Optimization in Chemical Engineering
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Lecture – 58
Software Tools for Optimization (Contd.)

Welcome, to lecture 58 in this final week of 12. We are talking about Software Tools for Optimization. In our previous two lectures, we have talked about Microsoft excel solver as an optimizer. We have seen how to use excel solver to solve unconstrained and constrained optimization problems. We have seen both single variable and multiple variable optimization problems. In today's lecture we will learn how to use Microsoft excel solver for solution of linear programming problems.

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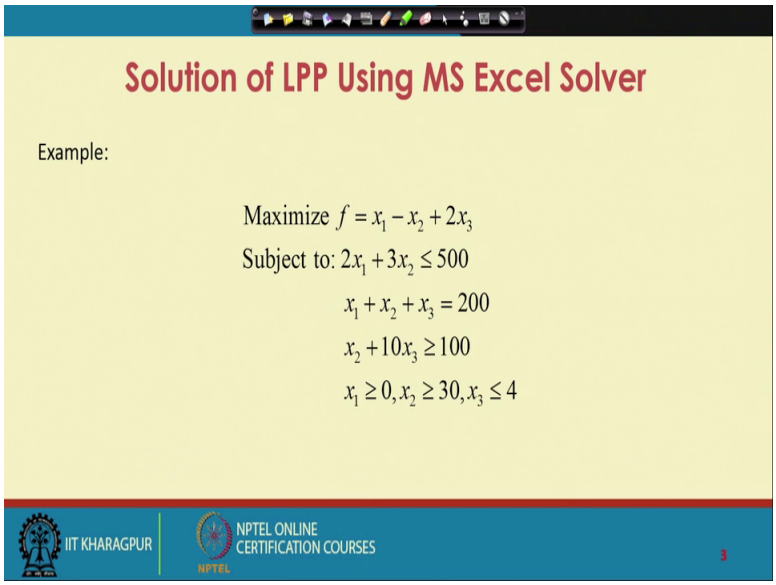

So, let us consider this linear programming problem. Maximize f equal to x 1 minus x 2 plus 2x 3 subject to three constraints; 2x 1 plus 3x 2 less or equal to 500, x 1 plus x 2 plus x 3 equal to 200 and x 2 plus 10x 3 is greater or equal to 100. So, you have one less or equal to type constraint, one equal to type constraint and one greater or equal to type constraints.

Then you also have x 1 greater equal to 0, x 2 greater or equal to 30 and x 3 less or equal to 4.

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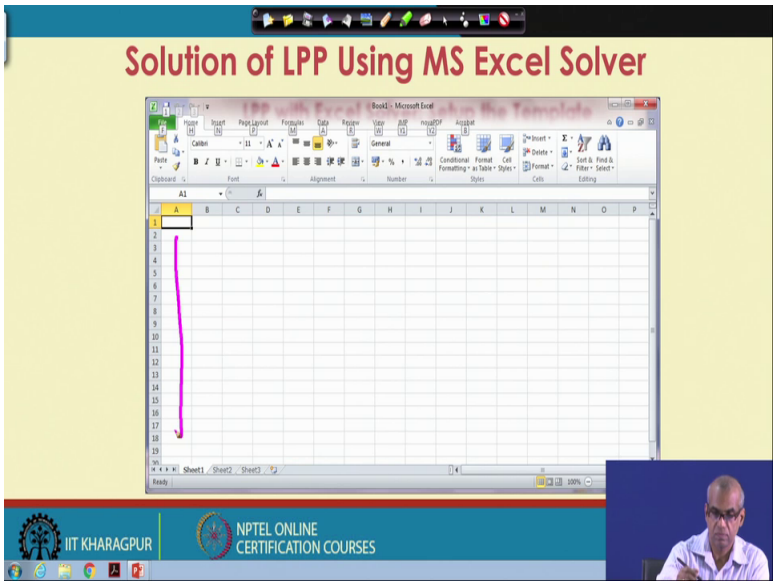

So, the first task will be you launch excel solver, maxi that solver is loaded. We have seen in previous lectures how to do that and look at this columns A, B, C, etcetera.

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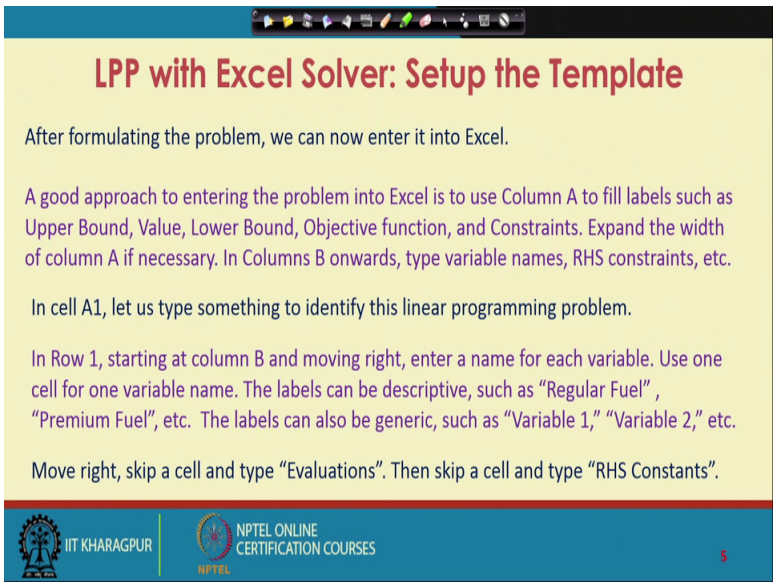

Now, after you have formulated a linear programming problem, we have to enter the problem in to excel sheet. A good approach to entering the problem into excel sheet is to use column A to fill labels such as Upper Bound, Lower Bound, Value, Objective function and Constraints. You can expand the width of the column A if necessary.

So, what I mean is that what you can do is you can use these column to write variables Upper Bound, Lower Bound of the values of those variables then the constraints. So, these labels we can entre in column A. In column B onwards, we can type the variable names. Suppose, there are three variables; x 1 and x 2 and x 3, so, we can write these things in column B, C and D.

We can also put the right hand side constraints on the columns after that. So, in cell A1 let us type something to identify that this is a linear programming problem. You can give a name to this linear programming problem. Suppose, in cell A1 you write linear programming problem example 1, so on and so forth.

In row 1, starting at column B and moving right, enter a name for each variable. Use one cell for one variable name. The labels can be descriptive such as Regular Fuel, Premium Fuel, etcetera or the label can also be generic such as Variable 1, Variable 2 etcetera. You can also give name such as x 1, x 2, etcetera. So, whatever is convenient, you can accordingly give labels for variable names. So, either you can give descriptive name such as Regular Fuel, Premium Fuel, Product A, Product B, etcetera or you can give x 1, x 2, x 3, so that you can easily identify them.

So, after you have entered the variable names starting from column B, you now move right, skip a cell and type evaluations. So, after you have completed writing, the variable names, you skip a cell and type evaluations then skip another cell and type right hand side constraints. Note that this skipping cell is not compulsory, but it will be convenient for us if you do that.

