Optimization in Chemical Engineering

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Lecture - 08
Optimization Problem Formulation (Contd.)

Welcome to lecture 8 this is week 2, we will continue our discussion on Optimization Problem Formulations.

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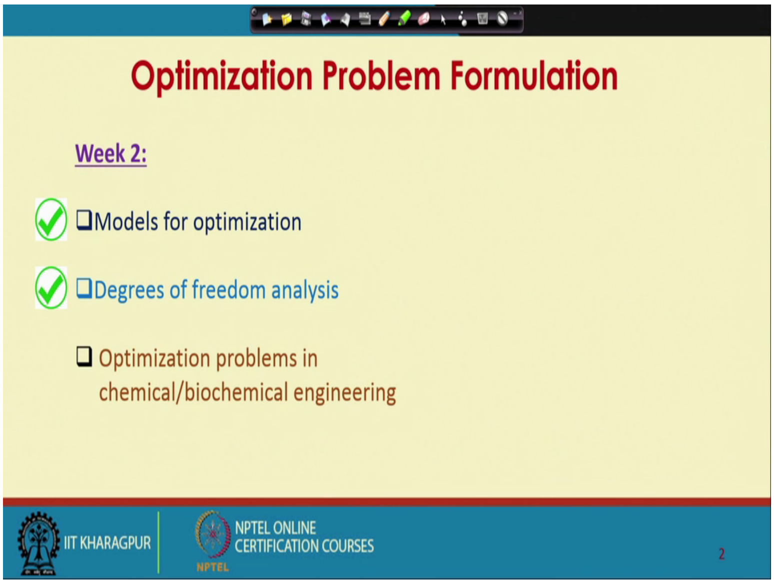

So, we have talked about models for optimization, we have also talked about degrees of freedom analysis. We have seen one or two problems of optimization and their formulations in this lecture also. We will continue our discussion on optimization problem formulation.

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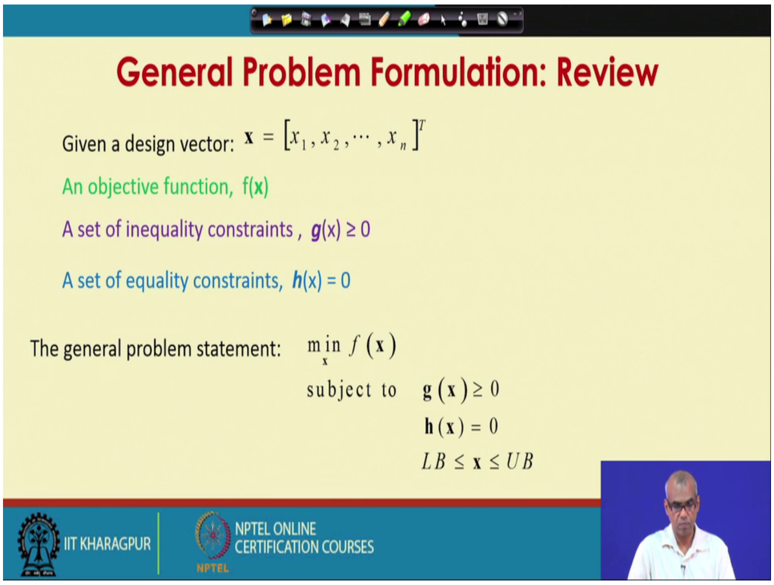

So, we have discussed this given a design vector x 1 x 2 up to x n given an objective function f X a set of inequality constant and set of equality constant the general optimization problem statement, is given as minimisation of objective function by finding out suitable values of design vector X. And the solution must respect, or satisfy set of inequality constant g X greater or equal to 0 set of equality constant h X equal to 0 and the bounds that exist on the decision variables X.

