Optimization in Chemical Engineering
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Lecture – 44
Linear Programming: The Simplex Method (Contd.)

Welcome to lecture 44, in this week we are talking about simplest methods. As of now, we have demonstrated with two examples, the working of simplest methods. In today's lecture, we will discuss some special cases of simplex methods. So, we will discuss how to handle minimization problem as of now, we have talked about maximization problem. But how do you handle minimization problems? We will talk about how the simplex tableaus will indicate when you are going to get an unbounded solution or the presence of alternate solutions.

We have seen such cases when we used graphical method of solutions. We will see such special cases in the context of simplex method for solution of linear programming problems.

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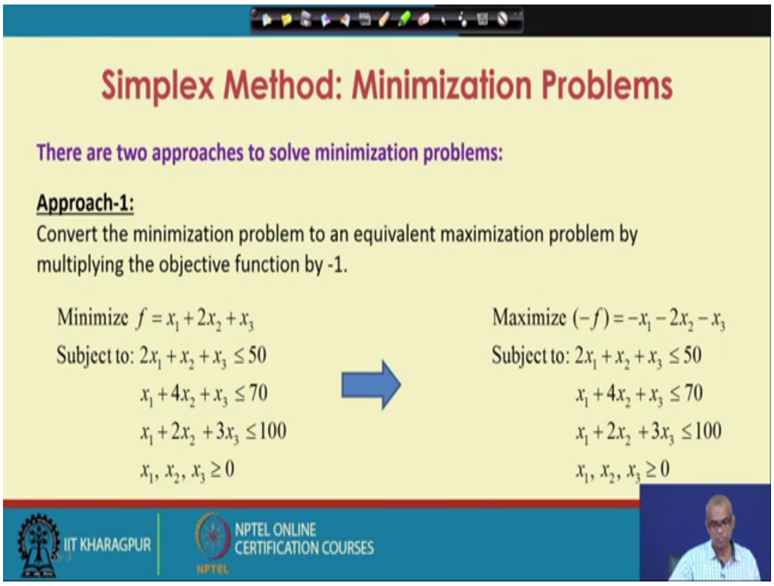

So, the first question, he asks, how do we solve a minimization problem? We have seen two important rules; one is inner product rule which helps us to determine which non basic variable will become basic variable.

The other important rule is minimum ratio rule which helps us to determine which basic variable will become non basic variable. So, how these rules will be changed whether there will be change or not? If there is a change what kind of change will there be? There are two approaches to solve a linear programming problem that is a minimization problem. First approach is straightforward; you can convert the minimization problem to an equivalent maximization problem by multiplying the objective function by minus 1.

So, all you have to do is to convert the minimization problem to an equivalent maximization problem by multiplying the objective function by minus 1. And then you treat it as maximization problem and follow the steps that we have demonstrated as of now. So, note that the minimize f equal to x 1 plus 2 x 2 plus x 3 becomes maximized minus f which is minus x 1 minus 2 x 2 minus x 3. Note that there want be any change in the constants.

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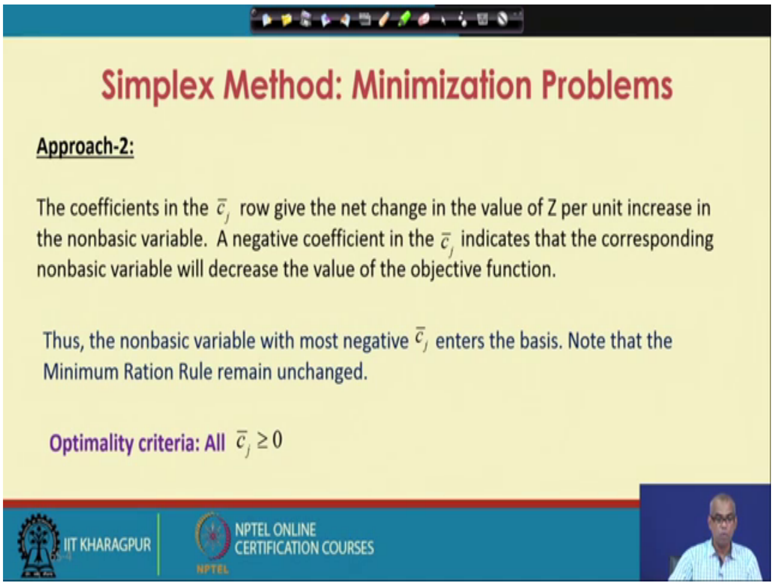

The second approach is actually about applying the inner product rule for a minimization problem. The coefficient in the C j bar row give the net change in the value of objective function that is z, but you need increase in the non basic variable that is what you have known as of now that the coefficients C j bar give the net change in the value of the objective function per unit increase in the non basic variable.

