Heat Transfer
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Lecture - 19
Tutorial on the Application of energy Equation

From the derivation of energy equation, I think we are in a position right now to solve for some standard typical cases, where we may have conviction may not or may have only conduction , but before we move into the formal solution of convective heat transfer situations there just one or 2 interesting problems that I would like to draw your attention too.

So, this part of the class is going to be tutorial, where I would solve a problem and also give you a problem to do it on your own. So, the problem that we are dealing with is a very interesting one.

Let us consider 2 cylinders which are concentrate to each other. So, we have one cylinder and another cylinder enclosed in between and the gap between the 2 cylinders is very very small ok. And, it is you get the in practical situations, whenever you have a bearing where one of the one of the surface is moving with a high speed in the other surface is stationary.

So, in order to reduce friction whenever there is a high speed in between high diluted speed in between 2 surfaces one moving and one stationary, in order to reduce friction we use lubricants. Most of these lubricants are oils which have high viscosity and they act in a way so, as to reduce or minimize the wire and tear of the moving system.

Now, as I have mentioned the gap between the moving part and the immovable part is generally very small. So, have so, have very little play in the moving part. So, it is it is moving smoothly continuously and at the same time without having direct contact with the with the stationary surface. So, that is a thin film of lubricant in between.

Now, when you think of this situation it is an ideal case for viscous heat generation, because we have as moving part at a high velocity a stationary part and a very thin gap in between. So, the velocity changes from the velocity of the moving part 2 0 over distance which is very small.

So, if you think of the gradient of velocity it is re of the moving part minus 0, which is that of the velocity of the stationery part divided by the distance between 2 which is very small. So, V the numerator is large the denominator is small. So, therefore, this ratio is going to have a large value. And, remember that we have seen and I have told you that the viscous heat generation is generally gradient square multiplied by viscosity.

So, a large number v by d where d is the separation between the 2 a large number is squared and multiplied with density, multiplied with viscosity, and the viscosity of the lubricating oil oil is generally high. So, this is potentially and ideal candidate for viscous heat generation. So, fine we have viscous heat generation, but what is; what are the implications?.

Let us say the surface which is moving has some temperature t t b the base temperature and the surface which is stationary has another temperature. So, whenever you choosing an a lubricant for this specific purpose, you have to make sure that the temperature of the of the of the entire system is not going to exceed the rated temperature for a specific lubricant.

Every lubricant has a maximum temperature that it can withstand. If you go beyond that temperature, the ingredients of the lubricating oil they start disintegrating dissociating. Therefore, the thereby reducing the efficiency of the lubricant, you loose your lubrication property and that is a disaster when you have a high speed rotating system in contact with a stationary system through a lubricant, which is which is failing which is which is which is not working anymore.

So, every lubricant has to be rated for the of a maximum operating temperature and then you choose the lubricant based on whatever be the temperature of the inner cylinder, because let us say the inner cylinder is that higher temperature, the outer is at lower temperature, but is that correct are you correct in assuming that the maximum temperature of the lubricant in between these 2 parts is going to be equal to the temperature of the surface, which has a higher temperature.

So, we are going to test we are going to analyze the situation to find out are we correct in assuming that the maximum the temperature is going to be the temperature hot surface. And therefore, if you know the temperature of the hot surface I can choose lubricant that can withstand that temperature.

But so, you would see whether we are correct or not and if not what is the remedial measure, what how do you how do we calculate and decide which lubricant to choose?.