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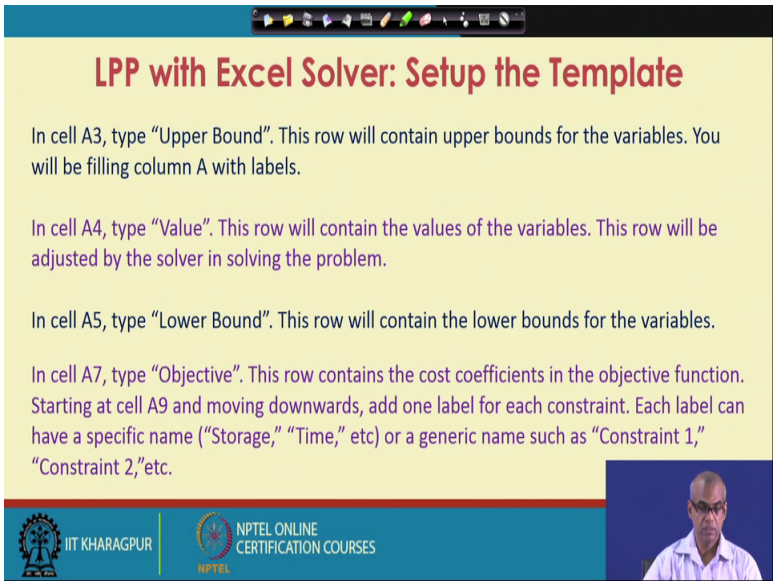

In cell A3, type Upper Bound. This row will contain upper bounds for the variables. You will be filling column A with labels. As we discussed in the beginning that column A, we will be filling with labels. Upper Bound is one such label. Similarly, we will put Lower Bound, we will put values. So, these three labels will use for variables in column A.

So, in cell A3, we type Upper Bound as labels for the variables, Upper Bound for the variables. In cell A4, we type Value. This row will contain the values of the variables. So, this is a values of the decision variables. This row will be adjusted by the solver in solving the problem. So, finally, this row will contain the solution of the problem and initially you supply the initial guess for the problem in this row.

So, in cell A5, type Lower Bound. So, this row will contain the lower bounds for the variables. Suppose, you have three variables; so, in cell A3, you have typed upper bound. So, B3, B4 and B5 columns will have the upper bound values for the variables. Similarly, in the cell A5 you have typed lower bound that means this is row 5. So, in the cells B5, C5 and D5 you will have values for the lower bound of the variables. So, this will be more clear when I show you the picture or the snapshot of the excel sheet.

In cell A7, type Objective. This row contains the cost coefficients in the objective function. Starting at cell A9 and moving downwards, add one label for each constraint. Each label can have a specific name, such as Storage, Time, etcetera; which describes the constraints or you can supply a generic name such as Constraint 1, Constraint 2 etcetera.

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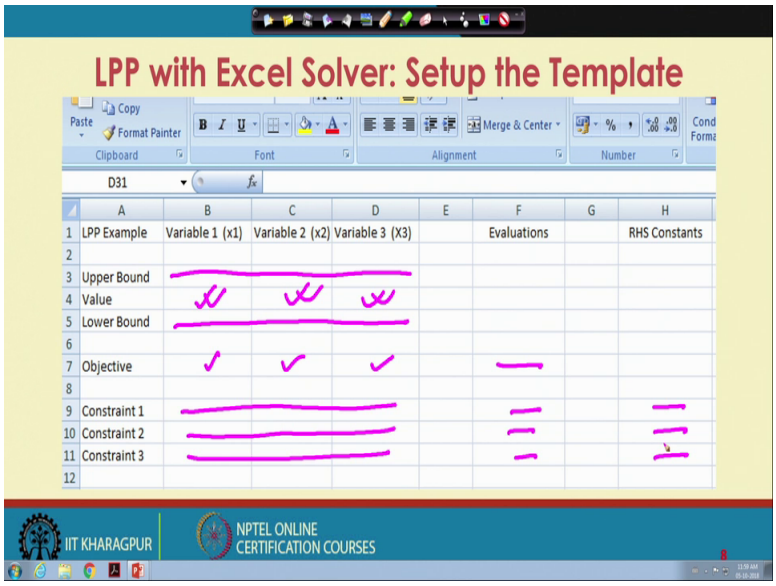

So, this is what the excel sheet will look like. In column A, we have given the labels; column B, C, D contains names of the variables; Variable 1 or x1, Variable 2 or x2, Variable 3 or X3. Column E, I have kept blank, column F contains Evaluations, contain G column G blank and column H contains right hand side constants.

So, this is more clear. So, you can note that the column A contains all the labels as we discussed and column B, C, D contains variable names, column E is blank, column F contains evaluations, column H contains right hand side constraints. So, these three cells will contain the upper bounds for variables x1, x2 and X3 respectively. These three cells will contains lower bound for decision variables x1, x2 and X3. These three cells will contain coefficients of the objective function and these three cells will contain coefficients of the constraints; Constraint 1, Constraint 2 and Constraint 3 respectively and these three cells initially will contain initial guess for the solution and finally, will contain the converged solution.

So, Evaluations will contain the processing of the objective function and the processing of the constraints. The right hand side constraints of the three constraints will be contained in column H under RHS constraints. So, this is the structure that we are following. Note that you can also use the different structure, but I think this is convenient and the labels that I am using, again and not fixed labels to be use, you can use your own labels according to your choice.

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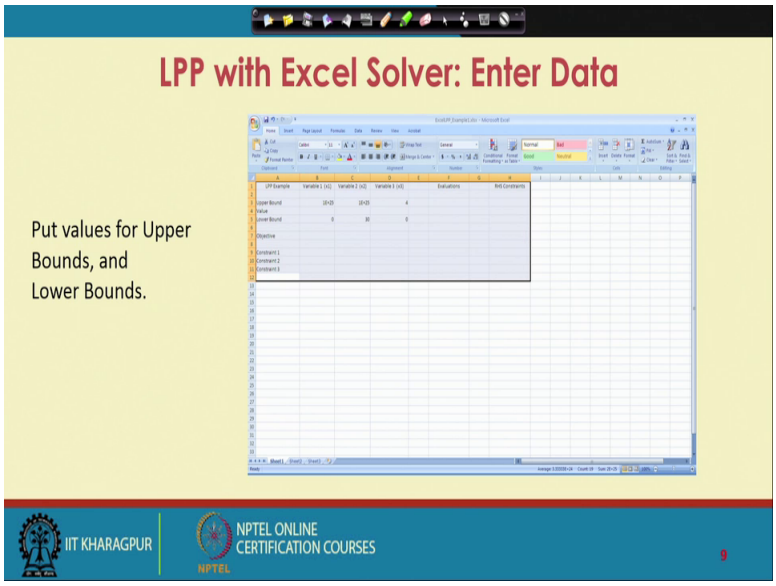

So, now, put values for Upper Bounds and Lower Bounds.

