Heat Transfer
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Lecture - 02
Introduction to Heat Transfer (Contd.)

So, by now we are familiar with the modes of Heat Transfer. And the fundamental relation of conduction, which is Fourier's law, which gives you the heat flux vector as consisting of components in 3 directions and the components in each direction can be expressed as proportional to the temperature gradient in that direction.

In other words q x, which is the heat flux the x component of the heat flux vector q is going to be proportional to del T del x, which is the derivative which is the change in temperature in the x direction and the proportionality constant being a property of the material which is known as thermal conductivity. What we are going to do next is fundamental relation, which is known to all of us which is conservation of energy. So, what does conservation of energy tell us and how the conservation of energy can be related to heat transfer, in order with an aim to obtain that temperature at a specific point at a given instant of time. Because in heat transfer that is what is required? You need to find out what is the temperature of an object at a specific point and at a specific instant of time.

Since, we are using conduction we are studying conduction right now we understand that conduction is essentially energy, which the flow of which depends on temperature gradient. So, we need to precisely know what is the temperature profile of a temperature profile inside an object, in order to find what is it is gradient? Which when multiplied with k would give us the flow of heat through a plane, which is perpendicular to the point where the temperatures are uniform.

So, if we have 2 points 2 different temperatures this point will call it as an isothermal surface where everywhere the temperature is equal to T 1 and this is the T 2 another surface, which is a T 2 when the amount of transfer between these 2 would simply be governed by Fourier's law, but the concept of control volume is going to be important.

Because not only you are going to have transport of heat between point 1 and point 2 there will be situations in which you are going to have heat generation between point 1 and point 2 as well. So, let us think of an current flowing through an electric wire. So, we are going to have some sort of joule heating which is present in between 2 points that you need to take into account in order to find what is the temperature of the object at any given point?

So, when you are generating heat and when you are having some amount of heat flowing in some amount of heat flowing out. And as a result of all this the total energy content of the volume that you see will keep on changing. In other words let us think of just block of copper, which you have placed on a hot surface and some amount of energy enters the block of copper, because of it is proximity with the hot surface.

And as it enters let us assume that no heat leaves from the copper surface to the surrounding, because let us say to begin with the temperature of the copper surfaces the same as that of the surrounding. So, no heat transfer takes place. If it is different as the copper surface becomes copper copper block of copper becomes more and more hot, it is going to loose energy from the all from the other surfaces to the surrounding.

So, you have some energy in by conduction, some energy which is going out of the copper block to the atmosphere again by conduction. And let us say by some means you are generating some amount of heat inside the inside the copper block and it is an unsteady state process.

So, if I think of an instant of time the amount of energy, which comes in to the copper block, the amount of energy which goes out of the copper block, the amount of energy which you are generating artificially inside the copper block, all these would result in the total amount of energy which is stored in the copper block.

So, we are going to talk that is what conservation of energy is all about the amount in minus the amount out plus any heat generation. So, that is taking place must be equal to the energy to the time rate of change of energy contained within the copper block. So, this copper block is therefore, is termed as the control volume. So, control volume is what in something it is an enclosed space, which has a specific mass and the form of conservation law, which is applicable for such a control volume am going to write that which is in minus out plus or minus plus is for where energy is generated inside the control volume.

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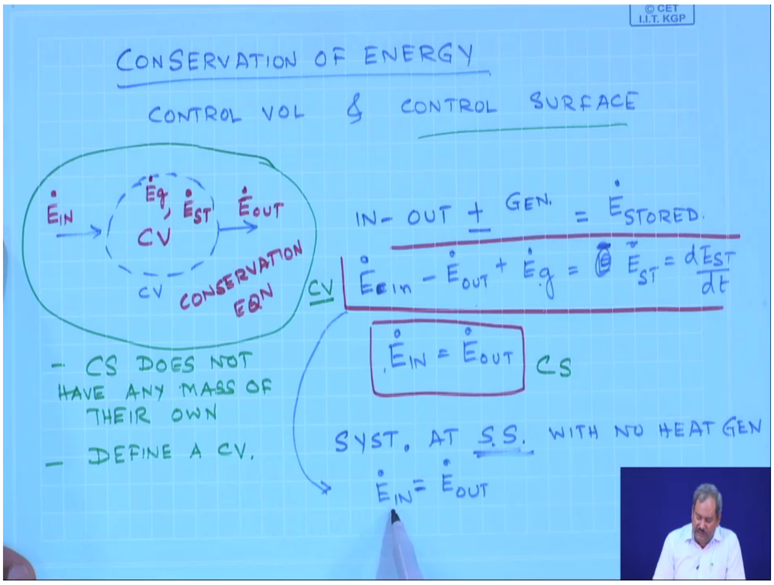