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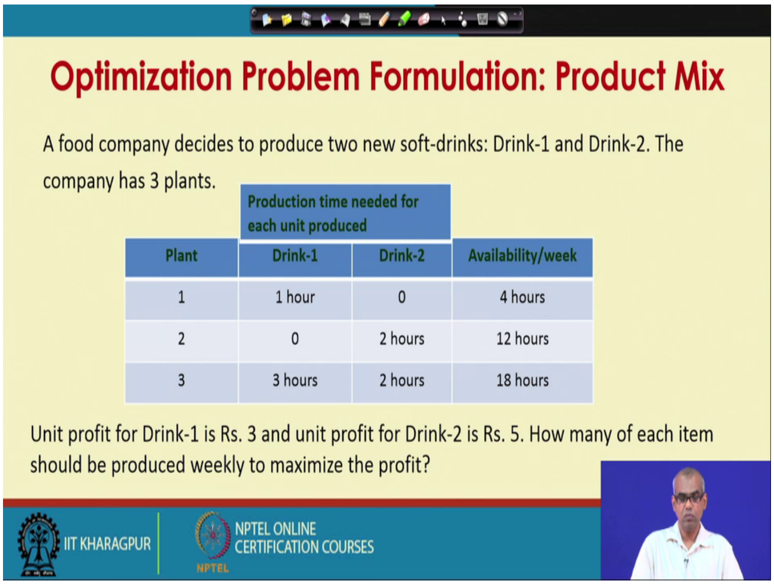

So, let us now talk about an optimization problem, this problem is known as product mix. A food company decides to produce two new soft drinks. Let us call them drink 1 and drink 2. The company has 3 plants, where they can produce this soft drinks. The data is shown on the screen for each plant, the production time needed for each unit of drink 1 and drink 2 are shown and, also a availability of the plant per week is also shown.

So, drink 1; 1 is produced in plant 1, the production time required for each unit is 1 hour in plant 1 the drink 2 is not produced, availability of plant 1 per week is 4 hours, in plant 2 drink 1 is not produced, drink 2 is produced and production time required for each unit produces 2 hours. The availability of plant 2 per week is 12 hours. In plant 3 both drink 1 and drink 2 are produced, in plant 3 for production time required for each unit produced for drink 1 is 3 hours and that for drink 2 is 2 hours, the availability of plant 3 per week is 18 hours.

Unit profit for drink 1 is rupees 3 and unique profit for drink 2 is rupees 5 let us say, unit profit means each unit of drink 1 will bring a profit of rupees 3 and each unit of drink 2 will bring a profit of rupees 5. The company wants to maximise the profit. So, they want to determine how many of each item should be produced weekly so, that the profit is maximised. So, you have to determine how many of drink 1 and how many of drink 2 should be produced weekly to maximise the profit. So, how do I formulate this optimization problem?

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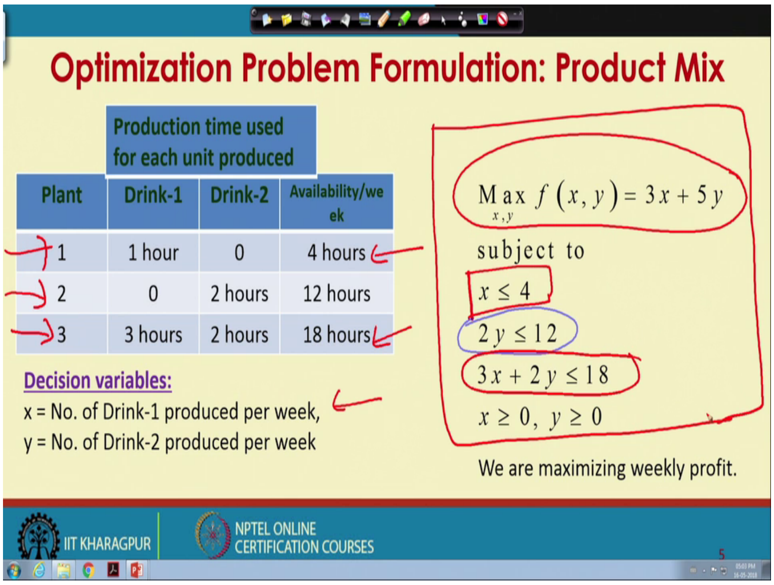

So, let us consider the decision variables as number of drink 1 produced per week and number of drink 2 produced per week. So, the choice of decision variables is natural, let us consider x equal to number of drink 1 produced per week and y equal to number of drink 2 produce per week.