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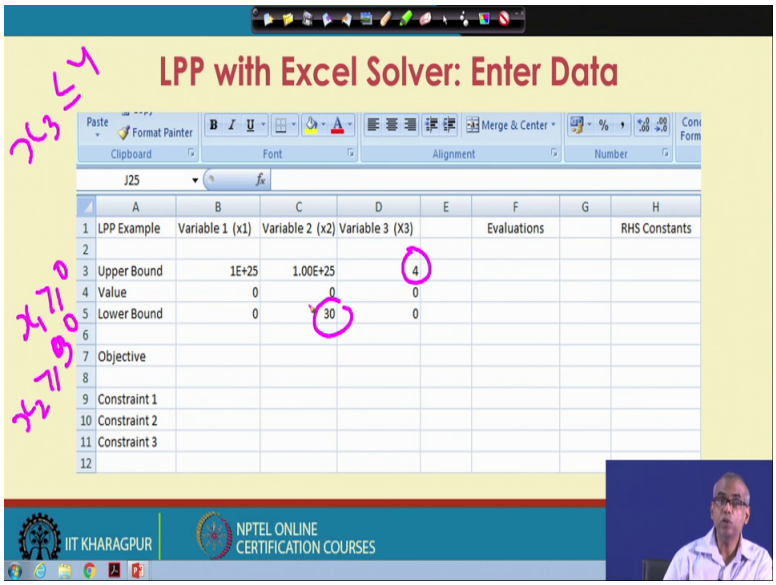

So, you can see a zoomed version here. Note that the Upper Bound for x 1 and x 2, I have put as 10 to the power 25 because there was no upper bound mentioned in the original problem. If you remember the problem percolation, it was retained x 1 greater or equal to 0 and x 2 greater or equal to 0; sorry, x 2 greater or equal to 30, x 1 greater equal to 0 and x 2 greater or equal to 30. So, there was no Upper Bound. So, I took a large Upper Bound 10 to the power 25; x 3 was given as x 3 less or equal to 4.

So, that is why Upper Bound for x 3 has been put as 4. You can supply the initial guess for values variable x 1, x 2, x 3 all as 0, if we keep blank it will be taken as 0 and the Lower Bound x 3 is greater or equal to 30; so, Lower Bound of x 2 greater or equal to 30. So, Lower Bound of x 2 is 30. No lower bound for x 1 and x 3 is given. So, you consider non negative values for the variables and consider x 1 and x 3 have lower values as 0. So, I have now put the values.

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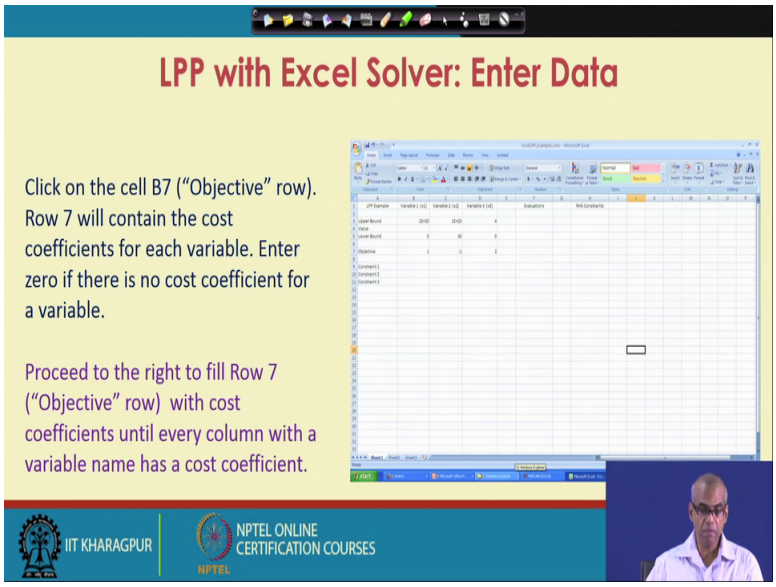

Now, click on the cell B7; that means the row which contains Objective label. Row 7 will contain the cost coefficient for each variable. So, enter zero if there is no cost coefficient for a variable. So, this proceed to the right to fill Row 7, that is, Objective row with cost coefficients until every column with a variable name has a cost coefficient.

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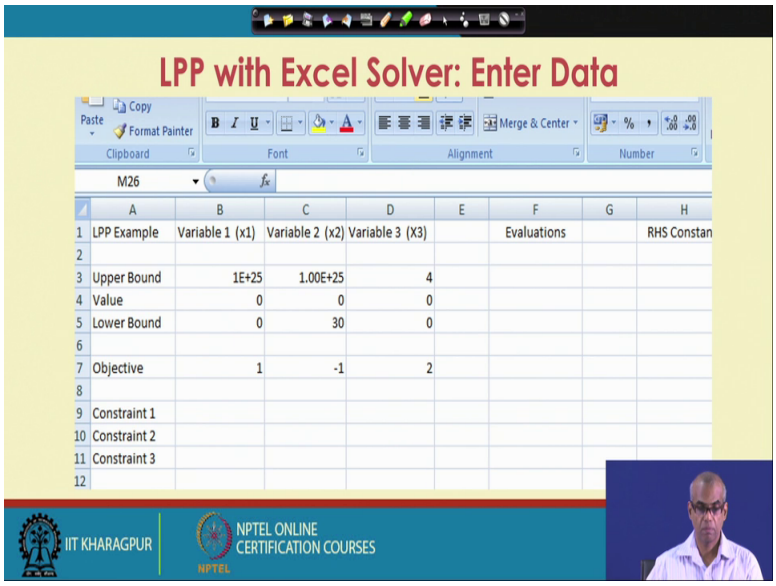

So, if you remember the objective function was maximize x 1 minus x 2 plus 2x 3, x 1 minus x 2 plus 2x 3. So, the cost coefficient of x 1 was 1, of x 2 was minus 1 and of x 3 was 2. So, those are the values I have put in row 7 where in column A objective variable objective label is written. So, these values we have to put.

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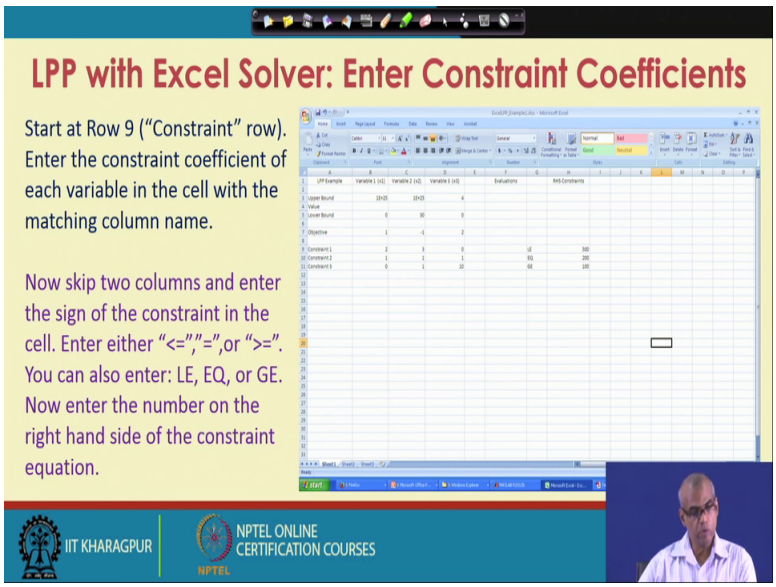

Now, we have to enter now constraint coefficients. We have as of now we have entered the variable names, then we have entered the upper bound, lower bound and initial guess

for the values. Then we have entered the cost coefficients of the objective function. Now, we have to enter the constraint coefficients. So, you have to look under Constraint row.

So, start from Row 9 which contains constraint 1. Enter the constraint coefficient of each variable in the cell with the matching column; that means, under variable x 1, x 2 and x 3 the column B, C and D. Now, skip two columns and enter the sign of the constraints in the cell. The constraints may be greater or equal to or less or equal to type. So, appropriately you put in the column G and then in column H which contains righter side constraints, you put the right hand side constraints for these three constraints. For constraint 1; it was 500, for constraint 2; it is 200 and for constraint 3; it is 100. So, this is how it will look like.