A negative coefficient in the C j bar indicates that the corresponding non basic variable will decrease the value of the objective function. A positive coefficient represents that the corresponding non basic variable will increase the value of the objective function. If C j bar equal to 0, it means that the corresponding non basic variable will not change the value of the objective function. Similarly, a negative value of C j bar will mean that the corresponding non basic variable will decrease the value of the objective function.

So, thus while solving a minimization problem, the non basic variable with most negative C j bar should be chosen to enter the basis. In case of maximization problem, we have seen that the non basic variable with most positive C j bar will enter the basis.

So, we can modify this rule appropriately for minimization problem such that the non basic variable with most negative C j bar will enter the basis. Note that the minimum ratio rule will remain unchanged because this rule is related to the feasibility of the variables concern, the variables has to be greater or equal to 0.

So, this condition is maintained when you change the non basic variable by this minimum ratio rule. So, minimum ratio rule remains unchanged, but the optimality criteria now changes. So, for maximization problem, it was all C j bar less or equal to 0. For minimization problem, it will be all C j bar greater or equal to 0. So, in a simplex tableau when all C j bar greater or equal to 0, it means that there is no other non basic variable which can be made basic variable. And further decrease the value of the objective function ; that means, we have reached the local optimal solution.

Since, the linear programming problem is a convex programming problem; the local optimal solution is also global optimal solution. So, the optimality criteria for minimization problem will be all relative profit coefficient values C j bar are greater or equal to 0.

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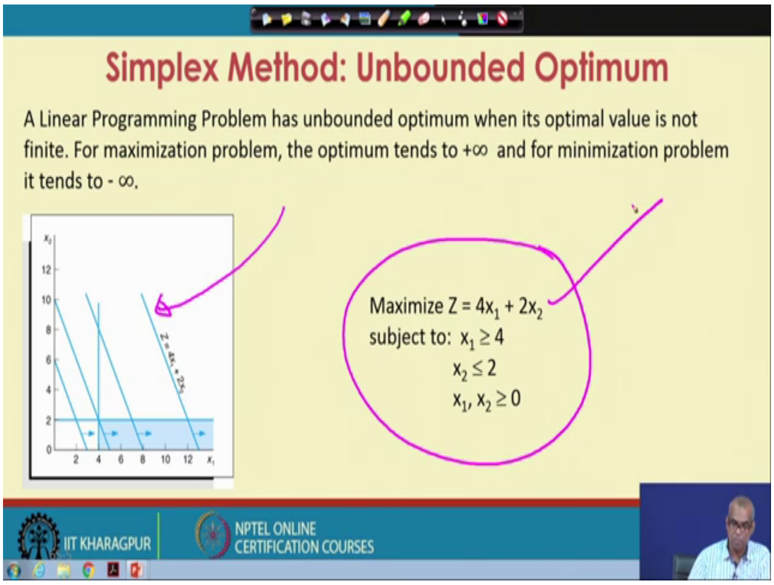

Next, let us consider unbounded optimum special case. We have discussed the unbounded optimum for linear programming problem when you discuss graphical solution method. A linear programming problem has unbounded optimum when its optimal value is not finite. For maximization problem the optimum tends to plus infinity and for minimization problem the optimum tends to minus infinity.

The figure shows that the given linear programming problem has unbounded optimum. This is the line corresponding to the objective function and you can check that you can go on moving the objective function line and increasing the value of the objective function infinitely. So, there is unbounded optimum. So, how is this indicated in simplex method?

