# Heat Transfer
**Prof. Sunando Dasgupta**
**Department of Chemical Engineering**
**Indian Institute of Technology, Kharagpur**

## Lecture - 20
## Nusselt Number of a Heated Sphere in Stagnant Air

So, we have covered the energy equation, it is derivation the significance of each of these terms in the equations. And, I have shown you how to use that equation for a specific example, in which the viscous heating was considered to be important for a planet system. So, I would show you the energy equation once again. And, then solve a few problems on it, after I explain the significance once again.

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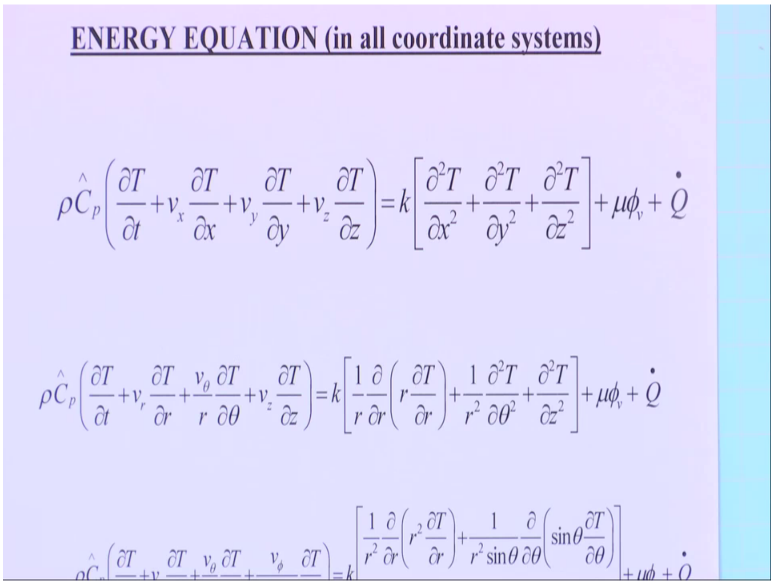


So, let us look at this energy equation, which I think now you would be able to see, and as you can see the energy equation is presented for all 3 coordinate systems. So, the first one that you see this is for Cartesian coordinate systems, this one is for cylindrical coordinate systems, and the last one is for spherical coordinate systems.

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$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]+\mu\phi_v+\dot{Q}$$

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right]+\mu\phi_v+\dot{Q}$$

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)=k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right]+\mu\phi_v+\dot{Q}$$

So, the first one is in x y z and the temperature is a function of x y x y and z and it could also be a function of time. So, the transient behavior is also included in it. And, if you look at the equation I have added this phi v term in all 3 equations, this phi v refers to the viscous dissipation, the energy generation due to viscous heat, which in most of the cases as I have mentioned before may be neglected. And it is easy to identify the viscous dissipation term in energy equation by the appearance of viscosity mu with all such terms.

So, in the expanded form of the equation, where the complete form of phi v is included, you should be able to identify them as referring to viscous dissipation energy generation by viscous dissipation, because of the appearance of mu in all those terms. What is here as Q dot; is the energy generated by some other means in the system. So, it could be electrical, nuclear or some other form of energy, which is generated inside the system.

So, the significance of phi v should now be cleared to all of you and the expanded form of phi v for all these systems would be provided to you. And, their also available in the text book, which I did not include in these equations. One more point here is I never expect you to remember this equation. So, whenever you need to use the energy equation, the equation in this specific firm would be provided in the question papers.

So, do not memorize this equation, but rather if the equation is given you should be able to identify what are the significants of each of these terms, in how to resolve them? How to neglect or take them into consideration based on the physics of the problem?

So, if I take the first equation, which is the energy equation in the Cartesian coordinate, what you see as the first term on the left hand side this del temperature by del time simply refers to the transient nature of the process. So, had this been a steady state process this first term would obviously, be equal to 0; so this first term would be equal to 0 for a steady state system all the other terms the remaining 3 terms on the right on the left hand side, they have velocity is the component of velocity v x v y and v z associated with them.

So, they collectively refer to the convective transport of heat into the system. So, the appearances of v x, v y and v z in their origin in their component form the referred to the transport of energy by convection in the x direction in the y direction and in the z direction.

So, the entire left hand side of the energy equation refers to transient term the first term being the transient term and the second third and fourth or the convective terms. When you go to the right hand side the appearance of thermal conductivity k, essentially signifies or identifies this terms that are associated with conductive heat transfer, heat transfer due to conduction. So, k del to del x square or del y square or del z square they refer to conduction.