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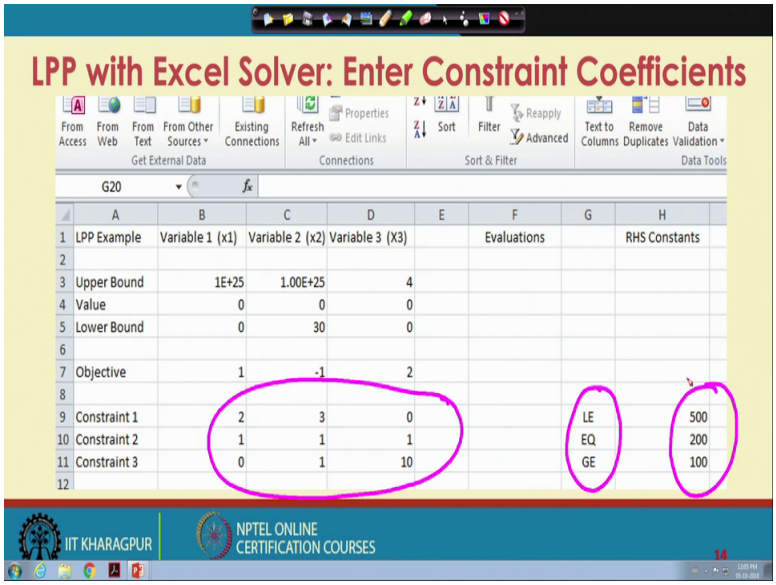

Note that I have put the constraint coefficients and then I have indicated what type of constraints are they; less or equal to, equal to, greater or equal to and the values of the right hand side constraints for Constraint 1, 2 and 3 respectively.

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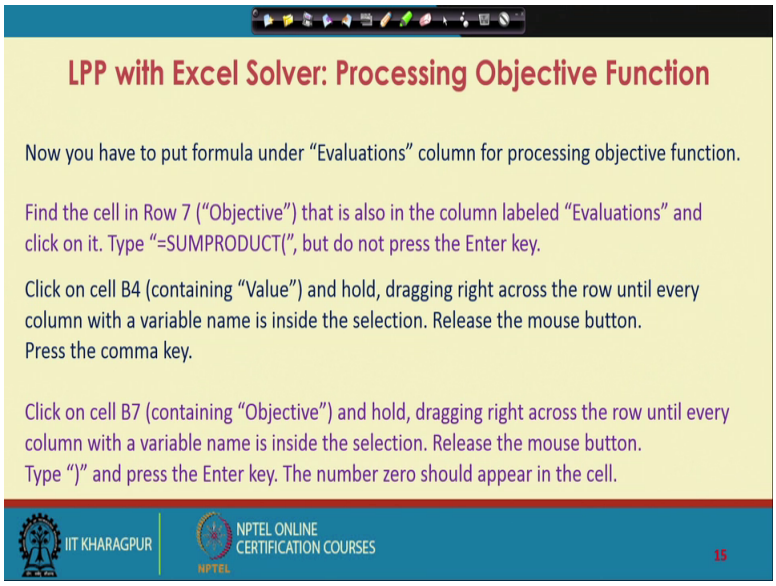

Now, we have to process objective function. Now, we have to put a formula under Evaluations column for processing objective function. Find the cell in Row 7 which contains the label Objective, you can also write objective function. So, find the cell in the Row 7 that is also in the column label Evaluations and click on it. Type equal to SUMPRODUCT, then first bracket, but do not press the Enter key. Click on cell B4 containing Value and hold, dragging right across the row until every column with the variable name is inside the selection. Release the mouse button. Press the comma key.

Now, click on cell B7 containing Objective and hold, dragging right across the row until every column with the variable name is inside the selection. Release the mouse button. Type closing first bracket and press the Enter key. The number zero should appear in the cell.

Basically, what we are doing is if let us say I have three decision variables; x 1, x 2 and x 3 and the cost coefficients are c 1, c 2 and c 3. You are basically finding out c 1 x 1 plus c 2 x 2 plus c 3 x 3.

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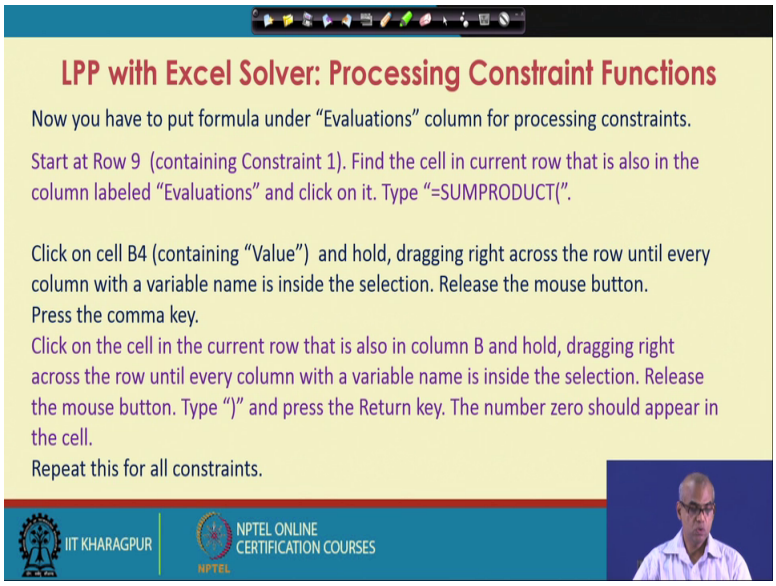

Now, similarly we have to process constraint functions. Now, we have to put formula under Evaluations column for processing constraints. Start at Row 9 containing Constraint 1. Find the cell in current row that is also in the column labeled Evaluations and click on it, type equal to SUMPRODUCT and starting first bracket.

Click on cell B4 containing Value and hold dragging right across the row until every column with a variable name is inside the selection. Release the mouse button. Press the comma key. Click on the cell in the current row that is also in column B and hold dragging right across the row until every column with a variable name is inside the selection. Release the mouse button. Type closing first bracket and press the Return key. The number zero should appear in the cell.

So, this is very much similar to what we did in case of previous step where we processed the objective function. So, similarly we have to process all the constraint functions. So, repeat what we did for constraint 1, for constraint 2, as well as constraint 3. So, basically you are multiplying the decision variables with their corresponding constraint coefficients.

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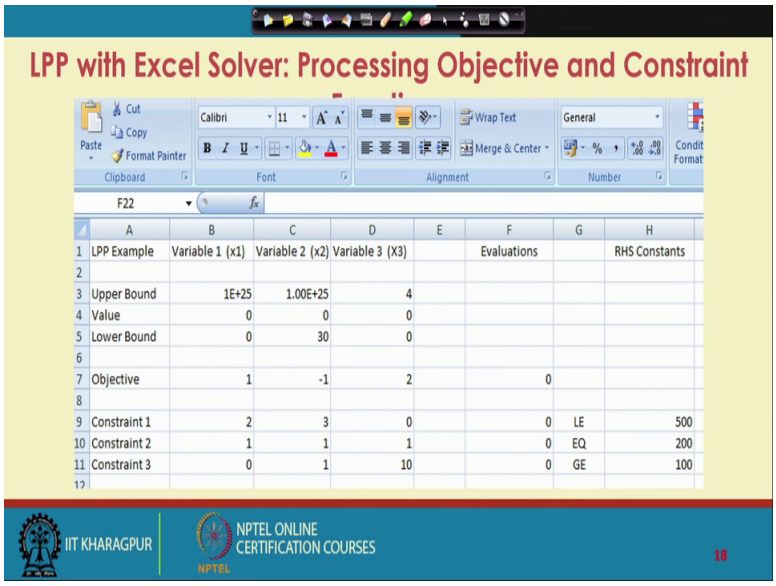

	A	B	C	D	E	F	G	H
1	LPP Example	Variable 1 (x1)	Variable 2 (x2)	Variable 3 (X3)		Evaluations		RHS Constants
2								
3	Upper Bound	1E+25	1.00E+25	4				
4	Value	0	0	0				
5	Lower Bound	0	30	0				
6								
7	Objective	1	-1	2		0		
8								
9	Constraint 1	2	3	0		0	LE	500
10	Constraint 2	1	1	1		0	EQ	200
11	Constraint 3	0	1	10		0	GE	100

So, this is how it you look like. Under evaluations all 0's are coming because the initial guess for x 1, x 2 and x 3 I have taken 0. So, everything every coefficient is getting multiplied by 0, so, we are getting sum product as 0.