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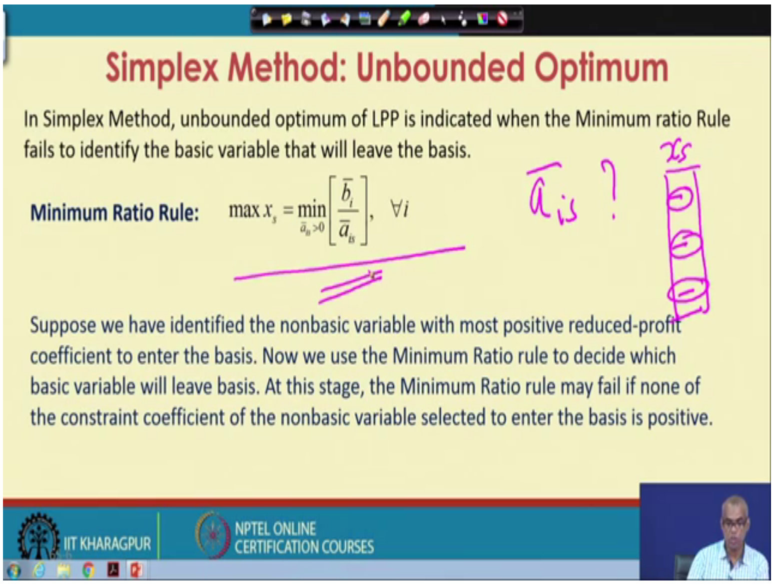

In simplex method, unbounded optimum is indicated when the minimum ratio rule fails to identify the basic variable that will leave the basis. In simplex method, unbounded optimum of linear programming problem is indicated when minimum ratio rule fails to identify the basic variable that will leave the basis.

Remember, the minimum ratio rule you basically divide bi bar by a is bar such that a is greater than 0. And you take the minimum ratio and that is the maximum increase in the non basic variable x s which will retain feasibility.

Suppose, you have identified the non basic variable with most positive reduced or relative profit coefficient C j bar to enter the basis. Now, we use the minimum ratio rule to decide which basic variable will lead basis. At this stage the minimum ratio rule may fail, if none of the constraint coefficient of the non basic variable selected to enter the basis is positive. So, to find out the minimum ratio, you evaluate bi bar divided by ai s bar. What is ai s bar? A is bar are the constant coefficients of the non basic variable selected to enter the basis.

So, a is bar are the constant coefficients for the non basic variable that has been chosen to enter the basis. Now, if all these constant coefficients are negative, then I cannot evaluate this ratio because a is bar has to be greater than 0. So, minimum ratio rule fails when all the constant coefficients of the non basic variable selected to enter the basis are negative.

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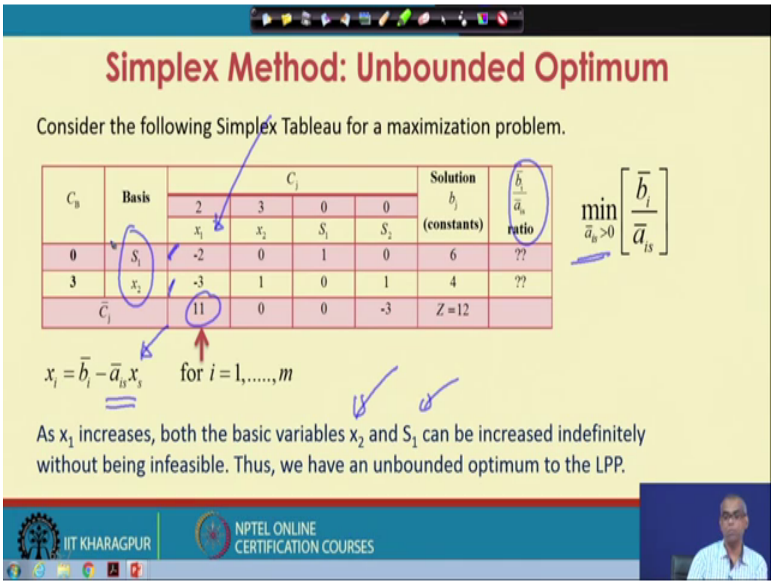

Consider the tableau shown the most positive C j bar is 11 corresponding to non basic variable x 2, sorry, x 1.