And minus when it is there is a some depletion of energy some energy energy, which is getting depleted inside the control volume and will see the examples of that in this. So, this must be equal to stored the time rate of change of energy stored in all these in and the out. So, this is E E dot in minus E dot out plus E dot generation is equal to E dot stored, which is nothing, but d E stored by d T.

So, this is the conservation equation and the enclosed space with the fixed mass is known as the control volume. So, this is a control volume and you have some amount of energy, which is coming in this is the time rate of energy, which is going in out and there may be some amount of energy, which is which is generated in here as a result of which you are going to have the rate of energy stored inside the control volume will also be different.

So, the conservation equation can be written for a control volume that has a fixed mass. So, this is what control volume is and this is the form of the conservation equation that you would expect you would you would you would expect for such a situation, then what is a control surface? Control surface is again an imaginary surface, which together form the boundary of the control volume. So, if you think of a copper block a rectangular copper block, then the 6 surfaces these and the 2 up and the top and the bottom the 6 surfaces of the copper block those are known as the control surfaces, which define the control volume, which defines what is the volume, what is the mass of copper, which is contained in the space defined by those 6 surfaces.

So, they are known as control surfaces by definition control surfaces do not have any mass of their own whereas, control volumes do. So, the major distinction between control volume and control surface is control surfaces do not have any mass ok. So, that is the difference between control surface and control mass. So, the conservation equation, which we are which we did right for the control volume has to be modified for control surfaces as they do not have any mass of their own. If they do not have any mass of their own, then they cannot store any energy and if there is no mass of the control surface then they also cannot generate any energy.

So, for a control surface the in the conservation equation, you need to drop the E dot g term the rate time rate of change of time rate of energy generated in the system, as well as the e dot stored that is the time rate of change of energy stored in the control volume. So, for a control surface the conservation equation would simply turn out to be E dot in is equal to E dot out. So, this is what you are going to get for a control surfaces. So, this is for a control surface and what you have here is for a control volume. So, that is the difference between a control volume and the control surface formulation as well as the conservation of energy is concerned.

So, this is primarily as a control surface as a C S does not have any mass of their own and they simply define a control volume. And these are the forms of the conservation equation. Now, if you look at this equation the conservation equation for a control volume there is what is going to be it is what is going to how what it would look like if I apply it for a steady state process. What if there is no generation which is present in such a system? It is a case of the blocking in where 2 surfaces are maintained at 2 different temperatures ok.

However, there is no generation of heat in there and you allow the process, allow the process sufficient time such that a steady state has reached. So, the moment has steady state is reached the amount of energy stored inside the control volume does not change cannot change ok. And, if there are if there is no heat which is generated inside the control volume then that term will also disappear. So, what you have for a system at steady state. So, for if you have a system at steady state with no heat generation, this equation would simply be equal to E dot in is E dot out.

So, when you when you when you look at these 2 equations the in absence of any generation heat and at steady state that is very important at steady state, this becomes equal in this becomes identical with the control surface equation.

However, fundamentally conceptually you must appreciate what is the difference between a control surface and a control volume ok. The next what am going to write is going to do is surface energy balance.

