Heat Transfer
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Lecture - 14
Heat Transfer from Extended Surface

So, today we are going to have our last formal class on conductive heat transfer, and we are going to end the study of conduction with a topic, which is the very common in everyday applications. We are going to talk about extended surfaces. So, what is an extended surface? An extended surface generally refers to a solid, where inside the solid the conductive heat transfer takes place and from the surface exposed surface of the solid, we are going to have convective heat transfer.

So, through the solid we have conduction and from the surfaces of the solid, exposed to a fluid environment we have convective heat transfer. Now one of the major examples of extended surface heat transfer is the case of fins. Now you probably have heard about this term before, what a fin generally refers to which is the thin solid surface through which is used for enhanced heat transfer. You would see those fins in various applications previously if you when you could look at the back of the refrigerator you would see those extended surfaces.

If you look at the radiator of a car or the radiator of ready or the engine block of a motorbike, you would see that there are protrusions of solid, which looks like thin plates and those thin plates provide the additional area available for convection. So, generally the heat transfer coefficient from solid to air is quite low. So, in order to; so we cannot change h without ex without expanding more energy; that means, making the fluid flow over the solid at a faster velocity. What we can do is we can enhance the area, increase the available area which is to be utilized for convective heat transfer.

So, q equals h a delta t, this a the area available for convection if it can be increased by putting certain fitments to the hot surface, which are to be used for additional heat transfer and they are collectively called as fins. Fins come in different shapes, sizes and in any case you have to justify the use of fins. Because if you have seen the radiator of a car their significant amount of fabrication cost involved in making the fin and also there is going to be some cost related to attachment of the fin to the hot surface. So, unless those fins meet certained fundamental criteria of additional or enhanced heat transfer, the use of a fin is never going to be prescribed.

So, one must know what are the benefits that one is going to get out of having the fin on the on the hot surface, what is its efficiency, what is its performance and based on these 2 parameters not only you decide whether to attach a fin, you also decide what kind of a shape, what are the design parameters of an ideal fin should be. So, we will in this class we will have a quick look of the fins, the governing equations, solution of the governing equation for some specific boundary conditions, which would give rise to an idea of what is the performance index of a fin and what is the efficiency of a fin. So, let us start with the very fundamental case of when a solid surface is experiencing convective heat transfer, and we would like to we would like to see the different shapes of fins which are commonly used in everyday applications.

So, let us first taken look at what a fin would look like, what an extended surface would look like.

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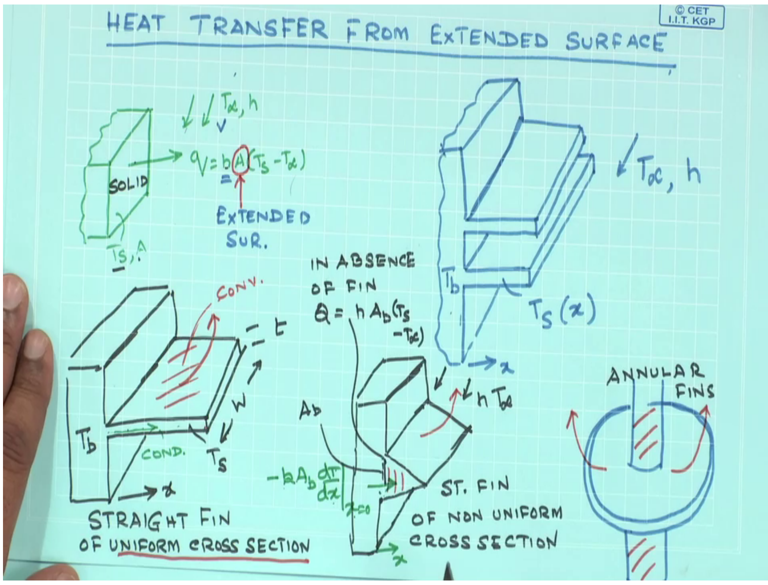

So, this is a solid, whose temperature is let us say at T S and we have some air which or some fluid which is flowing over it. So, this is a T S the temperature and the area at exposed for conductive for the convective heat transfer is A. So, the heat loss from the surface is simply going to be q is equal to h times a T S minus T infinity. So, as I said this h is more or less fixed, the moment you fix T infinity the moment you fix what is the velocity of the of the air which is moving what is the shape and size of the solid and so on.