So, the most positive C j bar is 11 and this is corresponding to the non basic variable x 1. Now, the constant coefficients, in this x 1 column are both negative. So, I cannot find this ratio, minimum ratio cannot be found because a is not greater than 0. So, what it means is, if a is negative, as we increase the non basic variable the values of the basic variables will increase indefinitely without being infeasible; that means, the objective function value can be increase indefinitely.

In this particular case as I increase x 1, both the basic variables x 2 and S 1 can be increased indefinitely without making them infeasible; that means, I can infinitely increase x 1 to make the objective function value larger and larger because x 2 and x 1 will be increase and they will still remain feasible.

So, the linear programming problem has an unbounded optimum. So, an unbounded optimum, it will be indicated when the minimum ratio rule fails because the constant coefficients under the non basic variable that enters as basis are all negative.

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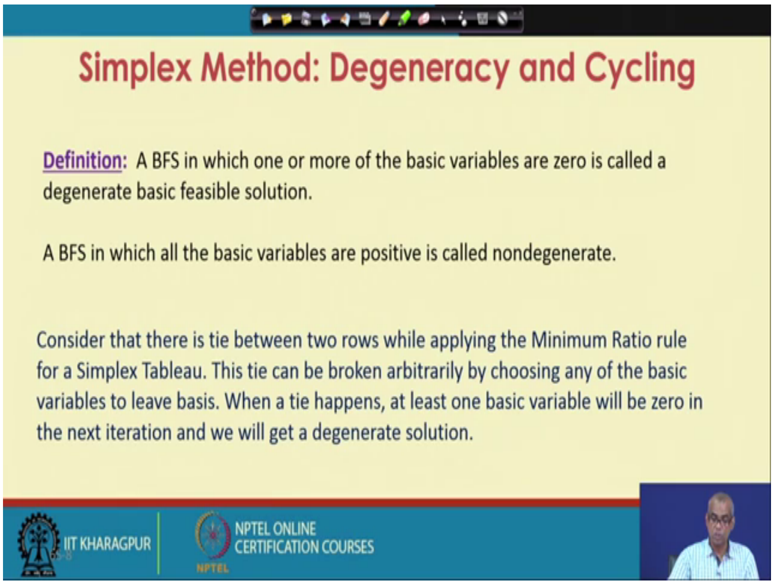

Next, you talk about another special case degeneracy and cycling. A Basic Feasible Solution in which one or more of the basic variables are zero is called a degenerate basic feasible solution. A Basic Feasible Solution in which one or more of the basic variables are zero is called a degenerate basic feasible solution. A Basic Feasible Solution in which all the basic variables are positive is called non degenerate. So, in case of degenerate basic feasible solution, there is at least one basic variable whose value is zero and in case of non degenerate solutions all the basic variables are positive.

Consider that there is a tie between two rows while applying the Minimum Ratio rule for a Simplex Tableau. This tie can be broken arbitrarily by choosing any of the basic variables to leave the basis. So, when there is a tie between two rows; that means, the value of the ratios are same for two rows. So, you can arbitrarily choose any one of these two rows to break the tie. But when a tie happens at least one basic variable will be zero in the next iteration. And then in the next iteration, we are going to get a degenerate solution.

So, once again if there is a tie between two rows while applying the Minimum Ratio rule, the tie can be broken arbitrarily by choosing any of the basic variables to leave basis. But when a tie happens at least one basic variable will be zero in the next iteration. What it means is that in the next iteration, we will get degenerate solution.

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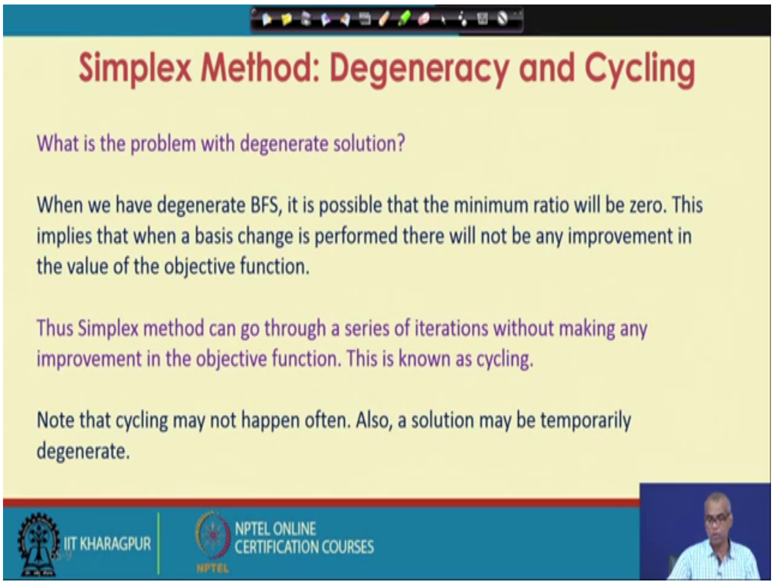