So, the right hand side contains the conductive terms, the viscous dissipation term and the energy generation inside the system. Should together this provides the complete form of the energy equation, which does not take into account the mechanical energy, because this equation was derived from the first law of thermodynamics considering both the internal and the kinetic energy. From that equation the kinetic energy equation was subtracted to obtain the energy equation in this form, which does not take into account are kinetic energy or the mechanical energy part of the mechanical energy part of the equation.

So, these 3 are the equations, which we would use in all our problem solution. So, if you look at the second and the third equation, the cylindrical and the spherical coordinates; in the cylindrical coordinates you have the independent variable independent space variable as r theta and z so, the radial direction the angular direction and the actual direction. So, these and their associated with velocity in the r direction, velocity in the theta direction, and velocity in the actual direction so, transient and convective terms in a cylindrical coordinates systems. The right hand side refers to conductive heat transfer in the r direction, conductive heat transfer in the theta direction and conductive heat transfer in the z direction.

In similarly, we have the dissipation function and the heat generation 1.2. Note here is that obviously, since this is the coordinate systems are different, the expression of phi v here and over here are going to be different and the complete expanded form of phi v for different coordinate systems are available in your textbook.

And, when we come to the cylindrical coordinate systems we have r theta and phi this these 3 terms refer to conviction, transient, the conductive heat transfer in the r direction, conductive heat transfer in the theta and in the phi direction. Again, we have this phi v and Q dot. So, I guess the significance of the terms in energy equation is now clear to you. So, what you need to do what is should do in solving problems of combined conduction and convection or convection alone is to see, which of these terms is significant. In therefore, the problem at hand and then you can cancel out the terms which are not present which are not relevant in that case.

So, if it is a steady state; obviously, this term would be equal to 0, if it is a one dimensional conduction only case, then v x v y and v z are 0. So, the entire left hand side would be 0 for conduction only case no convection. And, only these 3 terms would remain. And, normally phi v is neglect phi v is neglected in if we may or may not have Q the heat generated in the system.

So, for a one dimensional conduction only case, the only the first term will remain. One dimensional steady state, conduction case, with no heat generation; the governing equation would simply be k d 2 T d x square to be equal to 0. And, you know that this is going to give rise to a linear, temperature, distribution as you have done in your conductional analysis. If in addition you also have Q dot to be present, then the form of this equation be k d 2 T d x square plus Q dot is equal to 0. And, you should be able to solve the equation with appropriate boundary conditions as defined or as mentioned or as you can see in form from the statement of the problem.

So, this once you follow a methodical approach in identifying the equation, in identifying and cancelling terms which are not relevant in the equation. It should give rise to a compact equation describing the temperature distribution in the system, either in presence or absence of conduction or convection. And, mostly in absence of viscous heat dissipation and without with or without heat generation by some other means.

So, this is how you would solve any problem in heat transfer, where both conduction and convectional present. So, which should not be difficult, the equation looks too long and complicated, but once you understand their significance the solution of the problem should be in theory very simple. At least would be able to obtain the governing equation and identify the boundary conditions.

In some cases the and analytic analytical solution is possible, you would be able to integrate the differential the governing differential equation without much of a travel and then you get an analytic expression for temperature as a function of position and or time. In some cases you we analytics solution would not be possible. So, we may have to go for a semi analytics solution and in many cases a numerical solution.

So, there are several packages available for numerical solution of the heat diffusion equation and we will discuss them as we move along. One more point which is should be apparent when you look at the energy equation carefully is the appearance of v x, v y and v z, the 3 components of velocity. Since, you have the velocity component embedded into the energy equation it requires the complete expression of v x v y and v z.

In other words in to solve the Navier-Stokes equation or you need to solve the momentum equation, as you have done in fluid mechanics to obtain and expression of v x, v y or v z. So, once you plug the functional dependence functional form of v x into the energy equation, then the only dependent variable would be temperature and the independent variable should be x y z and possibly time.

So, the presence of the velocity terms in the energy equation mix the energy equation coupled with the momentum transfer equation. So, a pre requisite for the solution of energy equation, in cases where convection is important; that means, where you have these velocity terms is that you need to know the velocity expression from your momentum a momentum equation solution.

So, that way heat transfer is always coupled to the momentum transfer, but the reverse may or may not be true, if you look at your Navier-Stokes equation or equation of motion the momentum equation. In your fluid mechanics textbook you would see that there are no terms containing T in there. So, that equation is independent of temperature provided the thermophysical properties that appear in the in the momentum equation, namely the density and the viscosity. As long as they are they are not affected by a change in temperature, as long as they are constant for the temperature range of operation, the momentum equation can be solved in dependent of the energy equation.