So, this is how it will look like after processing objective function and constraint function.

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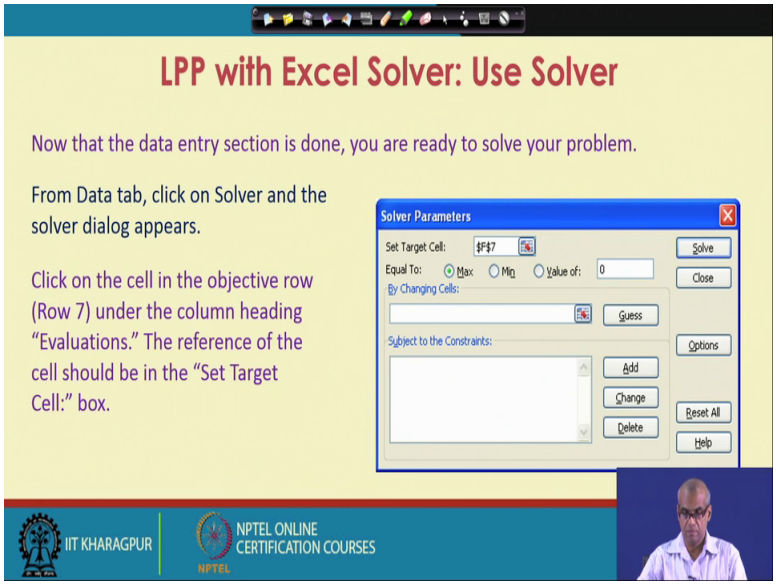

Now, you have entered all the data and we are now ready to solve the problem. From Data Tab, click on Solver and the solver dialog appears and what you see now is the

dialog box and this is for bit older version of Excel Solver. I will show you how the new version or the last say 2013 version will look like.

So, from the Data Tab you click on the Solver and the solver dialog box will appear. You have to set solver parameters here. Click on the cell in the objective row, Row 7 under the column heading Evaluations. This reference of the cell should be in the Set Target cell. So, the first thing that you have to do is you have to set the target cell. So, for setting target cell you have to click on the cell in the objective row, Row 7 under the column heading Evaluations. The cell reference of the cell should be in the set target cell box.

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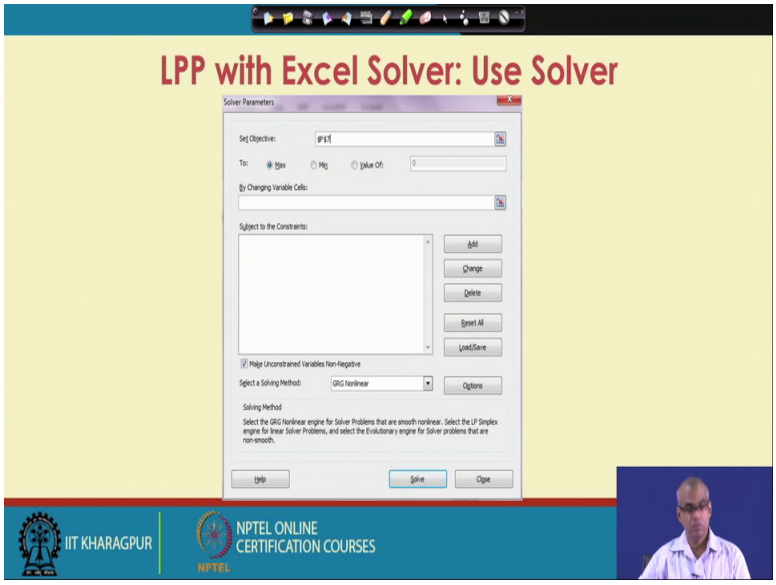

So, this is how the dialog box for the newer version of the Excel Solver will look like.

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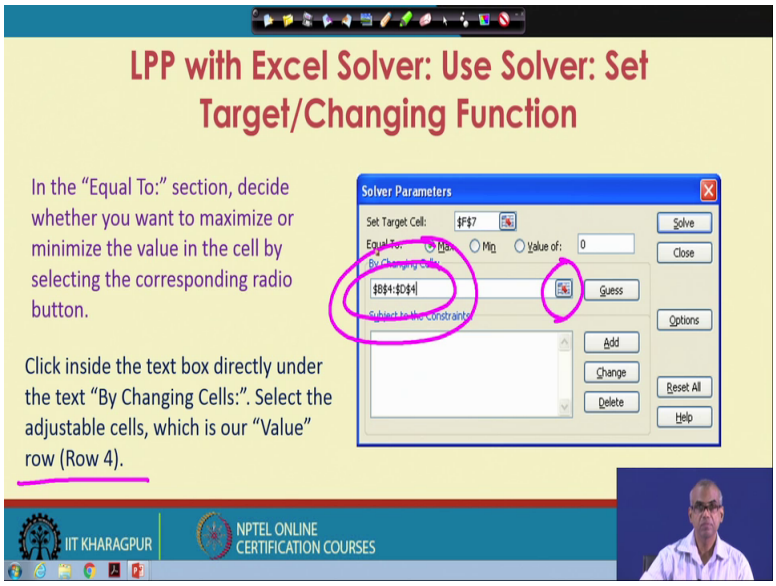

Again look at the dialog box. In the Equal To section, decide whether you want to maximize or minimize the value in the cell by selecting the corresponding radio button. We are maximizing our problem. So, you select Max radio button.

Click inside the text box directly under the text By Changing Cells. Select the adjustable cells which is our Value row, row 4. Remember, the Row 4 or the row with objective value under column A contains the adjustable cells, the variables x 1, x 2 and x 3. So, that is B4, C4 and D4. You can actually click on this red button and then click on B4, drag it up to D4, release the button, you will get this.

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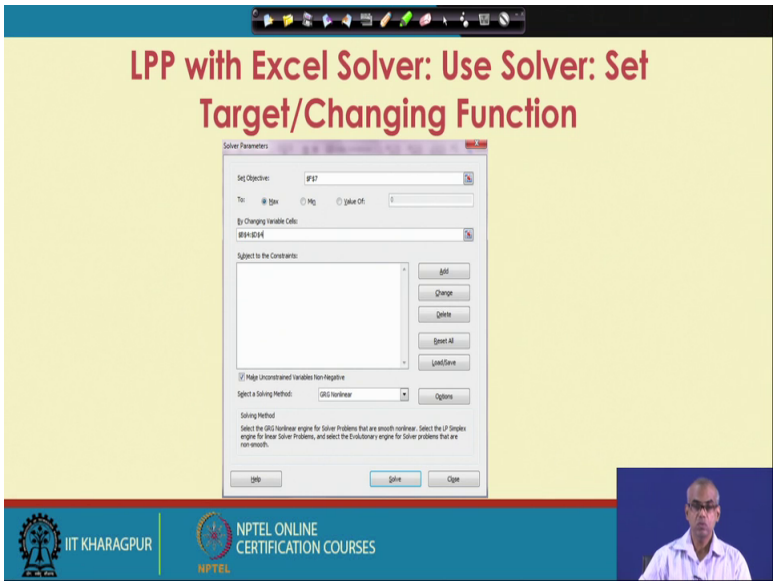

So, this is how the newer version will look like.