So, what is the problem with degenerate solutions? When we have, degenerate basic feasible solution, it is possible that the minimum ratio will be zero. This implies that when a basis change is perform, there will not be an improvement in the value of the objective function. So, and a when we have a degenerate basic feasible solution, it is possible that the minimum ratio will be zero.

This implies that when a basis change is performed there will not be any improvement in the value of the objective function. Thus, simplest method can go through a series of iterations without making any improvement in the objective function. So, it will be considered as if the simplex has entered a loop and from one iteration to another there is no change in the objective function value. So, Simplest method is going through a series of iterations without improving the objective function value, this is known as cycling.

Fortunately, cycling may not happen all the time, it does not happen often. Also, a solution may be temporarily degenerate.

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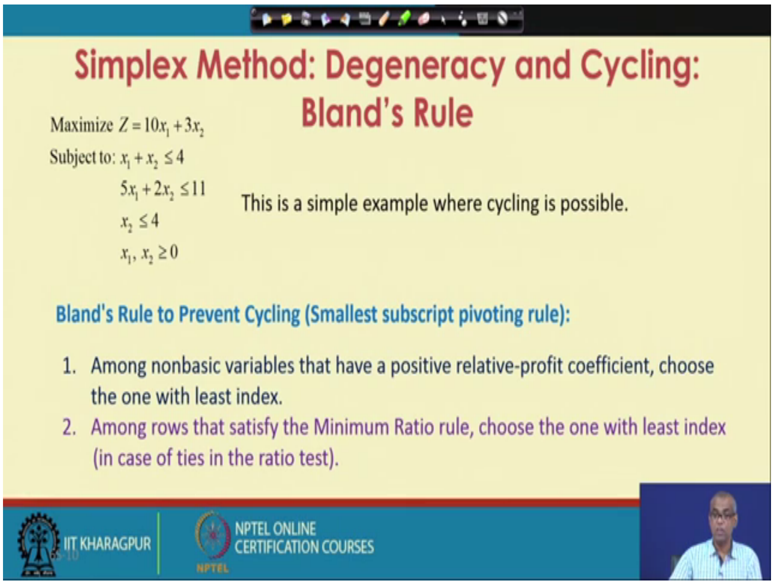

The example that you see is an example where cycling is possible, it is a simple example, but cycling is possible. Now, there is Bland's rule to prevent cycling, this is also known as smallest subscript pivoting rule. If we follow Bland's rule during pivoting, you will be able to prevent cycling. It can be proved that if we follow Bland's rule, your simplex will converge in a finite number of steps; that means, there will be finite number of Tableaus and you will reach the optimal solution in finite number of steps, you will not enter in an infinite loop.

So, Bland's rule states among non basic variables that have a positive relative profit coefficient choose the one with list index. So, among non basic variables that have a positive relative profit coefficient, choose the one with list index. Among rows that satisfy the Minimum Ratio rule, choose the one with list index, in case of ties in the ratio test. Among rows that satisfy the minimum ratio rule, choose the one with list index, in case of ties in the ratio test.

So, basically the Bland's rule to prevent cycling is about choosing smallest sub script during pivoting.