So, what will be the objective function, objective function will be the profit. Now, each unit of drink 1 drinks you have profit of rupees 3, we have seen in the previous slide. And each unit of drink 2 drinks, you have profit of rupees 5. So, if x unit of drink 1 is produced the profit is 3 x 1 for drink 1 and, if y units of drink 2 are produced, then the profit for drink 2 is 5 y. So, the total profit is 3 x plus 5 y. So, would like to maximise the quantity 3 x plus 5 y so that can be considered as an expression for objective function. So, if I represent objective function let us say y z.

So, objective function is z equal to 3 x plus 5 1, as I would like to find out x and y such that z is maximised, where z equal to 3 x plus 5 y, but x and y must satisfy certain constraints what are the constraints here, if you look at the production schedule, the information that is given about the availability of the plants per week we can frame the constants.

So, let us look at the constraints. So, objective function is this as discussed. Let us come to plant 1 first, in the plant 1 only drink 1 is produced. And the production time for each unit is 1 hour x units produced. So, the production time will be x hour availability per week is 4 hours. Remember we are producing x units of drink 1 per week, which is going to take x hour per week and, the availability per week is 4 hours for the plant 1.

So, the constant that we can write first is x must be less or equal to 4 because, it cannot exceed 4 at most it can be 4. So, x is less or equal to 4 becomes the first constant. Now, let us come to plant 2, where only drink 2 is produced and each unit takes 2 hours, we are producing y units of drink 2. So, it takes 2 y hours availability per week is 12 hours. So, the constant becomes 2 y is less or equal to 12. So, that becomes the second constant

Now, let us come to 3rd plant, in plant 3 both drink 1 and drink 2 are produced drink 1 takes 3 hours per unit. So, for x units it takes 3 x hours drink 2 takes 2 hours per unit. So, for y unit of drink 2 it takes 2 y hours. So, total time is per week is 3 x plus 2 y so, that has to be less than 18 hours that is available for week, that gives me the third constant which is 3 x plus 2 y is less or equal to 18. Of course, we have non negativity constraints on x and y x and y represents number of drink 1 and number of drink 2 respectively. So, x has to be greater or equal to 0 y has to be greater or equal to 0. So, this is the problem formulation for the product mix problem.

Note that in this problem the objective function is linear, all the equations there are no equations, all the constraints which inequality constant they all are linear. It is basically an example of linear programming problem we learn how to solve such a linear programming problems, when we talked about linear programming problems details later in the course.

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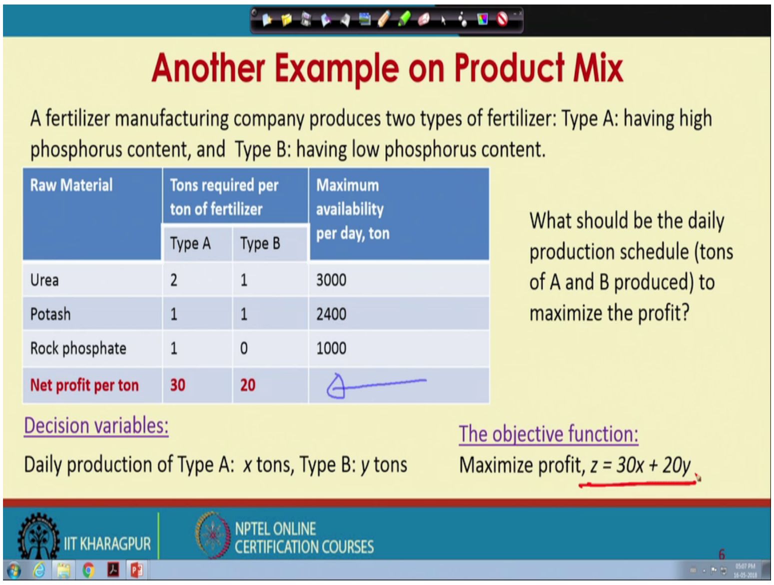

Let us take another example on product mix. A fertilizer manufacturing company produces two types of fertilizer, type A and type B, type B has high phosphorous content, type B has low phosphorous content. The raw materials used urea, potash and rock phosphate, number of tons required for producing per ton of fertilizer for type A and type B are given in these two columns. For example, to produce 1 ton of type A fertilizer, urea required is 2 tons, potash 1 ton, rock phosphate 1 ton. Similarly to produce 1 ton of type B fertilizer urea required is 1 ton, potash required is 1 ton and rock phosphate is not required to produce type B fertilizer.