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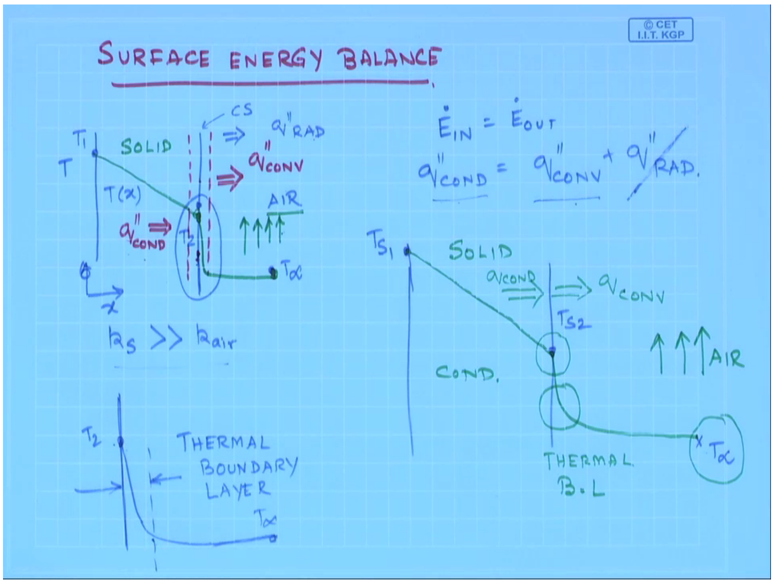

So, let us say that let us assume that I have a block this is my x direction and the over here I have the temperature this is the temperature axis and this edge I am taking as my control surface. So, the temperature is T 1 here and the temperature is T 2 at this point and over here it is exposed to an ambient air ambiance with the temperature equal to T infinity at this point.

So, when you so, between this is a solid and here you have as air. So, in the solid the principle mode of heat transfer in since there is there cannot be any motion of the molecules. So, in a solid the heat gets transferred from 1 point to the other by means of conduction only ok, as the as the molecules cannot on an average cannot move from their positions. So, in absence of any heat generation in a system and we will see why in a in subsequently, the in absence of heat generation and at steady state the temperature difference is temperature distribution is going to be linear ok.

So, the temperature distribution in such a case will be linear, and therefore this is going to be the difference in this is going to be the profile where this t is a function of x, but it is a linear function of x. Now let us say that this surface I am this is my control surface. So, the red line is essentially denotes simply denotes the x 10 denotes that this is my this is this is my control surface ok.

So, you are going to have q double prime conduction which is moving towards the control surface and over here from the control surface the heat is going towards air by q double prime convection. So, you have heat gets transported in the solid by conduction heat gets transported in the air by convection and this is a reasonable approximations since we realize that thermal conductivity of the solid is much more than thermal conductivity of air.

So, the principle mode of heat transfer on the air side of this solid interface is nothing, but convection. So, you can also this you can also assume that the air is moving with certain velocity along the sides along the sides of the along the exposed side of the block. Not only con convection you may also have if the temperature of the solid is large radiation which is going on ok.

So, this surface the control surface can lose heat as a result of convection as well as conduction as convection as well as radiation. So, the heat that travels to the control surface is; obviously, only by conduction and the heat which is lost from the surface is convection and radiation. And, since we know that the control surface has no mass of it is own the conservation equation would simply be E dot in is equal to E dot out.

And when you take this as the surface then q double prime conduction is simply going to be equal to q double prime convection plus q double prime radiation. So, most of the cases the radiation becomes important only when the temperature is quite large, in many real situations realistic situations the radiation does not play a significant role as compared to conduction and convection.

So, when that happens we can also drop this radiation in here since it may not be that significant. So, we have conduction equality of conduction and convection across a control surface 1 side of which faces the solid the other side is a flowing fluid of low thermal conductivity because this is a prerequisite low thermal conductivity and therefore, convection is the prevalent mode of heat transfer and your q conduction is equal to q convection.

But, how does how does the temperature profile go from here to T infinity. And therein lies something which I will have to explain to you later you will see more in greater detail that there exists something called something, which is called as a boundary layer or a thermal boundary layer. What is proposed is that all the temperature difference from T 2 to T infinity this drop in temperature from T 2 to t infinity takes place over a region, which is very close which is very close to the solid surface and then it asymptotically approaches the value of the temperature T infinity.

So, if I enlarge this section it would look something like this. So, this is my control surface this is T 2 and the temperature profile changes drastically now this is my T infinity. So, temperature profile changes drastically over a region, which is very thin and this region where the temperature changes from that of the base temperature to the temperature of the free stream to the temperature of the air the free flowing air which is T infinity this is known as the thermal boundary layer.

So, in a thermal boundary layer as a I have shown you the temperature drops suddenly the temperature changes suddenly till it reaches the temperature of the temperature of the free stream. We will discuss more about why in what is the significance of thermal boundary layer is there any experimental proof that they are there is something called a thermal boundary layer.