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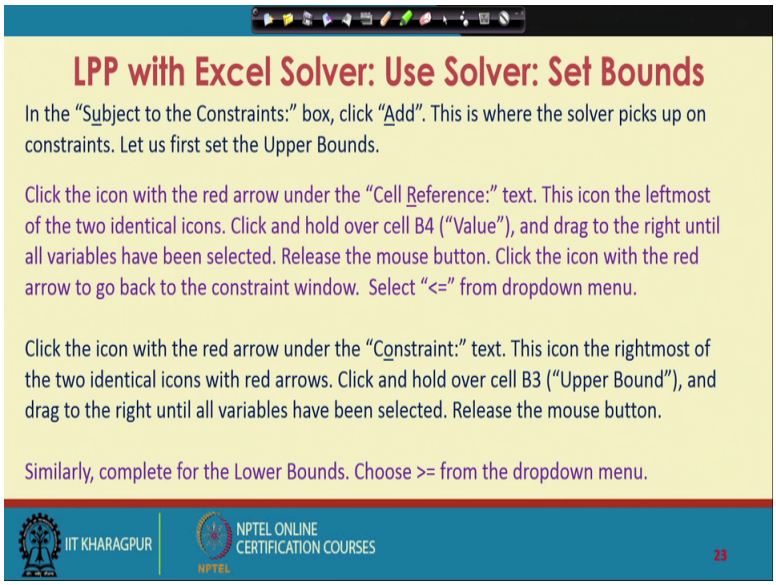

Now, we have to set the bounds. Actually, this will be done by setting constraints. If you look at here there is Add button in both the versions. So, this, Add button is used for selection of the constraints or for entering the constraint to the solver. So, let us first see how do we set the bounds.

In the Subject to the Constraints box click Add. This is where the solver picks up the constraints. Let us first set the Upper Bounds. Click the icon with the red arrow under the

Cell Reference text. This icon is the left most of the two identical icons. Click and hold over cell B4, that is the row with the objective Value and drag to the right until all variables have been selected. Release the mouse button. Click the icon with the red arrow to go back to the constraint window. Now, we have to indicate whether this is less or equal to, greater or equal to or equal to type constraint. This is less or equal to type. So, we select less or equal to from the drop down menu.

Click the icon with the red arrow under the Constraint text. This icon, the rightmost of the two identical icons with red arrows. Click and hold over cell B3 that is the label with Upper Bound and drag to the right until all variables have been selected. Release the mouse button.

Similarly, complete all the lower bounds. Similarly, we will complete for the Lower Bounds. While you do for the Lower Bounds you have to choose greater or equal to from the dropdown menu. Note that the Upper Bound is something like less or equal to type constraint, but Lower Bound is greater or equal to type. So, that is why you have to choose the appropriate sign; less or equal to or greater or equal to from the dropdown menu.

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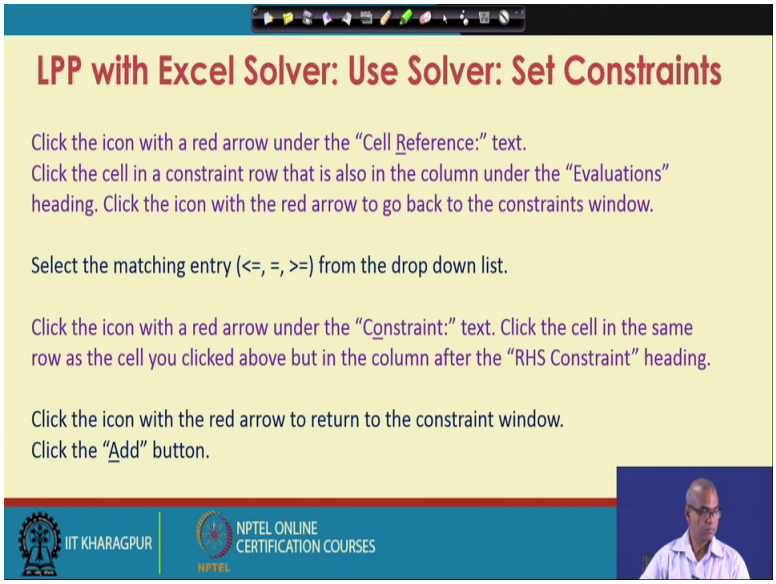

So, now in a very much similar way we have to set the constraints. Click the icon with a red arrow under the Cell Reference text, before that you must have put (Refer Time: 29:03) type add. So, click the icon with the red arrow under the Cell Reference text. Click the cell in a constraint row that is also in the column under the Evaluations heading. Click the icon with the red arrow to go back to the constraints window.

Select the matching entry less or equal to, equal to or greater equal to from the dropdown list. Click the icon with a red arrow under the Constraint text. Click the cell in the same row as the cell you clicked above, but in the column after the right hand side constraint heading. Click the icon with the red arrow to return to the constraint window. Click the Add button.

So, this way we can set the constraints. So, the all the constraints has to be set following the same procedure.

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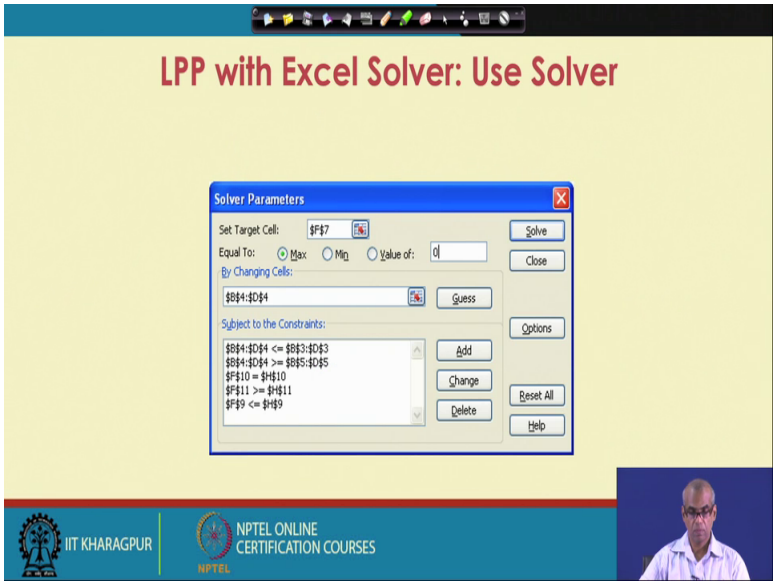

So, after you have put all the constraints your dialog box will look similar to what you see on the screen. This is for the older version.