So, I do not have any control on h, but I can enhance this area to increase the amount of heat transfer by the application of extended surface. So, these extended surfaces are commonly known as fins. So, you can have the fins generally look something like this, the shape of the fin with which we are mostly familiar with.

So, this is what the fins look like example of extended surface area. So, this is 2 fins which are connected to a base and you have flow of let us say here at T infinity which some velocity, which is going to give rise to a convective heat transfer coefficient of h and the temperature of the fin let us denote is denoted this as TS and if this is my x direction then my T S is going to be a function of x.

How far you are from the base and let us see the temperature of the base is T b. So, the problem that you that we deal with is to how to extract more heat from the surface using convective heat transfer. And also having these attachments the fins on top of the surface, which enhances the area available for convection, but making these fins fabricating these fins attaching it to the base obviously would involved some cost and we would look at what is the ideal situation, what is the basic requirement that you need to fulfill in order to prescribe the use of the fin.

So, there are several types of fins which are commonly you would see commonly. So, this is example of a straight fin, the one that I have drawn over here. So, this is an example of a fin and its width is let us say w, it has a thickness equal to t and this is the x direction. So, this is a straight fin of uniform cross section.

So, here you can see the cross sectional area of the fin is equal to w times t, which remains fixed and the temperature of the base is T b whereas, the temperature of the fin is let us call it is TS and this area is available for convection. The area at the top and the area at the bottom they are available for convection whereas, you are going to have conductive heat transfer conductive heat transfer through the fin. So, as the heat comes towards the tip of the fin, part of it is going to going to go out going to leave the surface by convection and so, there would be dimension amount of conductive heat, which is moving towards the tip of the fin.

So, it is the example of a straight fin with uniform cross section. On the other hand you would sometimes you also have fins which look like this, where the cross sectional area does not remain constant and you have fin whose cross sectional area is going to be different. So, if you look at this is what you would see is the cross sectional area of fin keeps on decreasing in the convection area, the area available for convection that is more or less fixed, but the area available for convection conduction keeps on decreasing.

So, it is an example of a straight fin of non uniform cross section this is another example of fin some of the fins which are commonly seen in applications. So, these are known as annular fins. So, these annular fins are connected to. So, this is the cylinder which is hot, whose temperature you would like to reduce. So, you attach an annular fin and this is the surface which is additional surface, which is available for convective heat transfer. So, what we see that the purpose of the fin is to provide additional surface area for convective heat transfer.

The fins can be of different shape, different size, its cross section which the area which is which is to be used for conduction may or may not remain constant. So, this kind of various variations in fin size and shape are possible. So, now, what we are going to do is we are going to find out we are going to drive a general formulation for the heat transfer simultaneous conduction and convection from the surface of a fin and using those that governing equation with appropriate boundary conditions, we would like to obtain what is the temperature profile inside the fin.

So, because if we if we can find out what is the temperature profile of the fin, then if I if I considered x at x equal 0 to be the base, where the fin is attached to the base. So, x equal to 0 refers to the point where the fin is attached to the base, from where you would like to extract the heat then minus k a suffix b, where A b is the is the area of the base at which the fin is attached times dt dx at x equal to 0 would give you the amount of heat that the fin extracts from the base.

So, if you look at this figure once again, if this area of the area of the fin over here; let us say this is the area is A b. So, the amount of heat which is taken up by the fin at steady state must be equal to k times area at the base times d T d x at x equal 0, where x starts from here.

So, minus k Ab which is the area of the fin at the base times dT dx at x equal to 0, gives you the amount of heat taken up by each of these fins ok. So, this would give you an idea this should give you an idea if you are making any additional benefit by having the fin. So, if you do not have the fin, then this area at the base would still be available for convective heat transfer. So, in that case the whole of A b would be exposed to whatever is flowing here with h and T infinity.

So, if I did not have the fin attached to this point, the amount of heat transfer in absence of fin in absence of fin the amount of heat transfer would simply be equals h times A b times T of surface minus T infinity ok. So, this is the amount of heat we should have been which I would have been taken up by the flowing air, if the fin is not there. The moment I attach a fin the amount of heat transfer must be equal to the conductive heat transfer at the base. So, its minus k Ab dT dx. So, if you look at these 2 quantities, this is when the fin is not present and this is when the fin is present. So, the performance the effectiveness all these definitions should have these 2 quantities either in the numerator or in the denominator ok.

