Optimization in Chemical Engineering
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Lecture – 43
Linear Programming: The Simplex Method (Contd.)

Welcome to lecture 43 or week 9. In this week, we are talking about Simplex Method for Linear Programming Problems. In a previous lecture, we have taken an example and demonstrated the steps for simplex method. In today's lecture, we will continue our discussion on simplex methods we will now take another example in three variables and three equations and we will solve the problem using simplex method.

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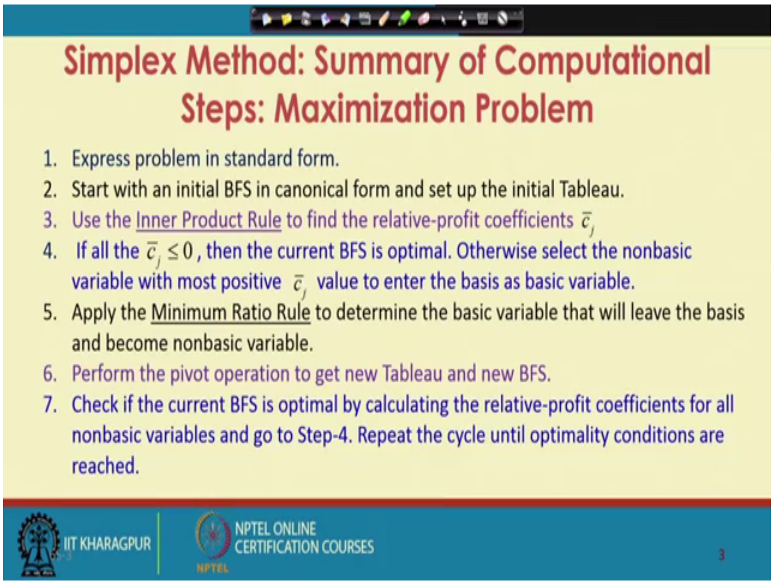

So, before that, let us summarize the computation steps that we have seen already and we are talking about a maximization problem. So, the first step towards solving a linear programming problem using simplex method is to express the problem in standard form. In standard form, all the constants will be of equality type, the right hand side constant will be non negative and all the decision variables are also constrained to be non negative.

In the next step, we start with an initial basic feasible solution in canonical form and set up the initial tableau. In the third step, use the inner product rule to find the relative profit coefficients that is C j bar for all non basic variables. Note that for basic variables, the relative profit coefficient C j bar is always 0; if all C j bar is less or equal to 0, then the current basic feasible solution is optimal.

So, when all the relative profit coefficients are less or equal to 0, then the current basic feasible solution is optimal; otherwise select the non basic variable with most positive C j bar that is relative profit coefficient and these non basic variable will enter the basis as basic variable. So, the non basic variable with most positive C j bar value enters basis as basic variable. So, one basic variable has to leave the basis and will become non basic variable.

For that, we apply the minimum ratio rule to determine the basic variable that will leave the basis and become non basic variable. Perform the pivot operations to get new tableau and new basic feasible solution, check if the current basic feasible solution is optimal by calculating the relative profit coefficients for all non basic variables and go to step 4, where you check whether all the relative profit coefficients are less or equal to 0. Repeat the cycle until optimality conditions are reached and the optimality condition for maximization problem is for all non basic variables the relative profit coefficients C j bar will be less or equal to 0.