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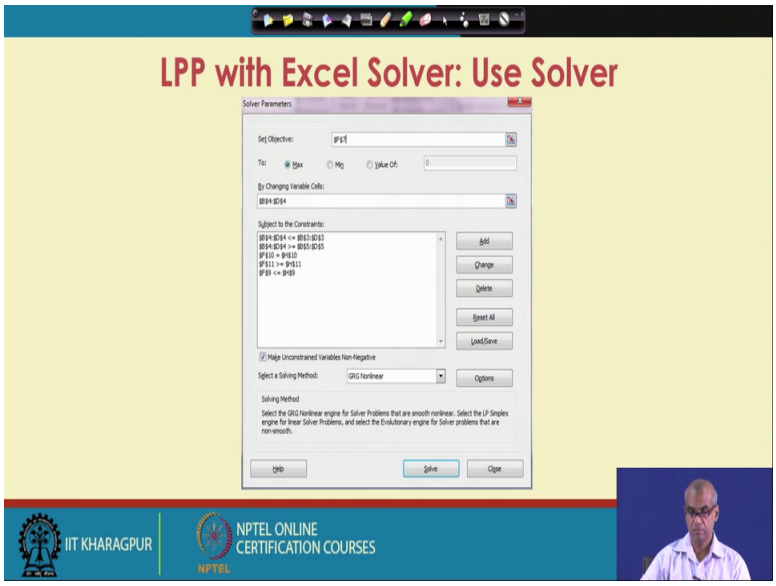

And, this is for the newer version.

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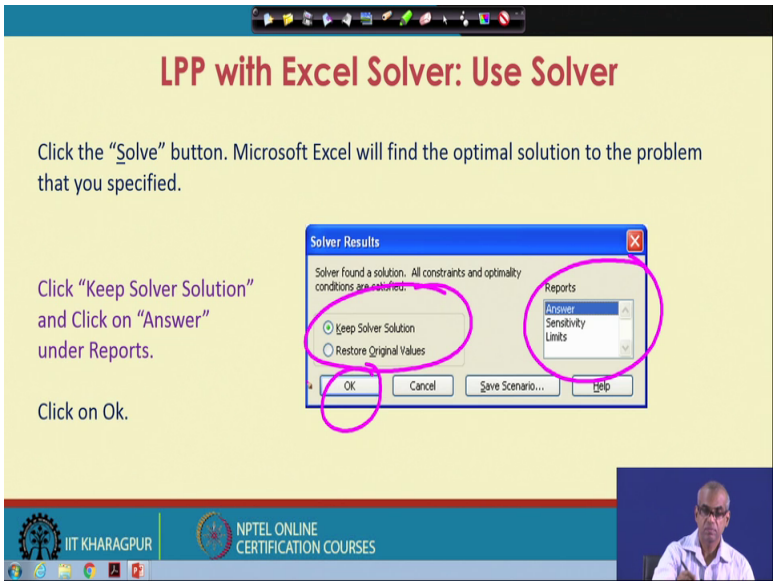

Now, click the Solve button. Microsoft Excel will find the optimal solution to the problem that you have specified. Click Keep Solver Solution option and also click Answer under Reports, then click Ok.

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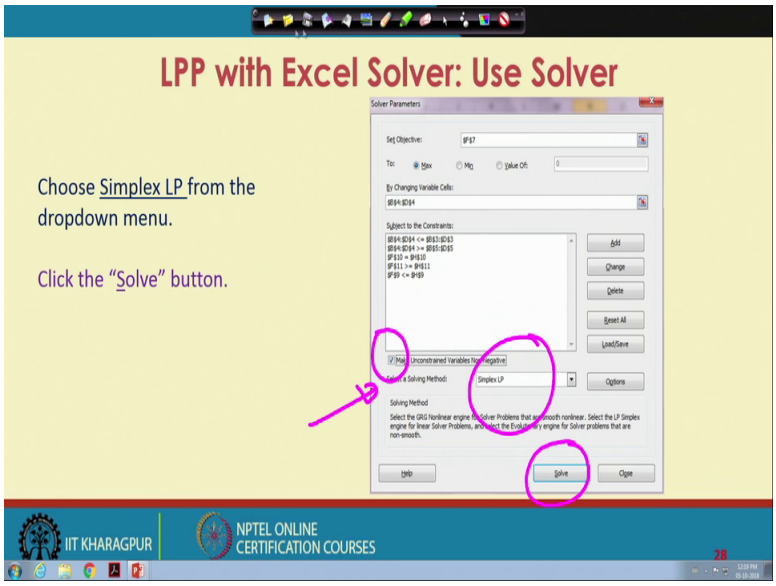

But, the newer version you choose Simplex LP from Select a Solving Method menu. So, choose Simplex LP from the dropdown menu then click Solve. You can check the box saying make unconstraint variables non negative.

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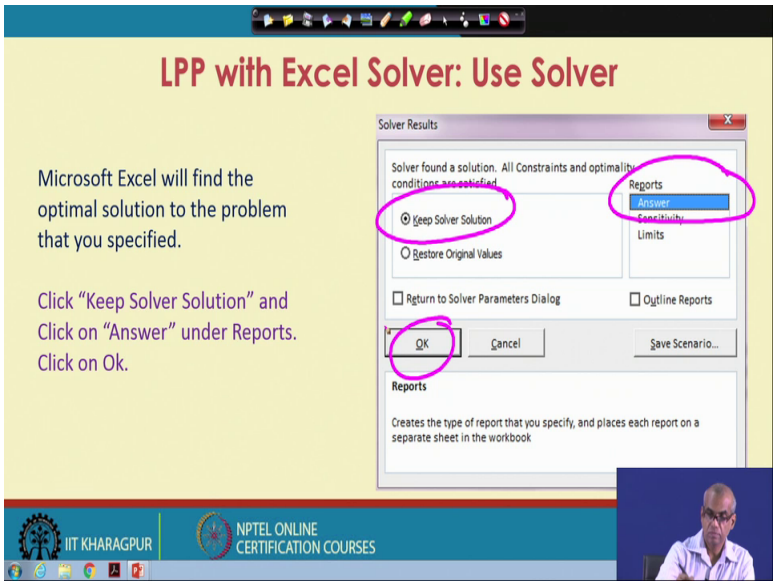

So, when you click solve Microsoft Excel will find the optimal solution to the problem that you specified. Click on Keep Solver Solution and click on Answer under Reports, then click on Ok.

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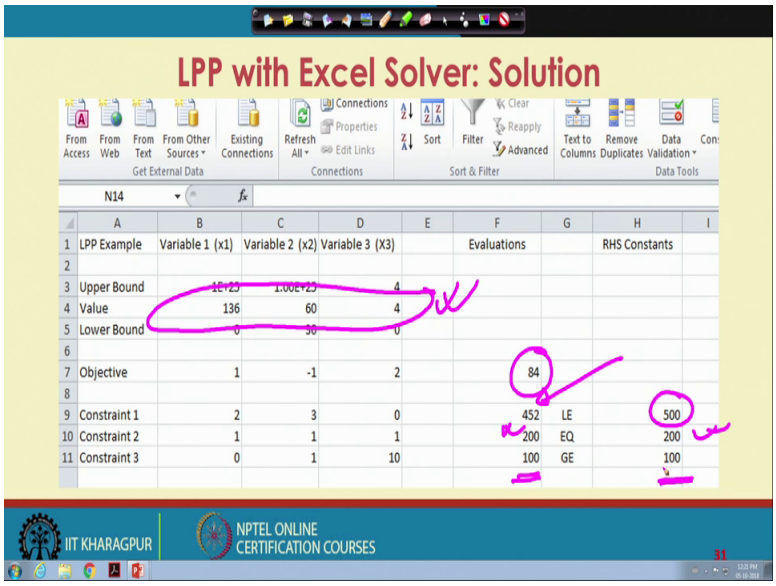

So, then this is the solution you will get. A zoomed version will be clear. So, the optimal values of x 1, x 2 and X 3 are 136, 60 and 4 respectively. Optimal value of the objective function is 84. Note that all the constraints are satisfied. First constraint was less or equal to 500. So, the value of the constraint for these x 1, x 2 and x 3 is 452 which is less than 500.