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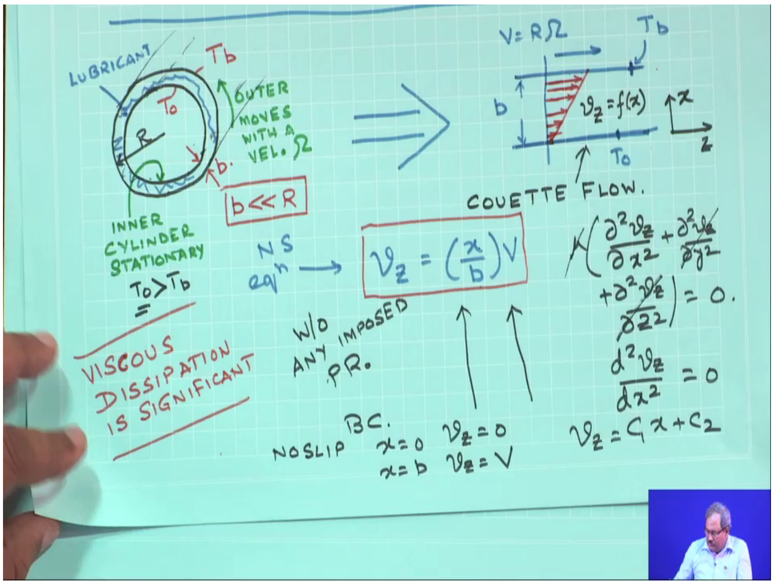

So, this system look something like this is a drawn over here, these are cylindrical systems. So, these 2 are essentially concentric cylinders ok. One inside the other and you are going to have the inner cylinder which is stationary the separation between the 2 cylinders is b, at the radius of the cylinder is equal to R and we are what we are saying is that this b the separation between the 2 cylinders is very small as compared to the radius.

So, this is what we have and the intervening space is filled with the lubricants. So, we have lubricant in here. So, this is the lubricant. The temperature of the inner cylinder the temperature of the inner cylinder is T 0 and the temperature of the outer cylinder is T b. And, we will assume that T 0 is greater than T b.

What, we need to find out is what is the temperature distribution that we have in the lubricant? I am I safe to assume that the maximum temperature of the lubricant is going to be T 0 and of course, the minimum is going T b. And therefore, my selection of lubricant should be based on T 0.

So, this is a system that I would like to modeled and I would like to analyze the first thing that one can do which you probably have studied in your heat in your fluid mechanics, that if the gap between 2 systems in rectangular in cylindrical coordinates is very small, then you can convert the cylindrical system into a Cartesian coordinate system without introducing significant or any appreciable error.

So, I have 2 cylinders like this ok. This inner one is rotating the outer one remains stationary and the gap is so, small in between the 2, this system can be analyzed without any loss of accuracy by opening them up and making them as 2 flat plates. So, the cylindrical system can be converted to a flat plate system where one of the plate is moving, the other one is stationary.

We can only do that when the gap is very small compared to the radius or in other words the curvature is going to be sm[all]- curvature is going to be small, and for large R and small gap this is a perfectly valid assumptions of converting a cylindrical coordinate system to a rectangular coordinate system, without any loss of accuracy. This helps us in whenever we have a Cartesian coordinate system we would always prefer a Cartesian coordinate system.

Because, the resulting equations are easiest to handled that is the only advantage, you can also do this problem by keeping this system as it is and choosing the equation for related to choosing the equation form of the form of the cylindrical coordinates form of the equation of energy, but since b is very small compared to R, I can convert this system from cylindrical to a rectilinear coordinate type of systems.

So, this looks like the system now then looks like it is 2 parallel plates ok, where the separation between the plates as in the problem is equal to b and the let us say this is the one which is the top plate. So, this has a temperature as in the figure this as a temperature equal to T b and it is moving with a velocity, which must be equal to R times omega where omega is the angular velocity. So, this linear velocity here is simply V is equal to R omega, and the temperature over here is T 0 temperature over here is T b.

So, you would expect this T b is more than T o. So, you would like to see if we have the maximum over here or that is not the case. Now, since the top plate is moving and the bottom plate is stationary, in your fluid mechanics you have probably read this to be as known as the Couette flow. Couette flow is a one in which you do not have any pressure gradient imposed on it, the top plate is moving and therefore, it is going to drag the liquid below and this layer of liquid is going to drag the layer below it and so on.