The maximum availability of urea potash and rock phosphates are also shown here. The maximum availability per day for urea is 3000 ton potash is 2400 ton and rock phosphate is 1000 ton, what should be the daily production schedule. So, that the profit of the company is the maximum; that means how many tons of A and how many tons of B must be produced per day so, that the manufacturer can maximise the profit.

So, what should be the decision variables, the obvious choice is number of tons of type A fertilizer to be produced and, number of tons of type B fertilizer to be produced. So, let us say the decision variables are x and y, where x represents daily production of type a and y represents daily production of type B, fertilizer both are expressed in tons.

What will be the objective function, again let us look at the data. The net profit per ton is shown here. So, the net profit per ton of type A is 30 in some unit and, net profit per ton of type B is 20 again in the same unit. We have not explicitly said what unit is let us say some unit.

So, the objective function will be the total amount of profit that comes from type A and type B, we are producing x tons of type A. So, the profit from type A fertilizer will be 30 x similarly profit from type B fertilizer will be 20 y. So, the objective function will be z equal to 30 x plus 20 y. So, our objective function will be maximise profit z equal to 30 x plus 20 y. So, now, the constraints so, the constraints can be put by looking at the maximum availability per day because, that is the restriction that has been imposed in the given problem.

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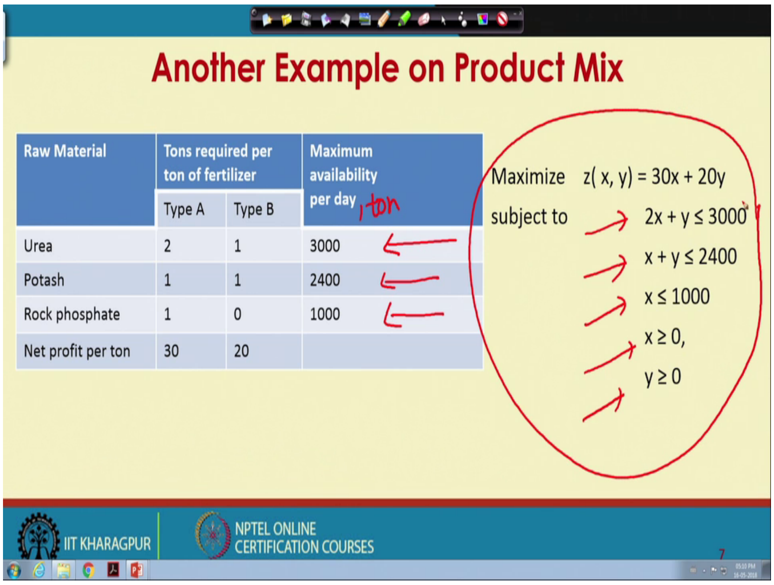

So, following the example that we discussed before we can now write down the constraints as follows: first let us look at urea the maximum availability per day for urea is 3000 tons, we are producing x ton of type A fertilizer y ton of type B fertilizer 1 ton of type a requires 2 tons of urea. So, x tons of type A will require 2 x tons of urea. And similarly y ton of urea will be required to produce y ton of type B fertilizer. So, total urea required will be 2 x plus y. So, that cannot exceed the maximum available quantity per day which is 3000 so, the first constant that you can write 2 x plus y is less than or equal to 3000.

Next come to potash 1 ton of type a requires 1 ton of potash. So, x ton of type a require x ton of potash. Similarly 1 ton of type B will require, 1 ton of potash. So, y ton of type B fertilizer will require y ton of potash. So, x plus y has to be less or equal to the maximum available quantity per day, which is 2400. So, x plus y is less or equal to 2400 is the next constant.

Now, let us come to rock phosphate, rock phosphate is used only in type a fertilizer and to produce 1 ton of type a fertilizer, 1 ton of rock phosphate is required. So, to produce x ton of type A fertilizer, x ton of rock phosphate is required and the constraint that you can write, then is x is less or equal to 1000 then of course, non negativity constraints on x and y. So, this is the formal problem formulation for the product mix problem on fertilizer manufacture.

