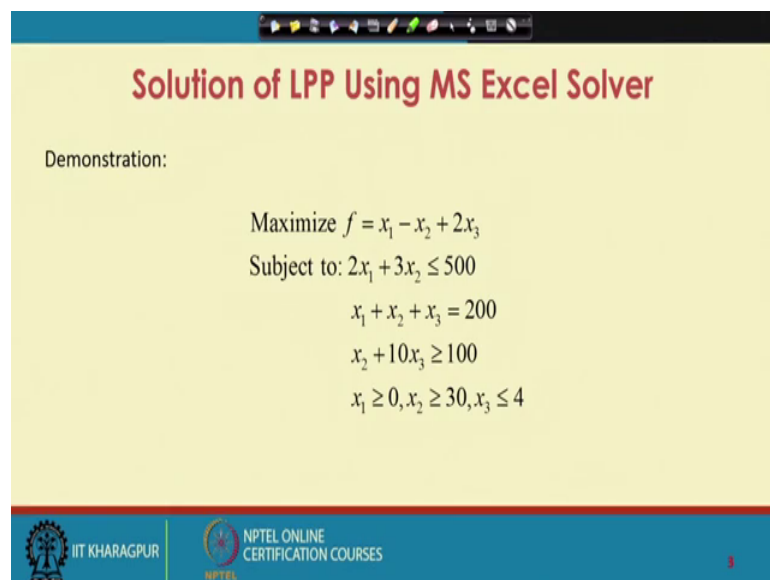


Optimization in Chemical Engineering
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Lecture – 59
Software Tools for Optimization (Contd.)

Welcome to lecture 59. See, in our previous lecture, we discussed how to solve a linear programming problem using excel solver. We went through all the steps. In today's lecture, we will actually solve the problem using excel on this computer, so that you can have a better feel of how to use excel solver for solution of a linear programming problem.

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Solution of LPP Using MS Excel Solver

Demonstration:

Maximize $f = x_1 - x_2 + 2x_3$
Subject to: $2x_1 + 3x_2 \leq 500$
 $x_1 + x_2 + x_3 = 200$
 $x_2 + 10x_3 \geq 100$
 $x_1 \geq 0, x_2 \geq 30, x_3 \leq 4$

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So, let us consider the same linear programming problem which we discussed in our previous lecture. So, maximize f equal to x_1 minus x_2 plus $2x_3$. Subject to $2x_1$ plus $3x_2$ less or equal to 500 ; x_1 plus x_2 plus x_3 equal to 200 ; x_2 plus $10x_3$ is greater equal to 100 ; x_1 greater equal to 0 , x_2 greater equal to 30 , and x_3 less or equal to 4 . So, let us follow the steps that we have discussed in our previous lecture. So, let me start with an excel sheet.

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	A	B	C	D	E	F	G	H	I	J
1	LPP Example	Variable 1 (x1)	Variable 2 (x2)	Variable 3 (x3)		Evaluations		RHS Constants		
2										
3	Upper Bound	1E+25	1.00E+25	4						
4	Value	136	60	4						
5	Lower Bound	0	30	0						
6										
7	Objective	1	-1	2		84				
8										
9	Constraint 1	2	3	0		452 LE		500		
10	Constraint 2	1	1	1		200 EQ		200		
11	Constraint 3	0	1	10		100 GE		100		
12										
13										
14										
15										
16										
17										
18										
19										

So, in the cell A 1, we said that let us write a label, so that we understand what kind of problem is this it is just a tag. So, we write say LPP Example. You can expand the width let us give a gap, and let us write here Upper Bound. Then we will write value we write Lower Bound. We give a gap and then we write objective or you can write Objective function if you want then let us skip one cell, then we write all the constraints Constraint 1, Constraint 2, and Constraint 3.

Now, we have put all the labels in column A. So, now, let us go to column B C D, and we set that it will be Variable 1 which is my x 1. Just expand it little bit Variable 2 x 2, you have to x 2 and then Variable 3 you have x 3 you need to expand little bit. So, we will give a let us skip one cell and then let us write evaluations. Again we will skip one cell, and write right hand side constants RHS Constants.