So, that way momentum equation is uncoupled from the energy equation, but the energy equation is coupled to the momentum equation. So, there is a one way coupling between momentum equation and energy equation as long as the thermophysical properties viscosity and density remain unaltered during the process.

So, it is important to identify and appreciate that there is a one way coupling between the momentum equation and the energy equation. Solution of energy equation requires the solution of momentum equation, but the reverse is not true. If the reverse it also true; that means, for cases in which the temperature variation is such that viscosity and density would also be changed would also vary based on the temperature, then there is a two way coupling between the momentum equation and the energy equation.

So, with their is a two way coupling between these 2 equations then both of them the situation is more complicated and you need to solve both the energy equation and the momentum equation simultaneously. So, that is very important to realize the importance of the assumption of constancy in viscosity and density to make the momentum equation uncoupled from the energy equation, which may not be true in all cases for which combined solution at the both of these equations will have to be simultaneously solved.

So, we would see some of the examples when a mostly when there uncoupled and you would be able to see you will be able to solve them separately. As we would see in the next class will be talk about the thermal boundary layer and so on. But, in today’s class we would see a very simple problem and which would establish the relation between convection and conduction. And, for that we would choose a system, which is very simple to visualize. Let us see a solid ball; spherical ball is at a temperature T which T 0 which is higher than the temperature of the room in which the ball is suspended in steel air.

So, in this room if there is a steel ball whose temperature is 100 degree centigrade let us see an it is suspended in this room, where there is no forced flow of air over the surface. So, if all the fans etcetera in the room is switched off all the windows are closed, then it is safe to assume that there is no flow of heat due to convection. Since, there is no external agency which forces the fluid to move fast the solid sphere, which is at a higher temperature than that of the air surrounding it. So, in room full or room of steel air a ball a spherical ball is being cold. So, you can think of it as the limiting case in which the convective flow is slowly brought to 0.

So, we would like to see what is going to be the convective heat transfer coefficient, the limiting value of the convective heat transfer coefficient when there is no flow. So, it is a hypothetical situation. All convictions require the presence of a flow, but if I am slowly reducing the flow velocity the flow past the cylinder. Then if I can make the velocity very small then all the heat is going to be transferred from the solid to the air by a conduction process.

And, if I equate that conduction by which heat is lost from the sphere to a hypothetical heat transfer coefficient at 0 velocity, then the value of each convective heat transfer coefficient or the value of the dimensionless number, which is in convection, which is common in convection, which is Nusselt number, which is defined as h L by k. H is the convective heat transfer coefficient, L is the length scale and k is the thermal conductivity of the fluid surrounding it, then this h L by k refers to Nusselt number. And, you would see that in almost all of the relations for convective heat transfer coefficient, the relation or correlation is expressed in terms of Nusselt number.

So, that way Nusselt number plays a very important role in quantifying convective heat transfer from any object, it could be a flat surface, it could be a spherical ball or it could be a bank of tubes through which a hot fluid is flowing and cold air is pumped over them, cold air is made to flow over them to cool the liquid which is passing through tubes. So, in all cases the heat loss or gain by the fluid is expressed in terms of a dimensionless number, which is called Nusselt number.

So, the problem that we are going to solve in this class looks at the value of Nusselt number when the value of the imposed velocity is reduced and it is slowly brought to a value equal to 0. So, at that point conduction convection stops and conduction takes over, but we would like to pinpoint to find out the asymptotic value of the heat transfer coefficient or the Nusselt number for the case of 0 velocity.

So, that is the problem, which we are going to look, which we are going to examine in this class.

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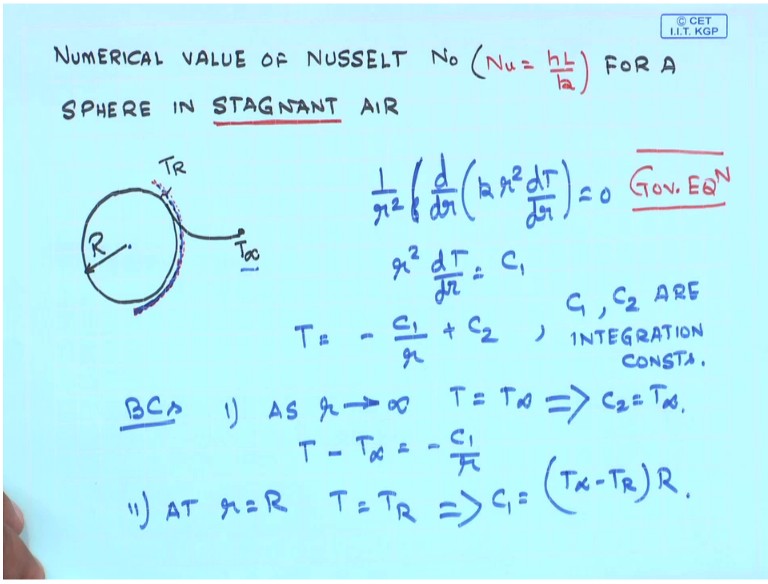