So, what would be the effectiveness of a fin? If it is there how much of heat it is extracting as compared to if it is not there. So, these 2 quantities these 2 quantities over here would give you some idea about what is what is the performance, what is the performance of the fin.

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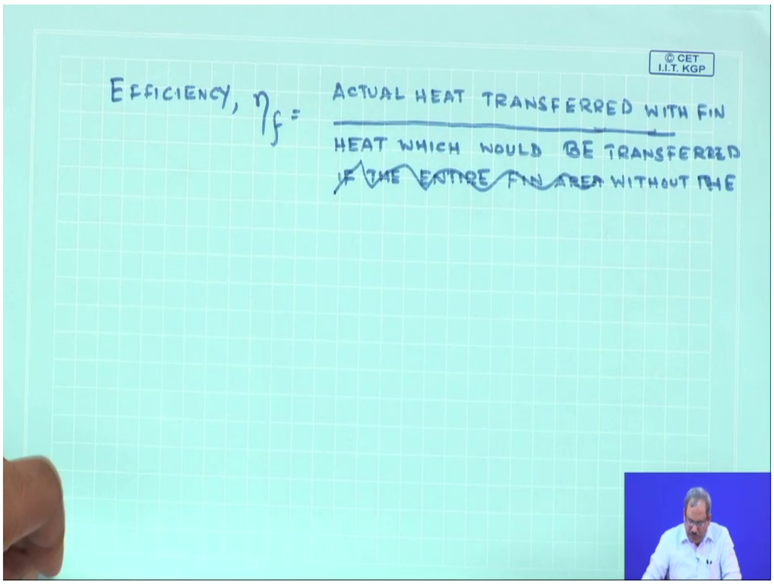

So, the efficiency of a fin; the efficiency which is which quality as eta times f is defined as the actual heat transferred divided by heat, which would be transferred if the entire fin area, if the actual heat transferred with fin, heat we should be transferred if the I would make it easy actual heat which would be transferred without the fin. I will since I have cut it so many times, I will write it once again. Just to make sure that the concept is clear to all of us.

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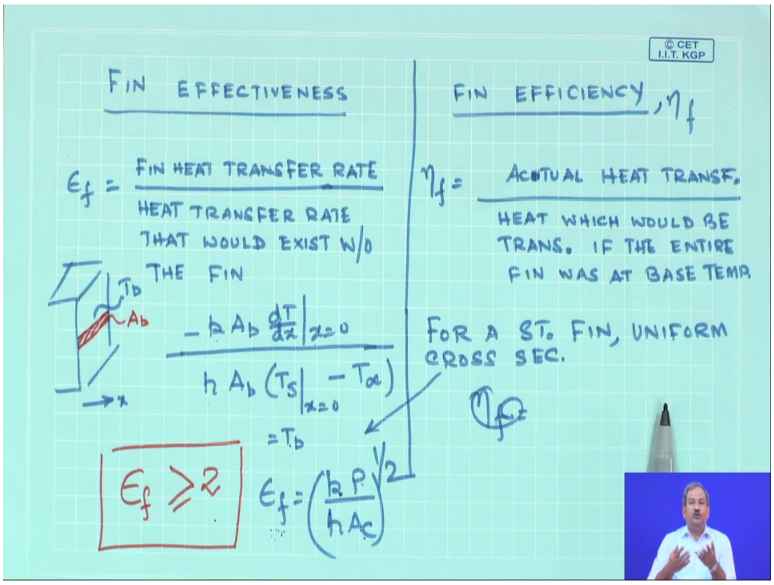

There are 2 concepts which are to be defined here. So, when I talk about effectiveness and efficiency, what exactly do I mean? In the case of effectiveness, I have to decide whether having the fin makes any sense should I have the fin or should not I have the fin. When I look at the fin efficiency, I am trying to find out how efficient is the fin. So, let us first talk about efficiency. When I have decided that yes I am going to use fin for a specific geometry for a specific operation of a specific application now, I am going to think when do you think the fin is going to be maximum effective.

Now the fin is the gradient which is responsible for a heat transfer from the fin to the atmosphere, is simply temperature of the surface of the fin at any location, which we denote as TS and temperature of the fluid which is a constant which we denote as T infinity.

So, the amount of heat transfer from the fin is dependent on T S minus T infinity. This temperature of the fin TS is a function of x, it is equal to T b; that means, temperature of the base and as I move if this is a fin and this is my base, this is at T b and as I move towards the tip of the fin, the temperatures gradually decreases ok. So, since the temperature gradually decreases. So, you start at 100 degree, in the air is blowing at 25. So, as you move towards the tip of the fin, the temperature gradient between the surface of the fin and the air we will continuously decrease.