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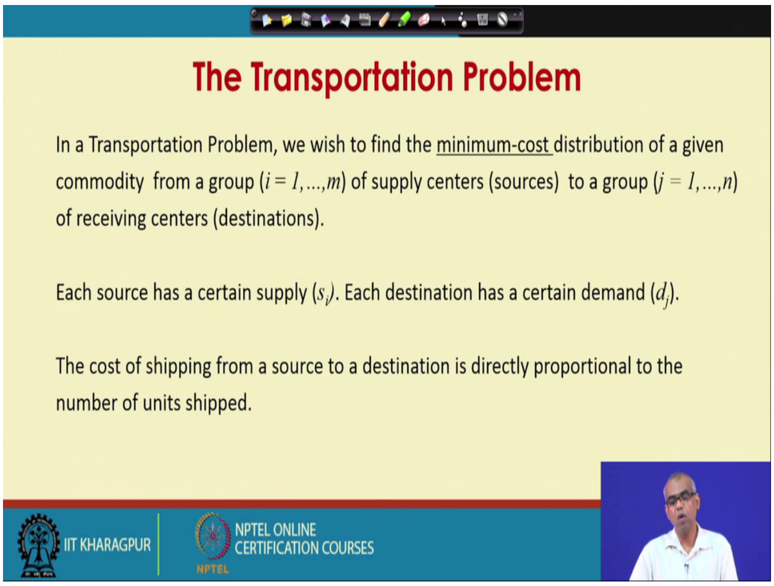

Next we will talk about another problem known as transportation problem. The first two problems we have seen, the objective functions a linear the constraints are also linear. So, both were linear programming problem. So, let us see what kind of problem we get for transportation problems. So, these problems we are talking about like product mix problem, transportation problem they are all classical problems. We will find examples of such problems in various textbooks and they are also related to real life applications.

The problems we are formulating here are smaller in dimensions, but the problems in real life will be much larger, may be there we have to mix not just three components may be 30 components. In the transportation problem you may talk about say transferring few commodities from one place to another, but in real life it will be a large scale problem, but you will be able to easily extend the idea that you learn here; that means, we are formulating a small size problem to a large size problem.

So, let us now look at the transportation problem, in a transportation problem, we used to find the minimum cost distribution of a given commodity from a group of supply centres, let us call them sources to a group of receiving centres let us call them destinations. So, we have i equal to 1 to m supply centres or sources. So, we have m supply centres or sources and, I have n receiving centres or destination, I want to transport a given commodity from these supply centres, or sources, this n supply centres or sources to n receiving centres, or destinations. Each source has a certain specified supply each destination has a certain specific demand. The cost of shipping from a source to a destination is directly proportional to the number of units shipped. So, how do I decide, how many items, how many how many how many items, I sent from a 1 source to various destinations.

So, what should be the distribution network, what should be the transportation network, that will be to the minimum cost. The cost of shipping from a source to destination is directly proportional to the number of units shipped. So, we have to decide how many units I ship from source 1 to say destination 1 2 3 so on and so forth.

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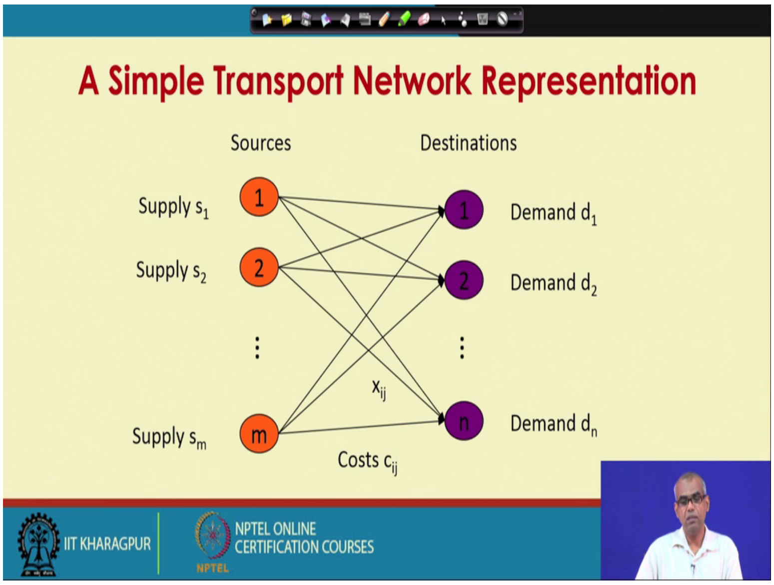