So, the motion of the top plate is being transmitted through the liquid due to the due to the presence of due to viscosity in it. So, if you can draw this it is been layers are going to move like this as move away from this as move away from the moving plate, the velocity progressively decreases and due to no slip condition the velocity is going to be 0 over here.

So, the velocity profile for a system like this looks something. So, this velocity let us say this is my Z direction and this is my x direction. So, the velocity is going to be only in the Z direction only in the Z direction and it is going to be a function of x alone.

So, the motion of the top plate is going to induce velocities at different layers of the lubricant present in between the moving part and the stationery part, and it is going to be linear distribution of velocity and the non-zero component of velocity is going to be in one dimensional flow. So, v is Z in the Z direction is going to be a function of x and is going to be linear function of x.

This type of flow is known as couette flow, which you would see and starting with the an equation of motion you would be able to obtain this velocity in the Z direction to be equal to x by b multiplied by v. So, this is from the navier stokes equation.

So, the navier stokes equation in the absence of any pressure and these are horizontal. So, the effect of effect of a gravity is not present, it simply going to be del square v z by del x square plus del square v z, by del y square plus del square v z by del z square multiplied by mu is going to be equal to 0 this is a navier stokes equation, I am I am I am sure that you have you are aware of it.

And, since v z is a function of x alone. So, this term is 0 this is 0 I cancel mu. So, what you have than is d 2 v z by d x square to be equal to 0, then this is than going to give as v z to be equal to C 1 x plus C 2 and the C 1 x and C 2 are the boundary are the integration constants and the boundary conditions are at x equals 0. That means, on the plate the v z is 0 which is known which is the no slip condition and at x equals b; that means, on this plate v z is going to be equal to capital V

So, these are the 2 boundary conditions which you would be able to solve for C 1 and C 2 it is a linear distribution and this would give raise to the for this form the linear form of the variation of the non zero component of velocity which is v z, as x by b times V. So, the this is couette flow and this is without any imposed pressure.

So, if there is an imposed pressure present in the system, then this is this straight line behavior is no longer going to be valid, and you will have a different type of different type of velocity profile I think I will not discuss that you have probably studied that in your fluid mechanics.

So, this is the relation, which is going to be useful to us when we solve for the energy equation. So, in this entire thing what we have done is we have converted the lubricant in lubricant ball lubricant bearing system, from cylindrical to cartesian coordinate top plate moving bottom plate stationary and we identify that the velocity is going to be like this.

So, let us see what is going to be the energy equation for this case. The energy and you can also see that the motion is going to be in this direction, and the energy is going to be transported in a different direction. And, this is a case where viscous dissipation is significant.

So, if viscous dissipation is significant in here then the viscous dissipation form of the equation of energy must be used for a rectangular system. So, what is that I will used this, this one once again and if you look carefully over here.

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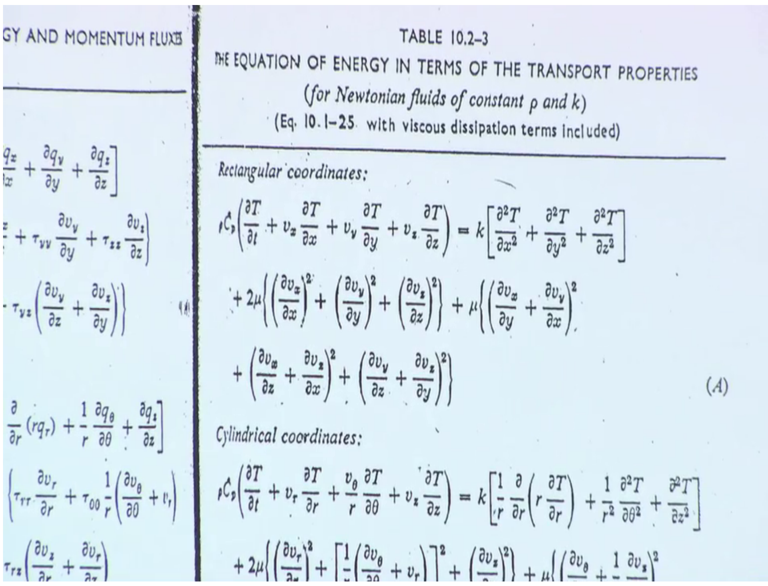