So, we would discuss that in the in subsequently, but what you have to accept now for the time being is that for any convection process, the temperature change is sharply near the solid fluid interface. And it reduces or increases depending on whatever be the case sharply enriches the value of the free stream temperature. And the thickness over which this change over takes place is commonly known as the thermal boundary layer. I think in your fluid mechanics you have you have already studied what is hydro dynamic boundary layer thickness.

So, if you have a flat plate and a liquid is flowing over it then due to no slip condition the velocity of the fluid on the solid liquid interface on the solid is going to be 0, that is the no slip condition, but if you go above slightly above the plate the velocity here is equal to the free stream velocity, which flows unperturbed over the solid surface if the distance from the solid surface is beyond certain value.

So, the region over which this change in velocity from 0 to the free stream velocity takes place is known as the hydrodynamic boundary layer. And you also probably know that the viscous forces are important inside the thin hydro dynamic boundary layer, outside of this boundary layer, the flow can be termed treated as in viscid or 0 viscosity. So, you have also probably have heard about 2 equations 1 is Navier stokes equation and the second is Euler's equation.

So, Navier stokes equation takes into account the viscosity the viscous forces, which are present in a flowing fluid, whereas Euler's equation assumes that the flow is in viscid and the famous Bernoulli's equation can be derived from Euler's equation and in it is true form the Bernoulli's equation is can be used only for in viscid fluids. So, inside the thermal inside the momentum boundary layer, inside the hydrodynamic boundary layer, viscous forces are present, outside of the boundary layer the viscous forces are unimportant are not present.

So, this is that is the concept of hydrodynamic boundary layer. So, which I think you know. So, based on same concepts the concept of thermal boundary layer is also prescribed, where the temperature is going to be the same as that of the solid, when we think about the fluid very close to the solid surface, but as we move along the temperature changes sharply. And over a small thickness it varies from TS the temperature of the solid to T infinity, which is the temperature of the fluid, which is flowing fast to the solid surface.

So, that is the thermal boundary layer, the important point here is what I need to show you is what would the profile the temperature profile look like for such a case. So, this is my solid and this is the temperature of the air and the side this side is maintained at a high temperature T S 1 and at a temperature T S 2 over here as there is no heat generation an it is a steady state case, it is going to be just a linear change from T S 1 to T S 2 inside the solid where only conduction is present, outside of which the air or the air is flowing with some velocity and a temperature constant temperature of T infinity which is lower than T S 2.

So, the temperature change will be very sharp near the solid wall. So, this is what is called as the thermal boundary layer and the complete profile would look like something like this. So, what you have here then is linear profile sharp change asymptotically approaching the value of T infinity. And, if you consider this point the flow due to conduction must be equal to the flow due to convection of course, assuming that there is there is no radiation is not important in such a case. So, what is the methodology that one has to follow in order to, so what is the conservation methodology analysis of application of conservation laws?

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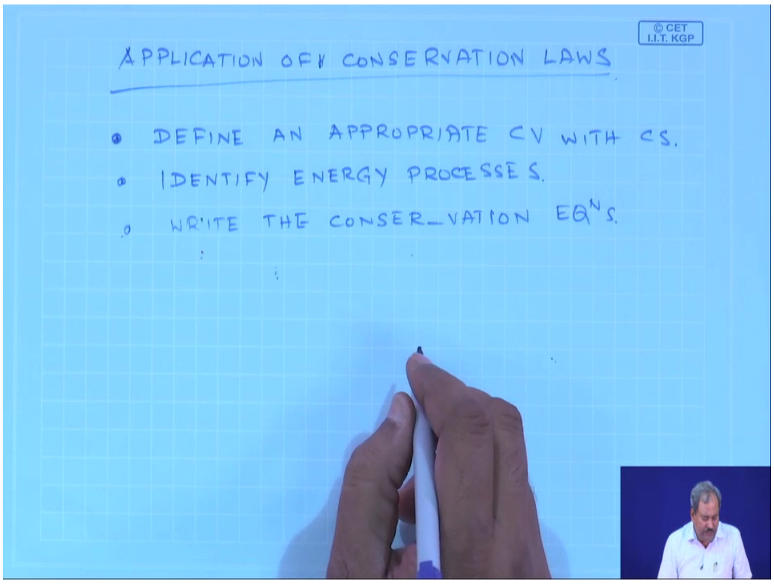

So, how do we apply the conservation laws? The first step is you define an appropriate control volume ok. Appropriate control volume with control surfaces ok. Then the second is identify the energy processes and write the conservation equation.