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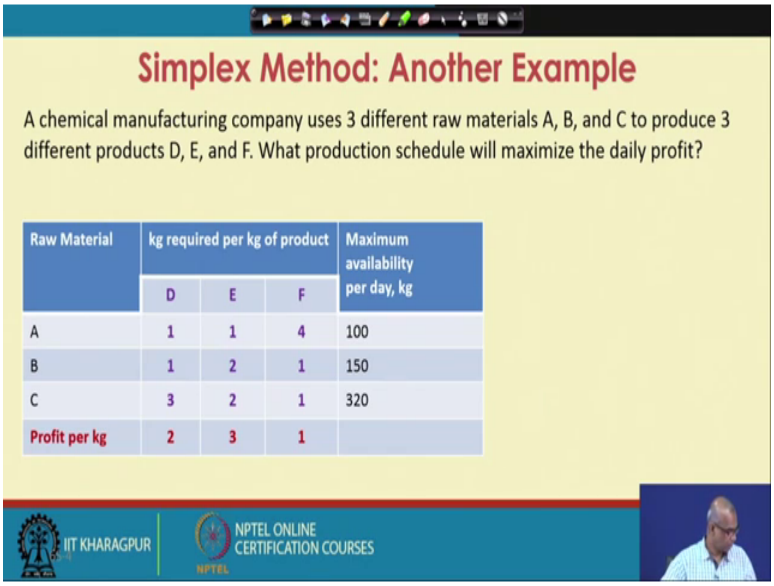

Now, let us take another example. A chemical manufacturing company uses 3 different raw materials A, B and C to produce the different products D, E and F. So, there are 3 different raw materials. Using these 3 different raw materials, 3 different products can be form D, E and F. We are not talking about reaction kinetics here, but how much of A and how much of B and how much of C are required to produce each kg of D, E and F are given in the table.

For example, for each kg of product D, 1 kg of A, 1 kg of B and 3 kg's of C are required; similarly, for E and F. This is a hypothetical situation, but we consider there these informations are true and we will formulate the optimization problem, so that we can get the optimal production schedule that will maximize the daily profit. The maximum availability per day for each of the raw materials 100 kg, 150 kg, 320 kg are given. Profit per kg of D, E and F are also given.

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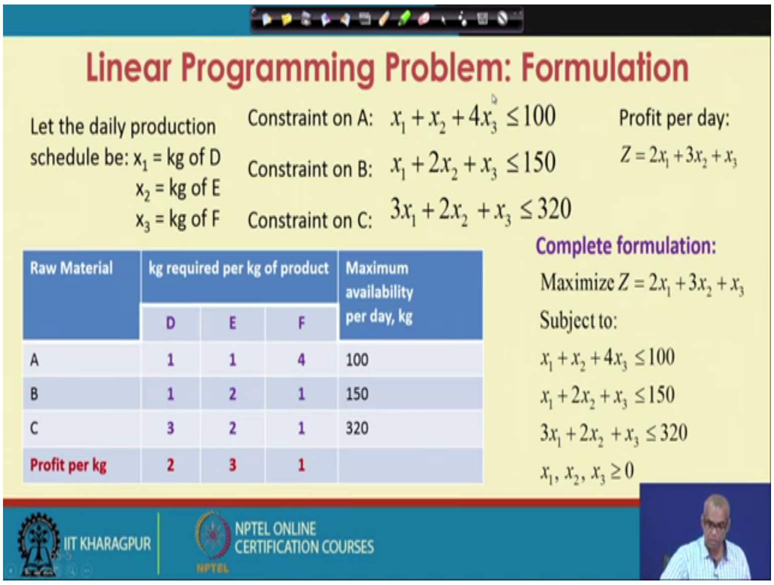

So, now, let us formulate the problem. Let us consider that the production schedule is will produce x 1 kg of D, x 2 kg of E and x 3 kg of F. Now, there are three constraints on the availability of raw materials A, B and C. We are considering we will produce x 1 kg of D, x 2 kg of E and, x 3 kg of F. To produce x 1 kg of D, we need x 1 kg of A; to produce a x 2 kg of E, we need x 2 kg of A and to produce x 3 kg of F, we need 4 x 3 kg of A.

So, x 1 plus x 2 plus 4 x 3 must not exceed 100. So, there is the first constraint that we get. Similarly, we get constraints on B which will be x 1 plus 2 x 2 plus x 3 is less or equal to 150. Similarly 3 x 1 plus 2 x 2 plus x 3 is less or equal to 320. So, these are the three constraints.