Since it decreases, you are going to have lesser amount of heat transfer from the fin as you move in the direction of plastics. Now how do you make the process maximum efficient, how would you how would how would how would you achieve maximum efficiency in the heat transfer process? That should only be possible if by some means or if by the nature of the material of the fin that you have used, the entire fin is as is at the temperature of the base.

So, instead of temperature of the fin wearing at points decreasing as you move away from the base, if the situation is such that the entire fin is it T b, then the heat transfer would be T b minus T infinity, which would be a constant and you are going to get the maximum amount of heat transfer from a given fin. So, that is how the efficiency of the efficiency of the fin is defined which is actual heat transferred divided by the heat that would have been transferred, if the entire fin is at its base temperature.

So, I am going to write the definition of the efficiency as actual heat transferred divided by heat which would be transferred, if the entire fin was at base temperature. So, this would give you an idea of what is the efficiency. Because we always defined efficiency in such a way, it has a maximum value equal to 1 or 100 percent. So, the actual heat transferred where the temperature is a decreasing function of x divided by the heat which would have been transferred if the entire fin was at T b the best temperature that would be the logical choice to define the efficiency of a fin.

So, now let us think about what is fin performance and this would tell us to this would allow us to decide whether or not fin use of a fin is prescribed. So, the fin effectiveness, what epsilon f is defined as fin heat transfer rate over heat transfer rate that would exist without the fin. So, if you do not have the fin, this entire area is at T b and of this much is the area where the fin was attached to. So, let us call it as area at the base.

So, the heat trans the fin heat transfer rate would simply be minus k times A b, dT dx evaluated at x equal 0, where this is the point where your x e where this is the direction of the degree of x. Now the heat transfer rate that would exist without the fin, it would simply be equals h times A b by TS at x equal 0 which is obviously, equal to T b minus T infinity.

So, this is the definition of the fin effectiveness, where the fin heat transfer rate is given obviously, by the numerator and the heat transfer rate that should exist without the fin is given by this is simply h A b T b minus T infinity. Now the rule of thumb is use of fin is rarely for rarely prescribe if epsilon f is less than 2. So, one can one can only prescribed the use of fin if its effectiveness is greater than or equal to 2. So, if this condition is made, then you go for a go for attachment of a fin to the surface, in once you do that then you look at what is going to be the efficiency of the fin.

So, in the I will very quickly tell you what would be the form of the fin efficiency, the efficiency of the fin and then we will see how that would give us an idea of what is the what should be the shape of the fin. So, for a straight fin, straight fin uniform cross section the eta f would be equal to the performance of the fin eta f would this eta f would be equal to I am sorry this epsilon f would be equal to k P by h A C to the power half ok. So, this is the fin effectiveness. So, this is over here.

So, what you see here that is epsilon f the fin effectiveness is a function of thermal conductivity is a function of P by AC and it is also going to be a function of h. You would like to have the largest value of epsilon f possible. The larger is the value of epsilon f as compared to 2 the better is going to be its performance. So, what would be the nature of the fin material? If you look at the expression for epsilon f once again, the material of construction of the fin should have a high value of thermal conductivity; that means, k has to be large, fin the value of h should be small. The value of h small means the use of fins are only prescribed when we have heat transfer from a solid to a gas, which traditionally has low values of convective heat transfer coefficient.

So, the fin material should be should be of large k such that the entire base entire fin is going to be at a temperature close to that of T b not exactly at T b, but close to that of T b. So, the material of construction of the fin should be large, the fin is prescribed when you have heat transfer from the solid to the vapor to the gas. So, that is why you would see use of fins mostly for the case of air not for when a solid is in contact with a liquid which has the high value of heat transfer coefficient. And finally, P by A c should be large; that means, forever for the ratio of the perimeter to the cross sectional area should be large; that means you are going to have a large number of thin fins attached to the solid surface from where you would like to extract the heat. So, it gives us idea about the material of construction of the fin, the shape of the fin and the situation where fins are to be used.

So, together they would give you a complete picture of a when and why and how fins are to be used. As very quickly go through the governing equation the derivation of the governing equation and then we can move on to the next part.