TABLE 10.2-3

THE EQUATION OF ENERGY IN TERMS OF THE TRANSPORT PROPERTIES

(*for Newtonian fluids of constant ρ and k*)

(Eq. 10.1-25 with viscous dissipation terms included)

Rectangular coordinates:

$$\rho \hat{C}_v\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + 2\mu\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \mu\left\{\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2 + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2\right\} \quad (A)$$

Cylindrical coordinates:

$$\rho \hat{C}_v\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right]$$

First of all it is a steady state system. So, del t by del temperature time this is going to be equal to 0. So, this term is going to be 0 this contains v x del T del x. Once again if you look at this is x and this is z. So, only have v z I do not have a v x. So, in this case v x and v y all are equal to 0. So, if that is the case then in this equation this v x term would be 0 v y term would be 0 and you have v z times del T del z.

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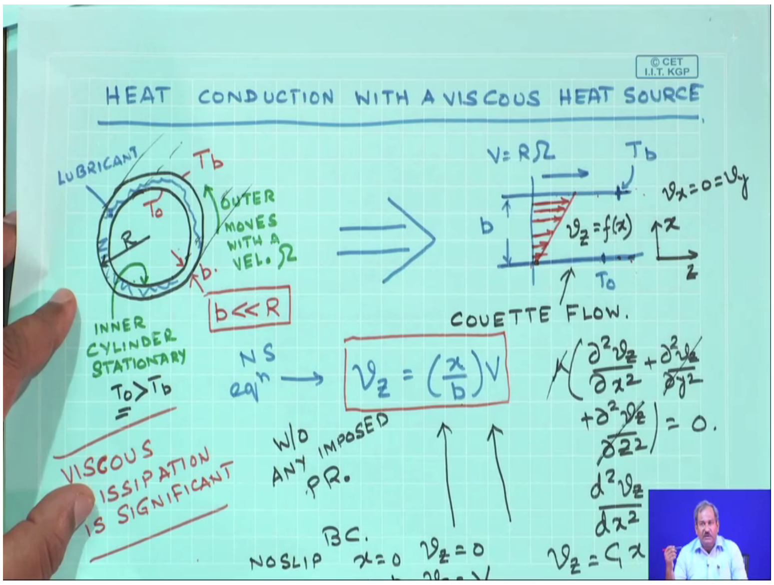

So, what I can write than from the left hand side of the equation is that rho C P v z del T del Z would remain on the left hand side. Let us see what I have on the right hand what will remain on the right hand side. The right hand side I have k del 2 T del x square k del 2 T del x square, if you look at this temperature is a function definitely of x write this is my x direction.

So, the first term would remain in the expression del T del y square temperature is not a function of y and del T del z square temperature is not a function of z as well. So, the only remaining term over here is going to be del T by del x’s square, then comes the viscous dissipation terms.

So, what are the viscous dissipation terms? That that you that you would see, the viscous dissipation is del v x del x velocity is not a function of the there is no x component of velocity, there is no y component of velocity, there is z component of velocity del v z, but that is not a function of z.

So, this is del v z del z whole square. So, since there is no variation of v z with z. So, this term would also be equal to 0. I have in this del v del v x del y plus del v x del z all the both of these terms are equal to 0. In here I have del v x del z which is 0 since v x is 0 I have del v z del x

Now, if you look at this figure once again I have a v z and this v z is a function of x. So, del v z z del x term must be present in this term I cannot neglect. Here I have del v y del z del v z del y both are equal to 0. So, what I than have on this side from my energy equation is del v z by del x whole square. So, the entire complicated looking equation the energy equation in rectangular systems, they resolved that they can be broken down to of this.

