refer as the diffusive transport of momentum.

So, what is diffusive transport of momentum? This is the transport of momentum because of the presence of viscosity, such that the velocities are transmitted through layers. The momentum is transmitted through the layers, which are moving at different different velocity. So, the velocity profile would probably look something like this. Where it starts at value equal to 0 and becomes equal to capital U asymptotically approaches capital U. So, this one is the diffusive transport of momentum.

Now, of these two since the variation in velocity v x with x is small variation of v x with x is small as compared to variation in v x with y; that means, in this direction. So, the overall contribution of the diffusive component of x x diffusive component of the x momentum is very small. And therefore, it can be equated to 0 I would I would explain it one more time. Both terms on the right hand side they refer to diffusive transport of momentum. They are diffusive transport which can easily be identified by the appearance of the viscosity in front of them, in the variation of v x with x is small.

Now, the diffusive transport depends on what is the velocity gradient? The velocity gradient in this direction is small as compared to the velocity gradient in this direction. So, the diffusive transport of x momentum in the x direction is very small, as compared to the diffusive transport of the x momentum in the y direction. So, this would allow us to drop the first term on the left hand side. And therefore, the governing equation which dictates the motion inside the boundary layer, the equates the reduced form of the motion of x component momentum transfer inside the boundary layer is two terms on the left hand side and only one term on the right hand side.

Now, let us apply the same logic to my energy equation. As, before this v x is very large in comparison to v y; however, the change in temperature with respect to y is going to be very large in comparison to the change in temperature with respect to x. So, T changes from T S, which is the surface temperature of the substrate 2 T infinity, where T infinity is the temperature of the fluid, which is flowing over the flat plate. Since, the thermal boundary layer is small in thickness compared to the overall length scale. So, del T del y would be much more than del T del x and which makes me which make which creates which leads to the conclusion like in equation of motion, that none of the terms on the left hand side of equation energy can be neglected can be made equal to 0.

So, both terms will remain in the energy equation. Now, we come to the right hand side. As, before the appearance of k the significance of these 2 terms are the diffusive transport of energy in the y and in the x direction, or this these are the conductive transport of energy in the x and in the y direction. The conduction depends on temperature gradient. Now, since the temperature changes less we in the x direction, as compared to the large change in the y direction, the contribution of this would be small and it can be equated to 0.

So, therefore, the only term which will remain on the left hand side would be this. And therefore, the you can see the similarity between these 2 equations, how equation of motion x component of equation of motion inside the boundary layer? And, the equation of energy inside the boundary layer can be reduced can be simplified by dropping the dropping the x component of diffusive transport, and the conductive transport of energy in the x direction in comparison to the y direction and none of the terms on the left hand side can be neglected.

So, this purely is from heuristics, this purely is from our understanding of what happens to the flow inside the boundary layer. Both in terms of momentum transfer as well as in terms of diffusive as well as in terms of heat transfer. These approximations or these understandings, these simplifications together are known as boundary layer approximations.

So, what we have done here is we have written the equation, the momentum and the energy as well as the as well as the equation of continuity over here. For a 2 dimensional flow, 2 dimensional steady state flow, 2 dimensional steady state flow without the presence of any imposed pressure imposed pressure gradient or without the presence of anybody force.

I have also written the equation of energy in here we did not considered the viscous dissipation, nor did we consider any amount of heat which is generated inside the system. And, then we have we have neglected the terms we have we have understood how I can drop these 2 terms? And therefore, I have the reduced form of the equations, which dictate the energy transfer and momentum transfer inside the boundary layer.

But, if you again if you look at the energy equation, which we are going to solve in order to obtain an expression of Nusselt number that is what our aim is the presence of v x and v y tells us that we need to solve these 2 equations first. Equation of continuity and equation of motion to obtain in order to obtain expressions of v x and v y which could then be substituted in here and then this equation can be solved to obtain T as a function of x and y. Only, when we obtain T as function of x and y, we should be able to obtain what is the expression of heat transfer coefficient, convective heat transfer coefficient, or what is the expression of Nusselt number?

So, that is going to be the next exercise, but before we before we moving to the energy equation, we need to solve this 2 equations to obtain v x and v y. I think you have done that part the momentum transfer part in your fluid mechanics, where the partial differential equation contain consisting of v x and v y, where solved by the method of combination of variables. So, what are you do in the next class is very quickly go through this combination of variables, without showing you all the steps.

Because, I think you have a fair idea of that from your fluid mechanics, if not please go back to your text like you can take a look at a fox and McDonald, which is a standard textbook for fluid mechanics and from fox and McDonald you would see how this was done? But, I would give you some of the pointers major pointers to see how the method of combination of variables. And, the introduction of stream function would allow us to reduce the equation to solve the equation, in a semi anal in a analytical method using an analytical method and the numerical solution of the of the changed the new governing equation, and use the results of that numerical solution for the solution of the temperature profile and Nusselt number it is not complicated.

Let me assured you that it is not going to be a complicated process. The fundamental concept must be clear to you the rest is available in your text book. And, once you have the concepts clear you should be able to follow the steps in between and arrived at the final results. I do not give much importance on get on the steps, but the concept is extremely important.

So, what I have done in today's class is the introduction is the boundary layer approximations the simplifications, that we can use and the simplifications, that we cannot use both are going to be important. So, in next class we will see how these equations? How, this is coupled equations can be solved to obtain an expression for Nusselt number?