So, look at the simple transport network representation, you have m sources in each of the sources, the supply that I had is s 1, s 2 up to and there are n destinations and each of these destinations I have demand d 1, d 2 up to d n. The cost of shipping is represented as c i j; that means, c 1 1 will be the cost of shipping from source 1 to destination 1 and so, on and so forth.

Now, you have to formulate an appropriate distribution network such that, the cost is minimised at the same time demand at each destination is fulfilled, you can consider the sources as factories that is producing a commodity. And the destination can be considered as warehouses, where the factory is shipping the commodity. And the number of units that will be shipped will depend, on the demand in the locality, where the warehouses located. So, how do I find the minimum distribution, the minimum cost distribution network?

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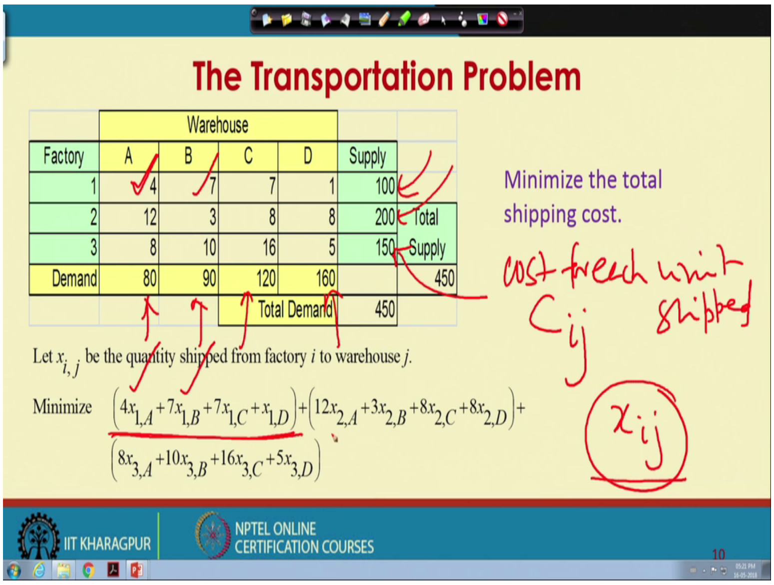

So, let us now look at a look at this problem with some data. So, the table shows the data for a transportation problem. So, let us consider we have three factory, factory 1 2 and 3. So, these are nothing, but the sources and we have 4 warehouses which are nothing, but destinations the warehouses are written as A B C and D.

So, the data source the cost C i j for each unit shipped, cost for each unit shipped. So, from factory 1 to warehouse A, the cost of shipping each unit is 4, in some unit. The cost of shipping each unit of the commodity from factory 1 to warehouse B is 7 and so, on and so forth. The supply in factory 1 is 100, the supply in factory 2 is 200, the supply in factory 3 is 150. Similarly the demand at warehouse A is 80 at B is 90 at warehouse C it is 120 at warehouse D it is 160.

So, let us now try to formulate the objective function first consider x i j be the quantity shift from factory i to warehouse j. So, x i j is the quantity to be shipped from factory i to warehouse j that is the solution to our problem actually, that that will determine the distribution network. So, how many units of the commodity are being shipped from 1 supply centre to destination centre. So, let x i j with the quantity shift from factory i to warehouse j so, i can now find out the cost required by looking at the data given in the table so, for 4 x 1 A.

Let us first look at factory let us first look at factory 1. So, factory 1 to warehouse A, each unit takes 4. So, 4 into x 1 A is the cost required for shipping x 1 A unit of the commodity from factory 1 to warehouse A. Similarly for shipping x 1 B; that means, from factory 1 to warehouse B. The cost required will be 7 x 1 B. Similarly you can find out for factory 1, factory 2 and then factory 3. So, this total cost must be minimised.