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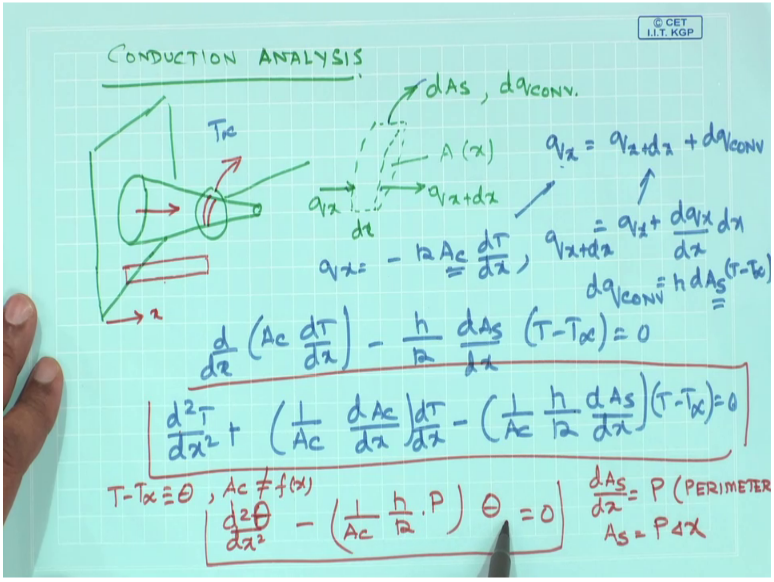

So, the governing equation this is a conduction analysis, where you have this being the fin, which is attached to a surface ok. So, this is the fin. So, this is my x direction and from here I have the conductive heat transfer and let us see this is the surface from where I have convective heat transfer. So, if I enlarge this, it is going to look something like this. So, this distance is d x, this area available for convective heat transfer is d A AS, which is going to lead to the convective flow of heat and this area this area would be a function of x as well this is a cross sectional area.

So, you have qx by convection coming into the control volume, q x plus d x which is the heat conductive heat going out of the control volume. So, by conservation I can write whatever heat is coming at this point, must be equal to whatever heat that is going out through this surface and whatever heat is going to be lost by the convection process. So, this is just a conservation equation, the heat coming in must be equal to the heat going out by conduction and heat going out by convection. And we also understand that my q x is equal to minus k AC, AX being the cross sectional area dT d x and q x plus dx can be written using Taylor series expansion as q x plus dqx by dx times dx. So, this is just the expansion of this. So, when you put this these into the equation, what you would get is and we also understand that dq convection must be equal to h d A S times T minus T infinity, where T infinity is the temperature of the fluid surrounding it.

So, this is AS and this is A C which we have to keep in mind. So, when you put this back in here, what I would what I am going to get is as my governing equation is as A C times dT d x minus h by k d AS by dx times T minus T infinity.

So, this becomes the governing equation and if I expand this, the final equation that I am going to get is. This is the final form of the equation that I am going to get. So, this would be the governing equation for heat transfer utilizing a fin. Now instead of this fin having a variable cross sectional area, if I assume that the fin is just like this a fin with constant cross section, then this term would disappear and what I am going to have is only the first term and the second term to be equal to 0.

So, if I define t minus T infinity to be equal to theta, and if I assume that A C is not a function of x, then the same governing equation can be written as d 2 T d x square A C is a constant. So, this term would disappear minus 1 by A C h by k times theta, and d AS dx is simply going to be the param the perimeter of the fin would be equal to 0. So, what I see is d AS by dx is P, whichever P is the perimeter of the fin. Because AS is simply P times delta x, P is the perimeter times delta x. So, this would be the reduced governing equation I call this as theta as well; so d 2 theta by d x square minus 1 by AC, h by k times P into theta to be equal to 0. So, for a fin of constant cross section, I will expand this equation bit more where you are going to get is that d 2 T dx square, minus h P by k AC times theta to be equal to 0.