So, the Upper Bound values we have chosen for Variable 1, and Variable 2 which did not have any Upper Bound we had chosen 10 to the power 25 so let me write 10 to the power 25. Variable 3 was x 3 less or equal to 4, so it is 4. So, this row the value will contain initially the initial case for the problem and finally it will contain the converse solution. So, the Lower Bound in the problem the Lower Bound was x 2 greater equal to 30 others did not have. So, let us consider 0 0 and then for x 2 consider 30. So, we are done with Upper Bound and Lower Bound. And we have also supplied initial case for x 1, x 2 and x 3.

Now, we have to supply the objective function coefficients. The objective function was maximize. Let us say maximize f equal to x_1 minus x_2 plus $2x_3$. So, for x_1 the coefficient is 1, for x_2 the coefficient is minus 1, and for x_3 the coefficient is 2. So, x_1 minus x_2 plus $2x_3$ this was the objective function. Now, look at the constraint coefficients. The Constraint 1 was $2x_1$ plus $3x_2$ is less or equal to 500, $2x_1$ plus $3x_2$. So, $2x_1$ plus $3x_2$ for x_3 it will be then 0, it is greater or equal to so, let us go to column G skip this two cells, and go to column G it is greater or equal to.

So, let us write G E for greater or equal to sorry the first constraint was $2x_1$ plus $3x_2$ is less or equal to 500. So, less or equal to 500 under right hand side constants let us write 500. Similarly, the constant 2 was x_1 plus x_2 plus x_3 equal to 200. So, x_1 plus x_2 plus x_3 equal to 100. And then let us come to Constraint 3. The Constraint 3 was x_2 plus $10x_3$ is greater or equal to 100 sorry the Constraint 2 it was equal to 200 not 100. So, it was 200.

So, Constraint 3 was x_2 plus $10x_3$. So, x_1 coefficient is 0, x_2 coefficient is 1 and for x_3 it is 10. So, x_2 plus $10x_3$ is greater or equal to 100. So we have now entered the upper bounds, the lower bounds, initial guess, objective function coefficients, and the constraint coefficients, type of constants, and the right hand side constants. Now, we have to enter formula into the excel sheet that evaluates the objective function. And then we will do the same thing for the Upper Bound Lower Bound and the Constants. So, let us first start with objective function. So, we will now enter a formula that evaluates the objective function. So, let us go to row 7, and come to the cells under evaluations column.

So, you come to the cell that is in row 7 that is objective row, and also under column evaluations. So, there you type SUM PRODUCT; note that I am not pressing enter now. So now what you have to do is, you have to select the values see the cells B 4 C 4 D 4 is contained. So, it is selected B 4 up to D 4 put comma. And then come to row 7 which contains objective. So, start with the cell in the row 7 under column B containing variable 1 or x_1 . So, from go right across x_1 , then x_2 , then x_3 . Now, B 7, C 7, D 7 selected.

Now, you close the parenthesis, so first bracket is closed. Now, you press enter so 0 is coming, because the objective the values that means the initial guess for the decision

variables are all 0. So, we are multiplying the cost coefficients with 0. Now, we have to do the same thing for the constants. So, come to row 9, which contains Constraint 1 come under revaluations, put equal to then type sum product start bracket do not put or do not press enter, go to values take B 4 to D 4 put comma.

Now, come to row 9 that is Constraint 1 row, come under variable x 1 in the same cell go to right across x 1, x 2 and x 3 selected. Now, close the bracket press enter. Similarly, you do for Constraint 2 equal to sum product I select x 1 x 2 x 3 I put comma and then I select coefficients for Constraint 2 close the bracket press enter. Similarly, do for Constraint 3, I writes equal to SUMPRODUCT, now select x 1, x 2, x 3 under values put comma come to Constraint 3 row select x 1 x 2 x 3 close the bracket press enter. So, now you have completed data entry you have, now completed data entry.