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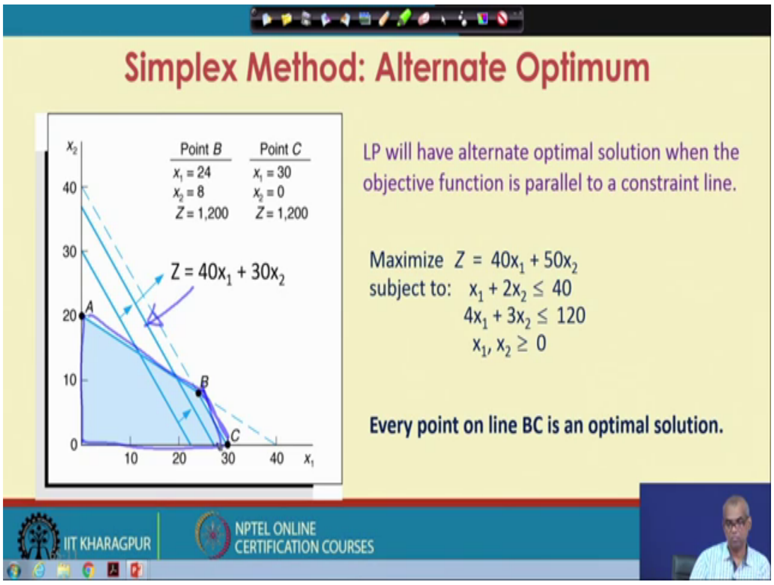

Next, this talk about the case where Alternate Optimal solution is possible in case of graphical solution of linear programming problem, we have seen that a linear program will have alternate optimal solution. When the objective function is parallel to a constraint line. So, in the objective function is parallel to a constraint line, the linear program will have alternate optimal solutions.

So, the figure shows that the objective function line is parallel to line BC. The feasible solution space, the feasible solution space for the linear programming problem is obtained by points OABC. And it is noted that the objective function line is parallel to the line BC, it means that every point on line BC is an optimal solution. So, you have alternate optimal solution. So, how is this indicated in Simplex Tableau?

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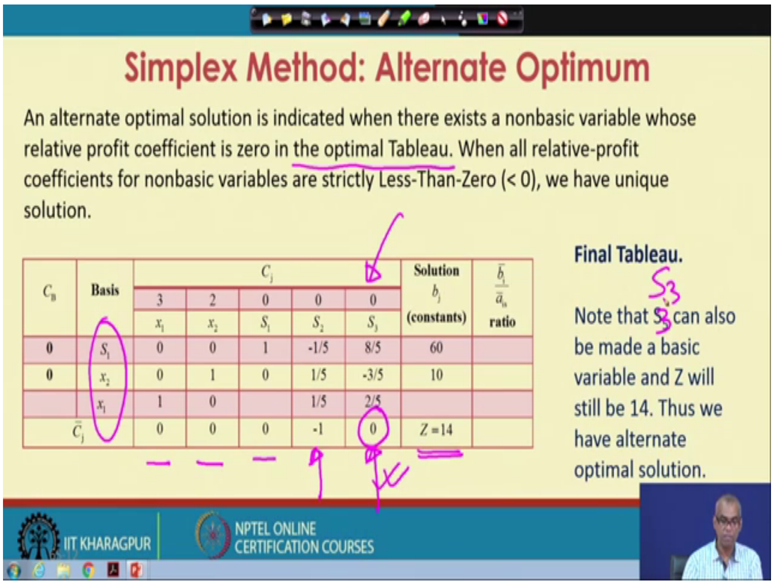

An alternate optimal solution is indicated when there exists a non basic variable whose relative profit coefficient is zero in the optimal Tableau. When all relative profit coefficients for non basic variables are strictly less than 0 we have unique optimal solution. But an alternate optimal solution is indicated when there exists a non basic variable whose relative profit coefficient is zero in the final optimal Tableau that is important what we are talking about final optimal Tableau.

So, this is an example of final optimal Tableau, your non basic variables are S 2 and S 3, x 1 x 2 and S 1 are basic variable. Now for the non basic variable S 3, the relative profit coefficient value C j bar is 0. So, if C j bar value is 0, it means that if that non basic variable is made basic variable, there will not be any change in the objective function because C j bar represents the net change in the value of the objective function when the non basic variable is changed.

So, if C j bar for the non basic variable S 3 is 0, it means that it S 3 is made basic variable, the objective function value will still the 14. So, it means that there is alternate optimal solution.