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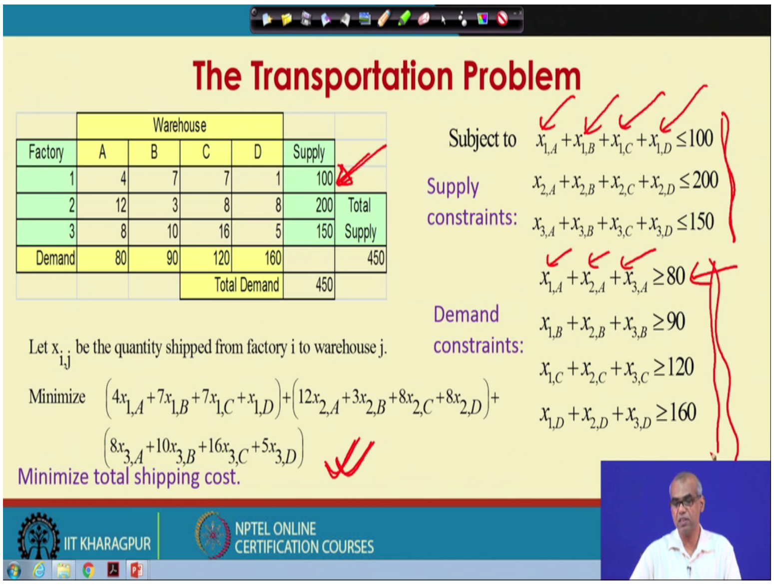

So, this is the same equation that I have written. Now, the constraints so, there are two types of constraints one is supply constraints another is demand constraints. So, the demand constraints must be made. So, the amount shipped to any particular warehouse from various factories must be greater, or equal to the demand at that warehouse. Similarly the total amount that is being shipped from a particular factory cannot exceed the amount of supply at that factory or the source.

For example if you look at factory 1, the supply is 100 x 1 A is the quantity supplied from factory 1 to warehouse A x 1 be is the quantity that is being shipped from factory 1 to warehouse B, this is to warehouse C this is to warehouse D. So, this total sum must be less or equal to 100. Similarly you can write for 3 different factories. So, 3 inequality constraints you can write down.

Now, there are 4 warehouses in each of these warehouses, there are specific demands. Let us consider warehouse A warehouse A receives x 1 A quantity from factory 1 x 2 A quantity from factory 2 x 3 A quantity from factory 3. So, their total sum must not be less than the demand at warehouse A. So, x 1 A plus x 2 A plus x 3 A should be greater or equal to 80, there are four warehouses you can write four inequality constraint like this.

So, you have this objective function, subject to this three supply constraints 4 demand constraints, that is the formal formulation of a transportation problem. If you have large, if you have many more factories many more warehouses you can imagine, the problem size will be quite large, but you can formulate using the same concept that I showed you here. Again look at it is a linear problem, linear programing problem because, objective function is linear all the constraints are also linear.

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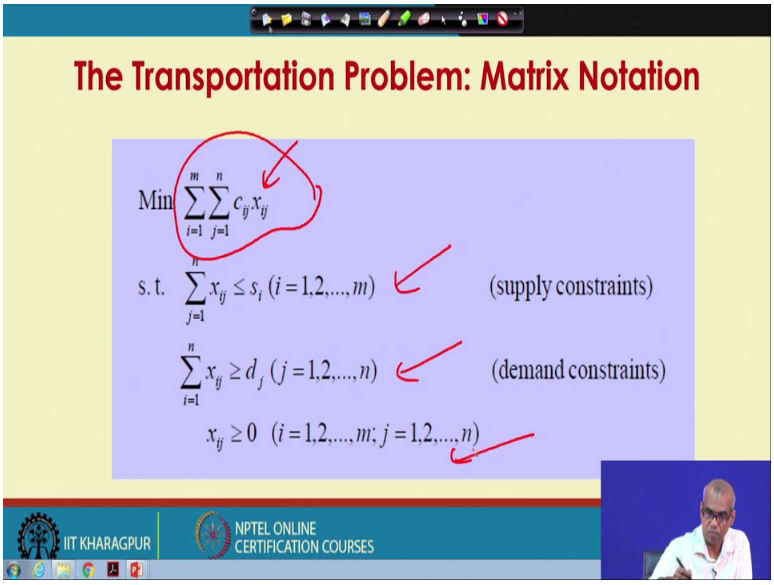

You can write down this transportation problem, very clear very cleanly using metrics notations, c i j is the cost required for shipping 1 unit from source i to destination j. So, x i j being the quantity that is being shipped from factory i to destination j. So, the total cost will be some overall m some overall n c i j, x i j that has to be minimum.