Now, we ready to solve your problem. So, to solve the problem you go to data tab, and you see the solver is loaded already, so click on solver. So, set objective you have to set objective here. So, go to row 7 which contains objective in this row you go to the cell which is under evaluations click there F 7 cell references selected we are maximizing the problem. So, the max button is chosen by changing variables. Now, click on the by changing variables we are changing x 1 x 2 and x 3 so that is in row 4 with label value. So, to choose x 1 x 2 x 3 we click on this red button and then we select x 1 x 2 and x 3. So, B 4 to D 4 is selected here click on the red button again so B 4 to D 4 is selected.

So, you have now selected the target cell which contains objective function we have set the objective. We have indicated there is a maximization problem and we have also chosen x 1 x 2 x 3 as decision variables this has been done by changing variable cells and choosing B 4 to D 4. So, now we have to add the constraints. So, you will add the Upper Bound, Lower Bound then Constraint 1 Constraint 2 and Constraint 3. So, now click on add button. So, we are now putting first upper bound. So, click on this cell reference on this red button and then value that means x 1 to x 3 use select click on the red button. So, B 4 to D 4 is chosen under cell reference. Now, this is upper bound, so less or equal to sign is selected from the dropdown menu.

The constraint click on the red button and then go to the row 3 which contains Upper Bound as label select Upper Bound for x 1 Upper Bound for x 2 and Upper Bound for x 3. So, B 3 to D 3 is chosen and then you click on add Upper Bound is chosen. Now,

similarly let us choose the lower bound. So, under cell difference click on the red button, let us choose x_1 , x_2 , x_3 this is Lower Bound, so greater or equal to we have to choose from the drop down menu. And then under Constraints again click on the red button and we now go to row 5 which contains Lower Bound as label, and select x_1 , x_2 , x_3 Lower Bounds, so click on add.

Now, similarly let us do for the constraint 1, 2, and 3 so for click on for Constraint 1 click on cell reference then come to the cell which is in the Constraint 1 row, and under evaluations column click there. This is less or equal to type, so less or equal to type, and the constraint, and the constraint click on the constraint. Then go to the right hand side Constraint corresponding to Constraint 1 click that cell you add. Similarly, for Constraint 2 go to the Constraint 2 row come under evaluation cell this is equal to type select equal to from the dropdown, and then click the cell under right hand side constants corresponding to row Constraint 2 add.

Now, again for Constraint 3 go to Constraint 3 row come under evaluations click on that cell. This is greater or equal to type constraint, so choose greater or equal to sign from drop down menu, and under constraint you click on the cell under right hand side constraints corresponding to constrain 3 row add it. So, all Upper Bound Lower Bound and 3 constraints have now been added. So, you can now put can you can now it cancel to go back to the dialogue box.

So, the problem is now completely define parameters have been set. The only now I have to do now is have to select a solving method. From the dropdown menu which I choose simplex LP you can look at under options let all be set as default value. So, to solve the problem you just have now to click on the solve button. So, I click on solve button. So, solver found a solution all constraints and optimality are optimality conditions are satisfied. So, keep solver solution button I choose and I also choose answer as well as sensitivity then I click on ok. So, this is the solution I get x_1 equal to 136 x_2 equal to 60, and x_3 equal to 4.

The same solution that we discussed in our previous lecture, Optimal solution is optimal value of the objective function is 84. Note that the constraints all the constraints are satisfied. For constraint 1, 452 is less than 500, 200 is equal to 200, 100 greater or equal 100. All the constraints are satisfied and the optimal solutions is contained in the row 4

which has label value. So, x_1 equal to 136, x_2 equal to 60, and x_3 equal to 4 at the optimal solutions, and the corresponding values of optimal values of the objective function is 84.