Now, once again I take your I will draw your attention to this where, the temperature is a function of x temperature is not a function of z this is my z direction. So, temperature is not a function of z temperature is a function of x. So, if temperature is a function of x then and not of z. So, T is not a function of z.

So, therefore, this term can also be set equal to 0. And, now I realize the temperature is a function only of x v z is a function only of x. So, therefore, the entire equation then can be written as k d 2 T d x square is equal to minus of mu d v z by d x whole square. And a so, this is the governing equation for the system that we are analyzing, what is d v z d x? My v z is this. So, my d v z d x is simply v by b.

So, therefore, since v z is equal to x by b times V, this would give me d v z by d x to be equal to x by sorry V by b. So, therefore, my equation becomes further simplified minus mu times V by b whole square or I can drop the k from this side and bring it over here.

So, you integrated to this is simple o d you integrate it twice and you are going to get minus mu by k V by b whole square times x square by 2 minus C 1, by k x plus C 2. These to C 1 and C 2 are the constants of integration in order to evaluate this you need to boundary conditions.

So, what are the boundary conditions you can see that the temperature at z equal to 0 temperature at x equal to 0 is t 0, temperature at x equals to b is T b. So, the boundary conditions are equal to 0 T is equal to T 0 and at x equals to b, T is equals to T b. Use these 2, use these 2 boundary conditions to evaluate C 1 and C 2.

And, what you should get and I think you should do it on your own to check that you are getting the correct form, what you would get is this form of the temperature distribution.

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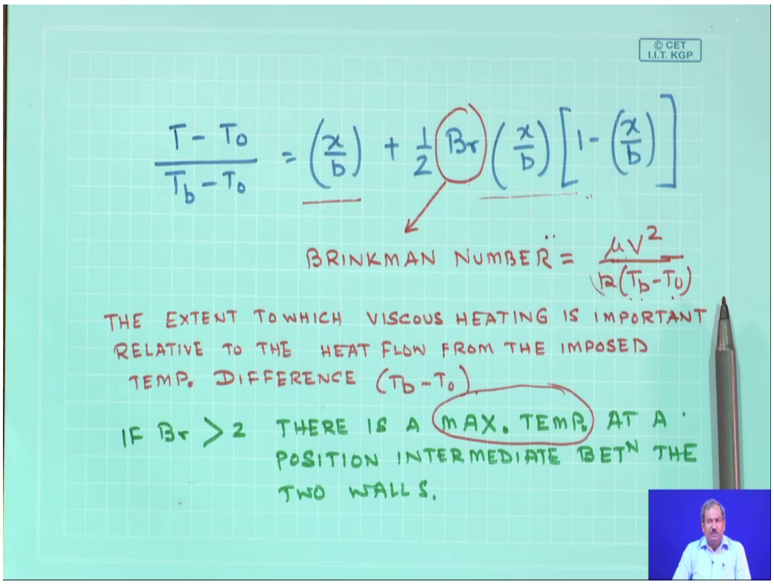

T minus T 0 by T b minus T 0 is equal to x by b plus half of B r and I will tell what B r is what B r is x by b times one minus x by b, please do it on your own and see that you are getting this form it is extremely important to practice these.

So, that you do not make any silly mistakes. We have a straight forward equation to integration constants to boundary conditions. So, evaluate C 1 and C 2 in order to get in order to get the form of the expression which I have written over here. Now, this B r is very interesting, this B r is known as brinkman number. And, you would see when you do it on your own this brinkman number is nothing V square by k into T b minus T 0. Once you solve it you are going to get this as your number.

So, brinkman number essentially tells you the extent to which viscous heating is important relative to the heat flow from the imposed temperature difference, which in this case is simply equals to T b minus T 0.