So, we can make S 3 a basic variable and that will not change the objective function value, objective function value is still remain 14. So, thus we have alternate optimal solution.

So, alternate optimal solution is indicated, if C j bar value in the final optimal tableau for a non basic variable is 0. In the given example C j bar for S 3 is 0. So, if S 3 is made basic variable, the objective function value still remains 14 which is optimal value. So, I have alternate optimal solution.

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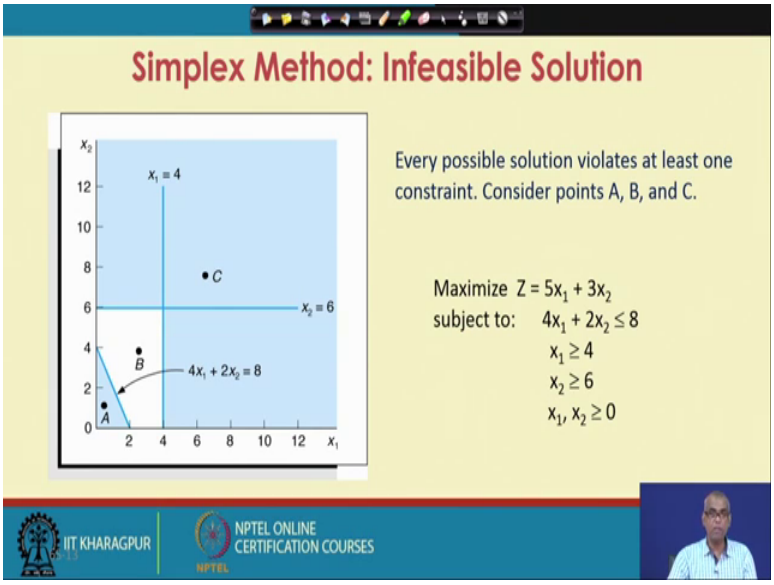

Next let us talk about infeasible solution. Every possible solution violates at least one constraint. Consider points A, B and C. Every possible solution violates at least one constrain. So, what you see is an example of infeasible LPP, Infeasible Linear Programming Problem, it does not have any solution.

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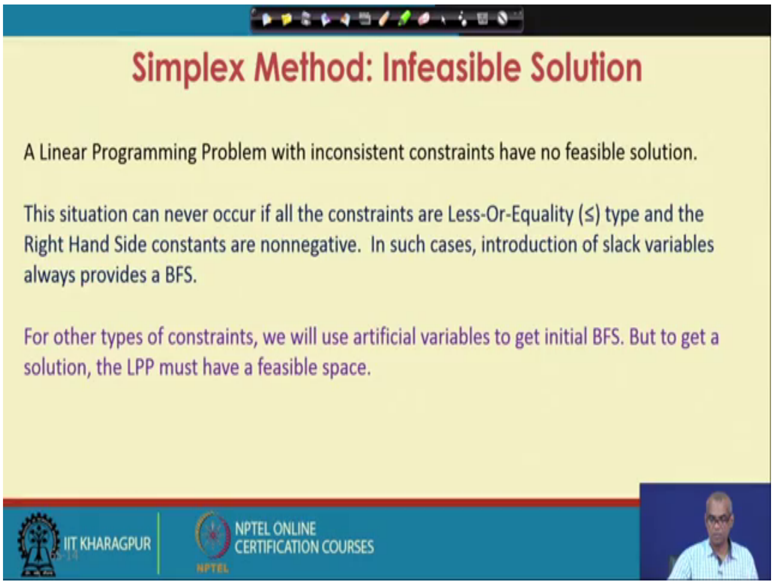

So, a Linear Programming Problem with inconsistent constraints have no feasible solution. A Linear Programming Problem with inconsistent constraints have no feasible solution. This situation can never occur if all the constraints are less or equality type and the right hand side constants are non negative because in such cases, we have already seen that introduction of slack variables always provides a basic feasible solution.

For other type of constraints, we will use artificial variables to get initial basic feasible solution. How to use artificial variables to get initial basic feasible solution, we learn in the next class. But to get a solution the linear programming problem must have a feasible space. So, a linear programming problem with inconsistent constants will have no feasible space and thus no feasible solution. With this we will stop lecture 44 here.