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The screenshot shows the 'Answer Report' generated by Microsoft Excel Solver. The report includes the following sections:

- Objective Cell (Max):**

Cell	Name	Original Value	Final Value
\$F\$7	Objective Evaluators	0	84
- Variable Cells:**

Cell	Name	Original Value	Final Value	Integer
\$B\$4	Value Variable 1 (x1)	0	136	Yes
\$B\$5	Value Variable 2 (x2)	0	60	Yes
\$B\$6	Value Variable 3 (x3)	0	4	Yes
- Constraints:**

Cell	Name	Cell Value	Formula	Status	Slack
\$F\$10	Constraint 1 Evaluations	200	\$F\$10:\$G\$10	Binding	0
\$F\$11	Constraint 2 Evaluations	100	\$F\$11:\$G\$11	Binding	0
\$F\$12	Constraint 3 Evaluations	432	\$F\$12:\$G\$12	Not Binding	48
\$B\$4	Value Variable 1 (x1)	136	\$B\$4:\$D\$4	Not Binding	81436
\$B\$5	Value Variable 2 (x2)	60	\$B\$5:\$D\$5	Not Binding	81436
\$B\$6	Value Variable 3 (x3)	4	\$B\$6:\$D\$6	Binding	0
\$B\$7	Value Variable 1 (x1)	136	\$B\$7:\$D\$7	Not Binding	136
\$B\$8	Value Variable 2 (x2)	60	\$B\$8:\$D\$8	Not Binding	60
\$B\$9	Value Variable 3 (x3)	4	\$B\$9:\$D\$9	Not Binding	4

So, now answer report is self explanatory, it tells you that it took 5 iterations it started with 0 values for the decision variables and converse to x_1 equal to 136, x_2 equal to 60, and x_3 equal to 4.

(Refer Slide Time: 28:25)

The screenshot shows the 'Sensitivity Report' generated by Microsoft Excel Solver. The report includes the following sections:

- Variable Cells:**

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$4	Value Variable 1 (x1)	136	0	1	10436	-2
\$B\$5	Value Variable 2 (x2)	60	0	-1	2	10436
\$B\$6	Value Variable 3 (x3)	4	21	1	10436	21
- Constraints:**

Cell	Name	Final Value	Shadow Price	R.H. Side	Increase	Decrease
\$F\$10	Constraint 1 Evaluations	200	1	200	24	136
\$F\$11	Constraint 2 Evaluations	100	1	100	48	36
\$F\$12	Constraint 3 Evaluations	432	0	532	10436	48

And it also contains sensitivity report. So, this is how you can solve a linear programming problem using Microsoft Excel solver.

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LPP with Excel Solver: Answer Report

Microsoft Excel 14.0 Answer Report
 Worksheet: [ExcelLPP_Example.xlsx]Sheet1
 Report Created: 2018-10-04 12:37:17
 Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

Solver Engine: Simplex LP
 Solution Time: 0.0 seconds
 Iterations: 3 / Subproblems: 0

Solver Options:
 Max Time Unlimited, Iterations Unlimited, Precision 0.000001
 Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume Non-Negative

Objective Cell (Max):
 Cell Name Original Value Final Value
 \$F\$7 Objective Evaluations 0 84

Variable Cells:
 Cell Name Original Value Final Value Integer
 \$B\$4 Value Variable 1 (x1) 0 136 Contin
 \$C\$4 Value Variable 2 (x2) 0 60 Contin
 \$D\$4 Value Variable 3 (x3) 0 4 Contin

Constraints:
 Cell Name Cell Value Formula Status Slack
 \$F\$10 Constraint 2 Evaluations 200 \$F\$10:\$F\$10 Non Binding 0
 \$F\$11 Constraint 3 Evaluations 100 \$F\$11:\$F\$11 Non Binding 0
 \$F\$9 Constraint 1 Evaluations 452 \$F\$9:\$F\$9 Non Binding 48
 \$B\$4 Value Variable 1 (x1) 136 \$B\$4:\$B\$4 Non Binding 10+3
 \$C\$4 Value Variable 2 (x2) 60 \$C\$4:\$C\$4 Non Binding 10+3
 \$D\$4 Value Variable 3 (x3) 4 \$D\$4:\$D\$4 Non Binding 0
 \$B\$4 Value Variable 1 (x1) 136 \$B\$4:\$B\$4 Non Binding 136
 \$C\$4 Value Variable 2 (x2) 60 \$C\$4:\$C\$4 Non Binding 30
 \$D\$4 Value Variable 3 (x3) 4 \$D\$4:\$D\$4 Non Binding 4

Now, so this was the answer report and this is the sensitivity report.