Second constraint was left hand side was equal to 200; so, satisfied. The third constraint was Constraint 3 was greater or equal to 100. So, that is also satisfied as equality.

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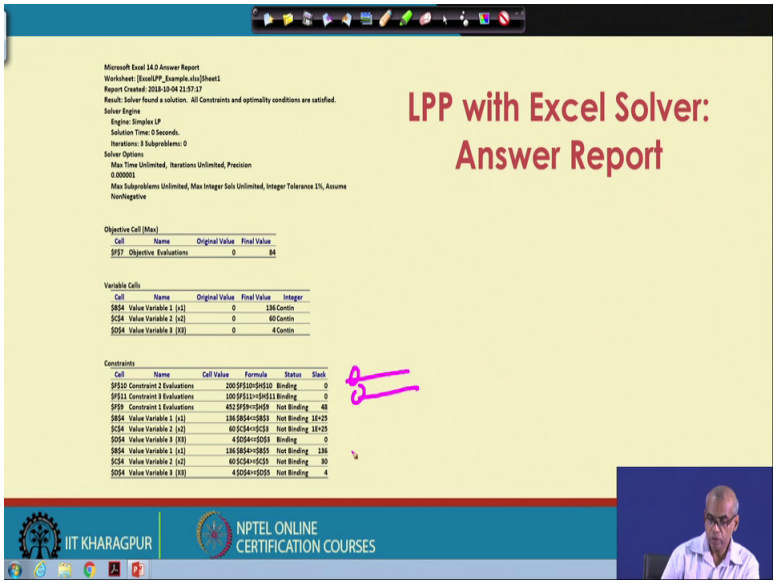

So, we have clicked on Answer Report. So, this is the Answer Report which is self explanatory contains the solutions and information's about the solution such as three iterations were required. It tells you that which constraints were binding which are not etcetera.

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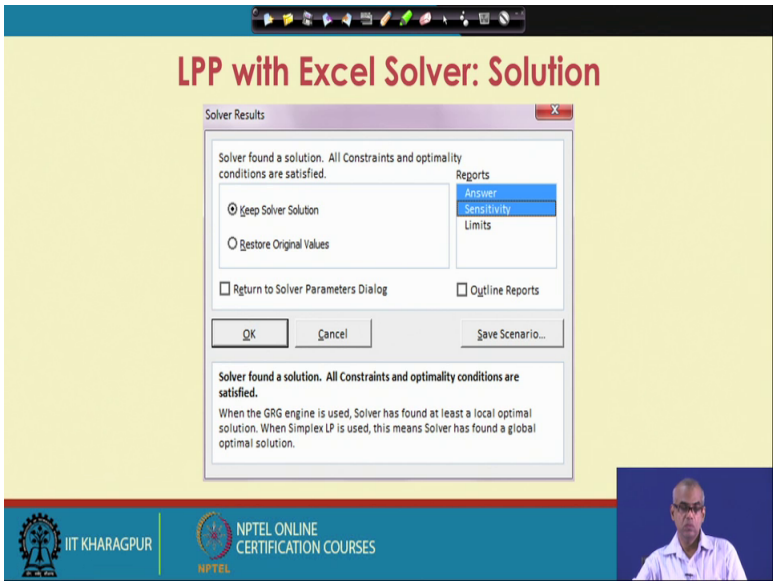

Now, apart from Answer under Reports you also have Sensitivity. You can also click on Sensitivity to get a sensitivity report.

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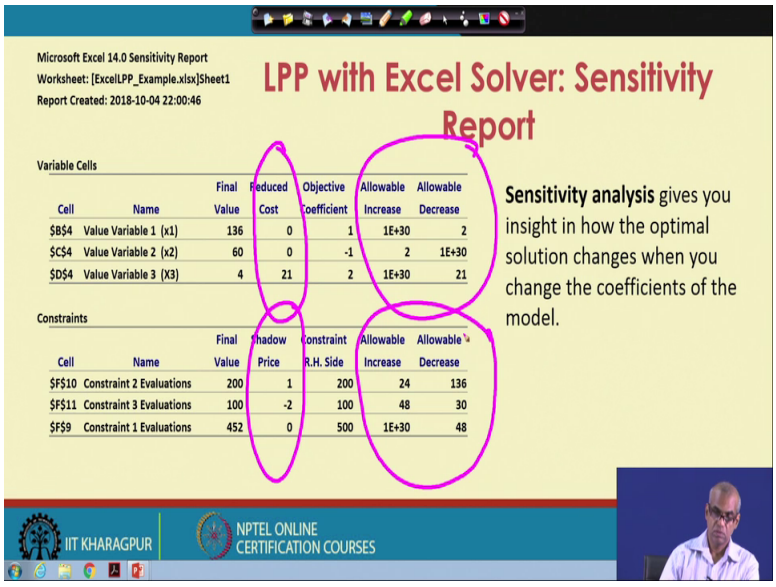

And, this is what will get. Sensitivity analysis gives you insight in how the optimal solution changes when you change the coefficients of the model. So, if you change certain data in the model, how the objective function value optimal values of the objective function will change that insight will get from sensitivity analysis. There are three important things to look at; Reduced Cost, Allowable Increase, Allowable Decrease and the Shadow Price also Allowable Increase and Allowable Decrease.

So, you have to study these to understand the sensitivity report and we will do this in the next lecture, when you demonstrate the solutions of these linear programming problem.

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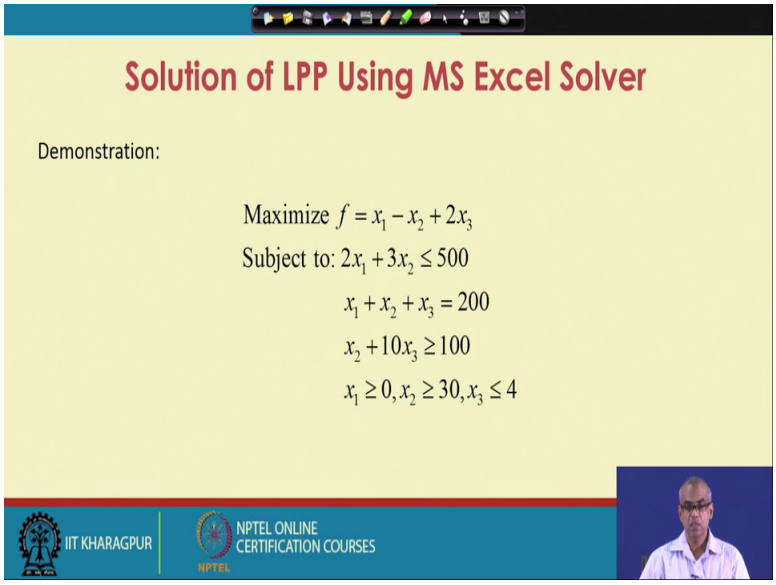

So, with this we stop lecture 58 here.