So, these are the 3 steps which one should follow in order to use in order to apply the conservation laws. So, what you do then is first you identify the control volume and the control volume is defined in terms of control surfaces, you identify the energy processes is it a conduction is it convection is it both conduction and convection, whether radiation is important or not you make those judgements and then you write the conservations equations.

And we so, far we know only about what to write for conduction? We are not sure what to write for convection or for radiation, since we have not discussed that, but I will show you what would be what would be the form of the rate equations for that. So, identify whether it is conduction convection or radiation and express the heat flow in utilizing certain laws for example, Fourier's law for the case of heat conduction.

And, then you are what you are going to get is an governing equation for the process and I will show I will discuss how this governing equation for any system can be derived can be can be developed. One more point before we conclude is that the conservation equations is valid for at differential control volumes, as well as for an integral control volume.

So, we can we can assume that my control volume it is size is d x d y and d z. And then if we write it appropriately what you are going to get is a differential equation. And, if you can solve that differential equation based on a energy conservation, if you can solve that the differential equation you get temperature as a function of space coordinates, that is x y and z and if it is an unsteady state process also as a function of time.

So, your temperature T as a function of x y z and time T is going to be the result of the differential equation, which you have obtained for a differential control volume and the approach by which the temperature that every point and at every instant, if it is point if it is possible to obtain that the approach that gives you that is known as differential approach.

There is there is another approach, which does not care about how temperature varies at every point in the in the control volume or at every instant. In though in that approach we are satisfied will be satisfied with how does the system behave in an overall sense to obtain what would be the overall behavior of this system as it gains energy, loses energy, and when it comes towards steady state.

So, the conservation equations can also be also be applied to an integral volume, which unlike the differential approach has a finite length, height, and width. So, you have L 1 length width and height as well as the time as well as it is a function of time. So, the approach which will give you an overall approach, which is which will give you an overall value of the temperature at a plane not at every point is known as the integral approach. When a times you are more interested in how does the system behave generally rather than what is going to happen to each point in the system.

So, on one hand you have a detailed approach, which is a differential approach that we would see also you have an approach which can also be termed as an engineering approach, which is a control volume approach, where you are not interested in obtaining the specific value and average value would do ok.

So, you do not want to know how temperature at every x varies, it just want to know what is the temperature of this plane the average temperature of this planer. So, the approach that gives you that is known as the control volume approach, if the control the control volume approach integral approach and you also have the differential approach, and we would see what those are so, but the bottom line is using your conservation equations somehow you have to write the governing equation of the governing equation for the system.

Once you have the governing equation you should be able to solve it utilizing some of the boundary conditions, because you know when you are integrating when you are solving a differential equation you end up with integration constants. These integration constants will have to be evaluated utilizing the physical description of the system. So, what is going to be those physical descriptions of the systems that will act as boundary conditions?

So, something has to be known for to convert the physical principles into boundary conditions. One example could be. And I will give you all the detailed examples in the next class one example could be that the temperature at a specific point in your control volume is known to you.

So, let us talk about the you have this pen whose you would like to find out what is the what is the how do you express the heat transfer taking place in this pen, but let us say that this edge of the pen is firmly in contact with a solid surface which is maintained at a constant temperature of a 100 degree.

So, what you can say then is at when x equals 0 the temperature is known let us say 100 degree temperature is known. So, at a special coordinates that the specific value of temperature may be known to you and that can act as boundary can act as a boundary condition.

So, expressing the description of the problem expressing the understanding of your problem in physical terms would give you boundary conditions, which you are going to which you need to solve the governing equation. So, what we would do in the next class is first identify the common boundary conditions that one would expect and then what is going to be the form of the boundary condition considering only conduction because in their initial part we will restrict ourselves to conduction.

So, we will write what is the differential equation? What is the governing equation the differential governing equation for conduction in a solid and what are the possible boundary conditions? So, with the help of the equation and the boundary conditions we should be able to obtain temperature profiles in different geometries under different conditions and they that would give rise to some very interesting and useful results which we will take up in the next class.