So, the problem that we have now is then find out the numerical value of Nusselt number, which is denoted by N u is equal to h times L by k, where h is a convective heat transfer coefficient, L is the length scale of the system that we are discussing and k is a thermal conductivity of the surrounding fluid for in stagnant air it is assume the fluid is air and important one is stagnant. So, there is no flow in the air.

So, I draw the system as this spherical ball has radius equal to R. And the surface temperature is maintained at T r, where is the temperature at a point far from it is T infinity and the spherical ball is going to lose heat by you can by a combination of conduction and convection, and then you would like to see how it is going to lose it is heat?

So, far this let us assume a thin shell of air surrounding the hot sphere ok. And, I am going to write the energy equation for the air which is situated in this thin shell of air completely surrounding the sphere. So, if you look over here for the energy equation for the for the energy equation and if you see there is no velocity, the velocity there is no imposed velocity and it is steady state case.

So, the first term would be 0 v r v theta and v phi all are 0 since it is stagnant air. So, therefore, the entire left hand side of this equation would be 0. Since, there is no flow or very little flow to talk about in their no question of viscous dissipation and therefore, this term would also be 0. And, there is no heat generation in the thin shell, that in the thin shell that I have drawn over here. So, therefore, the Q dot term would be 0. So, the terms which would remain are the terms that represented represent conductive heat flow.

Now, let us look at a what is the variation of temperature with r theta and phi. Obviously, the shell that we have drawn over here it in the shell there is going to be no dependence of temperature with theta and no dependence of temperature with phi. Temperature is only going to only going to depend on how far this shell is from the centre of the sphere? Or in other words temperature is going to be a function only of r and nothing else.

So, this equation even though it looks long and complicated, once you understand the significance of the terms then you would be able to cancel the whole of left hand side the second term on the right hand side, the third term the fourth term and fifth term. And therefore, the governing equation for as is for a spherical system, which is which is dissipating is energy into a room full of stagnant air can simply be written as one by r square the only non-zero term in the energy equation times k r square d t d r to be equal to 0. So, this becomes your governing equation.

So, I can again it is so, simple to start with the general equation in obtaining governing equation without just by looking at the equation looking at the system and cancelling the terms. So, is my thermal conductivity is constant I am simply going to write r square d T d r is some constant C 1, in once you integrate you should be able to integrate it would be minus C 1 by r plus C 2, where C 1 and C 2 are constants are integration constants.

So this is required to boundary conditions. So, what could be the boundary conditions; obviously, you would be we can see that the temperatures at 2 locations are provided to you. The temperature on the surface of the sphere is known is equal to T r and temperature at a point far from the sphere is the temperature of the air, which is T infinity. So, the boundary condition the first boundary condition would be as r tends to infinity at a firm, that is a for from it T would be equal to T infinity and this would give you C 2 as equal to T infinity.

Therefore, the equation would becomes T minus T infinity is equal to minus C 1 by r. And, the second boundary condition would tell you at r equals capital R; that means, on the surface of it T is equal to T R, which would give you C 1 to be equal to T infinity minus T R times R.

So, these are the 2 integration constants which you can evaluate by using appropriate boundary conditions. So, therefore, using the expressions of C 1 and C 2 you would be able to write the temperature profile.

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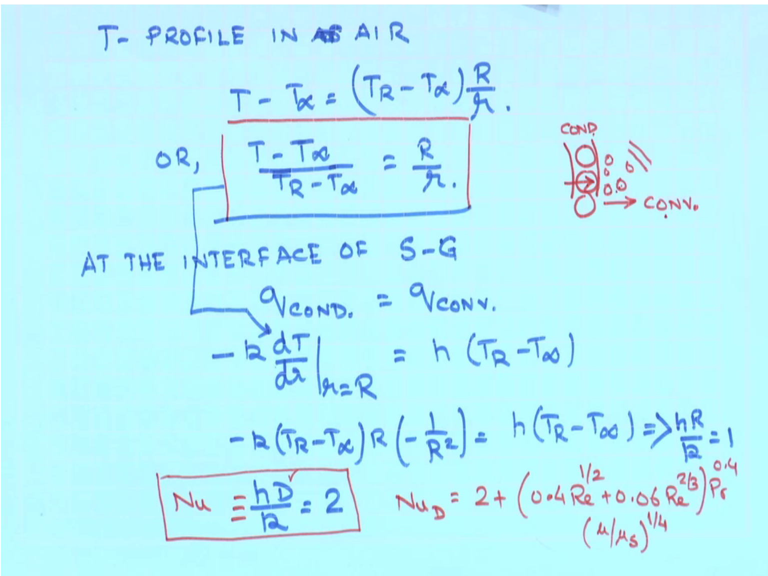