Then supply constraint total quantity that is being shipped for a particular factory, must be less than the supplied A and the demand constraint. The total quantity that the warehouse, or destination is receiving must be greater or equal to the demand there and, the non-negativity constraints on the quantity of commodity that is being shipped from factory i to destination j.

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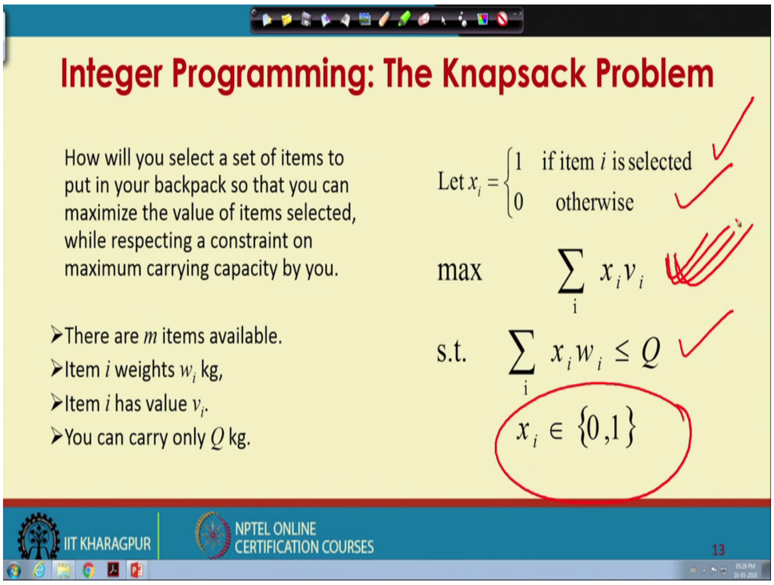

Next we will talk about integer programming. So, far we talked about linear programming problem. This is the special class of linear programming problem, where in case of linear programming problem that we have talked about so far, the variables could take on real values. Now, the decision variables are restricted to take only integer values

Let us consider we have to select a certain items that you want to put in your backpack. So, that you can maximise the value of items selected, while respecting a constant on the maximum carrying capacity by you. So, you have a backpack and, you have a set of items from which, you want to select certain items which will maximise the total amount of value for the items that you have selected, but there is a constant which is which comes from the maximum amount that you can carry. So, the question is how will you select a set of items to put in your backpack. So, that you can maximise the value of items selected and at the same time, you respect a constraint on maximum carrying capacity by you

Let us consider there are m items available item i was w i kg and, item i has value nu i, you can carry only Q kg. So, how do I formulate the problem? Let us consider x i with item, or x i with the decision variables so, x i equal to 1 if item i is selected x i equal to 0 otherwise. So, the decision variable x i can take only 1 or 0.

So, item i has value nu i so, the total amount of value x i into nu i has to be maximise. So, the objective function is sigma x i nu i, but i have the maximum i have a limitation on the maximum carrying capacity. So, item i was w i so, the total weight is x i w i. So, x i w i sigma x i w i must be less or equal to Q and x i has to be 0 or 1. So, x i is 1 if the item is selected otherwise it is 0. So, the maximum value is x i times nu i summed overall i. So, that is the objective function that you are going to maximise and, the constant comes from the amount or load you can carry. So, x i into w i summed overall i is the amount or the load. So, that is less or equal to Q. So, that is the integer problem formulation and the classical problem known as knapsack problem

So, today we checked the linear programming problem, three linear programming problem we also checked 1 integer programming problem. And, you have seen here also all the decision variables decision variables can take are only 0 or 1 and of course, the objective function is linear.

So, we stop our discussion for this lecture here.