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LPP with Excel Solver: Sensitivity Report

Microsoft Excel 14.0 Sensitivity Report
 Worksheet: [ExcelLPP_Example.xlsx]Sheet1
 Report Created: 2018-10-04 22:00:46

Variable Cells:

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$4	Value Variable 1 (x1)	136	0	1	1E+30	2
\$C\$4	Value Variable 2 (x2)	60	0	-1	2	1E+30
\$D\$4	Value Variable 3 (x3)	4	21	2	1E+30	21

Constraints:

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$10	Constraint 2 Evaluations	200	1	200	24	136
\$F\$11	Constraint 3 Evaluations	100	-2	100	48	30
\$F\$9	Constraint 1 Evaluations	452	0	500	1E+30	48

Sensitivity analysis gives you insight in how the optimal solution changes when you change the coefficients of the model.

So, sensitivity analysis gives us the insight in how the optimal solution changes, when we change the coefficients of the model.

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LPP with Excel Solver: Sensitivity Report

A reduced cost tells us how much an objective function coefficient must be improved before the corresponding decision variable gets a value different from zero.

Objective function coefficient ranges.

Excel defines a shadow price as the amount by which the objective function value changes when the corresponding constraints right side increases by one unit.

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The slide features a yellow background with a blue header and footer. A small video inset of a man in a white shirt is visible in the bottom right corner.

So, if you look at it, there is reduced cost shadow price, and this allowable increase decrease columns. And there are 2 tables one is for variable cell that is for x_1 , x_2 , and x_3 . And this is for constraints those 3 constraints. Now, what do this mean a reduced cost tells us how much an objective function coefficient must be improved before the corresponding decision variables gets a value different from 0. So, reduced costs tells us how much an objective function coefficient must be improved, before the corresponding decision variable gets a value different from 0. So, my decision of the decision variable is 0.

So, if a decision variable is 0, then the entry under reduced cost column will tell you how much an objective function coefficient must be improved so that that decision variable gets a value different from 0 say 0 to 1. Now, the objective function ranges, now these are the objective function coefficient. If you remember, the objective function was x_1 minus x_2 plus $2x_3$, so these are those coefficients.

Now, allowable increase is 10 to the power 30 that means you can consider to be very large or infinity, let us say allowable decrease is 2. What it means is that you can the amount of increase or decrease that you can make to these objective function coefficients and still the optimal solution will not change. So, allowable increase tells you by how much you can increase the object that objective function coefficients without changing the objective function value.

The same interpretation holds true for allowable decrease. How much you can decrease that particular objective function coefficients without changing the optimal solution for the problem. So, in this particular case you can decrease by 2 only, but increase you can increase by any amount. Now, excel defines a shadow price as the amount by which the objective function value changes when the corresponding constraint right hand side increases by one unit.

So, the shadow price is the amount by which the objective function value changes when the corresponding constants right hand side increases by one unit. So, what it means is that the shadow price for this particular Constraint, Constraint 1 is 1 that means if I increase the right hand side Constraint that means 200 for this constant to 200 and 1 my objective function value will be increased by 1. And this interpretation is valid within these Allowable Increase or Allowable Decrease.

So, within this range, if I change the right hand side constant by unit amount, the shadow price will tell you the amount by which the objective function will change. So, this is how you can interpret the sensitivity report of the Excel Solver. So, this is how you can solve a linear programming problem using Excel Solver.

So, what you do it? You did is we took an example where there are 3 constraints. All the constraints are of three different times greater or equal to lesser or equal and equal to. So, we have learned how to setup the problem in the excel sheet, and how to use the solver, how to set the parameters and solve the linear programming problem. We also tried to learn briefly although how to interpret the sensitivity report that the solver also gives you with this would like to conclude lecture 59.