In air that surrounds the hot sphere as T minus T infinity is equal T r minus T infinity times capital R by small r. Or, we can rearrange it a little bit more to express within a more elegant fashion T minus T infinity, by T R minus T infinity is equal to R by r.

So, this is what to be temperature distribution in the air is non. So, if you look at the interface of the solid and the gas which is air so; that means, at this point right, at this point at the interface between of the solid and the gas which is air. So that means, at this point right at this point at the inter face between the solid and the gas the heat, which comes in as I heat with it comes in by conduction must be equal to the heat which goes out by convection.

So, at the interface between the solid and the liquid, the solid and the and the gas the gas is immobilize immobile due to the due to the no slip condition. So, all the heat transfers through this immobile gas layer must take place by conduction and on the outside of this gas layer delete the air is mobile. So, therefore, you can have conviction from this immobile there. So, if I think of this as my immobile layer of the gas molecules, then heat is going to get transferred by conduction over here and over here the molecules are mobile. So, heat is going to transfer by convection for the layer of molecular which as situated just outside of the interface and are free to move.

So, in order to maintain steady state this conductive heat must be equal to the convective heat. So, whatever be the amount of heat that goes through the molecules by a conduction process must be equal to the convective process by, which this heat is being picked up by the mobile gas molecules. So, if we understand that I am simply going to write that at the interface q conduction is equal to q convection.

So, what is the expression for q conduction? By Fourier's law it is going to be d T d r at small r is equal to capital R and the conviction is simply h times T R minus T infinity, this is the heat transfer coefficient. And, then when you substitute d T d r from here to this point, what you would get is minus k T R minus T infinity R times minus 1 by R square is equals h times T R minus T infinity which gives you that, which would give you h R by k to be equal to 1 or h D by k is equal to 2. And this h D by k is defined as the Nusselt number.

So, this is a very important relation which tells you what would be the value of Nusselt number. The limiting value of Nusselt number, when there is no flow of air surrounding the hot sphere. So, therefore, you get a numerical value of Nusselt number in the limiting case when there is no convection. Or this is very interesting result as the expression of h for velocity equal 0 is something, which is proven experimentary this is an asymptotic value which would which would give you the limiting value of Nusselt number for the case of heat loss from a sphere.

And, further calculations, further analysis showed that in presence of velocity; obviously, the value of each would increase, because more the velocity more would be the value of convective heat transfer each will increase an as h increases the value of Nusselt number will also increase.

So, the expression of Nusselt number which was obtained in a semi analytical fashion and using certain correlations, the that relation is Nusselt number based on diameter the length scale is taken as the diameter of the sphere is 2 plus 0.4 Reynolds number to the power half plus 0.0 6 Reynolds number to the power 2 by 3 times Prandtl number to the power 0.4. And then you also have a viscosity ratio to the power 1 by 4.

So, what this tells this is this is a correlation, which was obtained you; obviously, do not need to remember this, but what it shows is when the velocity is equal to 0, velocity is set to 0, the Reynolds number would be 0. And the entire second term on the right hand side would be 0 and the value of Nusselt number would approach to and this is what you have obtained over here? That the limiting value of Nusselt number at steady state for as sphere which is losing heat in a stagnant medium stagnant air is equal to 2.

So, that there is a simple, but elegant example of the use of the energy equation to obtain the governing equation, use of appropriate boundary conditions known temperatures in this case. To obtain the temperature profile in air and define a Nusselt number for the case when there is no flow of air, no flow of the surrounding air which is you can assume that it is the limiting case you can see that is it can be expressed as the case where you have conviction with 0 velocity.

That is a hypothetical limiting value of Nusselt number to be equal to 2 and numerous experiments and analysis has proved this to be correct. And the expression of Nusselt number generally used for flow of air over hot sphere we say, which is equal to 2 plus some function of Reynolds number. So, when Reynolds number is 0 Nusselt number asymptotically, which is a value equal to 2? So, this is one example of use of energy equation. We will see some more examples of that in the coming classes.

But, a in the next class we will start with flow over a flat plate and how to obtain the Nusselt number for that case in laminar flow.