So, it is clear that mu V square by k into T b minus T 0, you can divide this side by l square and the denominator by l square and from the definition of viscous heat you can see that that the top refers to the viscous heat which is generated, where as a bottom is the impose temperature difference.

So, they refer to the conductive flow of heat. Now, let us see if we get back to something which we would expect. Let us say that in this there is no viscous heat generation, there is absolutely no viscous heat regeneration only conductive heat flows from the high temperature from the high temperature to the low temperature. So, this a case in which 2 n temperatures are known it is steady state and there is no heat generation.

So, for a case where you have steady state no temperature difference and the 2 n temperatures are known, the temperature distribution will be linear that is what we have seen so, many times. That 2 n temperatures are known nothing no heat generation between. So, that is going to be linear distribution of temperature. So, how do I see if our analysis is correct as I mentioned you to you previously always look at the limiting form of the solution, look at the limiting form and see if it reverts to at form which is known to be correct to view.

So, how do I make sure that there is no viscous heat generation? Let us say that there is no velocity. So, if there is no velocity then this second term on the right hand side what you see you were here my the brinkman number let us put this to be equal to 0 V to be equal to 0. So, the entire second term will be 0 and what you get is a linear distribution of temperature in the medium.

So, this part is for linear distribution, which is caused by conduction only and the second one is due to viscous generation of heat. So, this brinkman number therefore, refers to viscous generation of heat in a system in a system where the thickness is very small, but even this is not interesting this is not there is something more interesting to it. What you can do it you can do it on your own to see that if brinkman number is greater than 2, there is a maximum maximum temperature at a position intermediate between the 2 walls between the 2 walls.

This is probably the most important part of the exercise, you have a temperature distribution find out what is the maximum days maximum temperature that you can get. So, find out what is d T d x set it equal to 0. And, see that for when you find out d T d x and set it equal to 0 from this expression you would see that brinkman number if it is greater than 2, then you are going to get a maximum temperature at a position in between these 2.

So, if you do not have any flow if you do not have any viscous dissipation the maximum temperature is going to be at T b, but if your brinkman number is greater than so, the profile in that case would look like this a linear distribution , but if you get a brinkman number greater than 2 than the profile may look like this. And therefore, the maximum temperature would lie at a point in between the top plate and the bottom plate. This is the interesting result, why is this interesting result, because you have chosen you lubricant based on T b the maximum temperature that you think is present in the system which is the temperature of one of the plates.

But, due to viscous heat generation there is a point where the temperature exceeds that of T b. So, your choice of lubricant should be detected by this temperature and not by this temperature. So, the intermediate position temperature if it exceeds the temperature of the upper plate, then it would be wrong for you to choose the lubricant based on T b, rather you have to see what the temperature would be from this expression and find out and specify the lubricant.

So, in any lubrication system, where there is viscous heat generation you must find out what is the value of the brinkman number. If the brinkman number is greater than 2, you must do further calculation to find out what you where is the maximum in what is the value of the maximum temperature for such a system

So, a simple analysis a simple problem can give you so, many ideas new ideas about how to treat conduction? How to how to use first of all how to use equation of energy? How to cancel the terms which are not relevant? See the importance of conduction; see the importance of conduction when there is flow present, but in a direction perpendicular to the heat flow.

Therefore, you are not considering conviction explicitly; however, the presence of the velocity in your equation or rather the presence of the velocity gradient in your energy equation, tells you that you need to probe further in order to obtain how would the shape of the temperature profile look like, more importantly are you going to get a maximum in the profile due to generation of viscous heat in between the top plate and the bottom plate.

And, the number the dimensionless number which tells you the importance of viscous heating relative to the temperature impose temperature gradient relative to the heat flow due to the impose temperature gradient, which is known as brinkman number that plays a critical role, and you must find out what is the value of brinkman number before you chose a lubricant. So, concepts from concepts to practical applications, this is one example which I think would be interesting to anyone who studying heat convective heat transfer.