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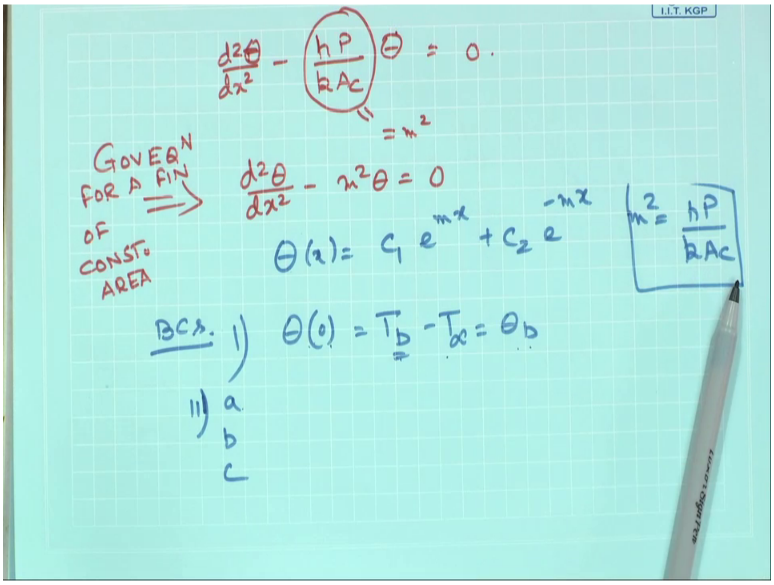

And if I assume this to be equal to m square, then its d 2 theta by d x square minus m square theta would be equal to 0.

So, this is going to be the governing equation for fin of constant area. And the solution of this equation solution of such kind of equation, these are homogeneous second order differential equation. So, your theta x would simply be equals C 1 e to the power mx plus C 2 e to the power minus mx where m square is h P by k AC so that is the standard solution which you can obtained. So, you need to boundary conditions for this, one is theta at 0 at x equal 0 is the temperature is known which is T b minus T infinity and let us call it as theta b. So, the first boundary condition is that the temperature at the base is known to me which is T b. So, T b minus T infinity I call it as theta b.

The second boundary condition can have different variations like A B and C. So, what would I get in as the second boundary condition can vary? Let us say I have a fin which is very long. So, what is going to happen if the fin is very long in this direction, then the temperature of the tip most likely is going to be very close to that of the air. So, for a very long fin, the boundary condition could be that as the x tends to infinity, temperature of the fin is equal to T infinity, that is one condition what which one can use ok.

If the fin is short what is thermal its thermal conductivity is very high, then what is going to happen is the entire fin is going to be at a constant temperature, which would be very close to that of the base. So, a shot fin of very high thermal conductivity, the boundary condition at x equal to l; that means, at the h is going to be T is equal to T b so, that is another possible boundary condition what that can happen.

The third type of boundary conditions that can happen is that, whatever heat comes to the tip by conduction must be convicted out through the cross sectional area, which is exposed to the ambient environment. So, what is the amount of heat which comes to the tip? It must be minus k AC times dT dx at x equal to l, that is the amount of heat which comes to the tip by conduction through the fin and if all of that at steady state.

Since it has to dissipate out of the cross sectional area by a convective process, the amount of heat loss by convection from the tip of the fin would be h times AC times T at the end which is T l minus T infinity. So, these different types of boundary conditions are possible in based on that the fin question has been solved, you can look at the textbook for the various forms of the equations, various forms of the temperature variation the temperature formed that form of the temperature for a fin of infinite length of short length, but very high thermal conductivity or for a fin whose tip is active. That means, the amount of heat coming to this to the fin and must be dissipated out.

So, these combinations the first condition is always fixed. That the temperature at the basis known it is that is at x equal to 0 the boundary condition is known, what happens at x equal to l? We have three or four different types of boundary conditions and for each of those boundary conditions, these solutions are available in your text. So, you should take a look at them, but you should also be very careful about the alternate conduction analysis and realize that the assumption of constant cross sectional area may not be valid in all cases. For example, in the tapered fin which I have drawn the area is not a area available for conduction is not a constant so, which could give rise to interesting problems. And the fin material should be of high thermal conductivity, the shape of the fin should be thin and closely spaced and you are going to use fins only when you heat transfer when you when you have heat transfer between the solid and the gas generally not between the solid and the liquid.

And there are 2 parameters which define whether or not a fin is to be used or what is the efficiency of the fin. The fin should can be used only when its performance is the index is greater than equal to 2, and it is going to be most effective when the entire fin is a at its base temperature.So, that is all I wanted to cover in this class.

I guess you have had some idea about and what is involved in heat transfer from an extended surface. There are vast literature and information available in many textbooks and many hand books about how to design fin, when to add fin, the fabrication techniques the way you attach the fins to a surface and so on. So, if you are interested you can take a look at them, but always remember the fins are useful way; is a the attachment of fins is a useful way to enhance heat transfer and this simply provide and extended surface. Then therefore extended surface heat transfer is an important topic in conduction heat transfer.