So, now we have formulated the three constraints. Let us write down the objective function. The objective function is to maximize the profit. So, x 1 kg of D will give 2 x 1 kg of 2 x 1 amount of profit plus 3 x 2 plus x 3 is the total amount of profit. So, Z equal to 2 x 1 plus 3 x 2 plus x 3 is the objective function, we in to maximize this. So, the complete formulation now is maximize Z equal to 2 x 1 plus 3 x 2 plus x 3 subject to these three inequality constraints.

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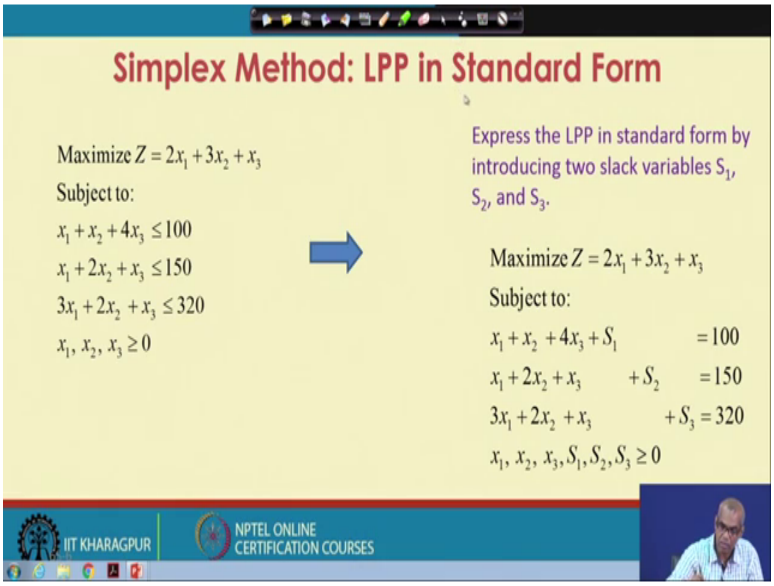

So, now to solve this problem using L P using simplex method, the first thing that we have to do is to express the problem in standard form. Note that, there are three inequality constraints. So, we have to add 3 slack variables. So, you have to add 3 slack variables, S 1, S 2 and S 3 to convert this linear programming problem to this linear programming problem which is in standard form

So, if you look at this, you see that this part helps you to start with an initial basic feasible solution. So, S 1 and S 2 and S 3 becomes basic variable x 1, x 2, x 3 becomes non basic variable and you have a basic feasible solution in hand to start the initial Tableau.

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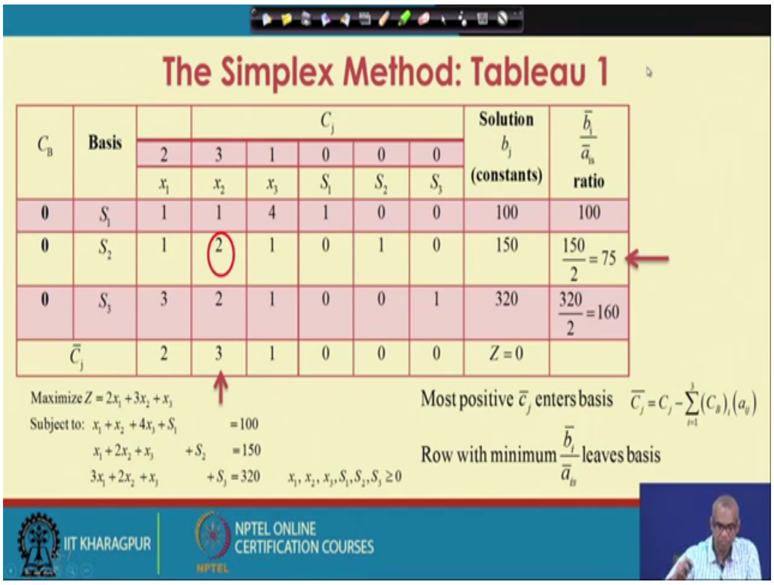

So, this is what you get as Tableau 1. So, S 1, S 2 and S 3 are the basic variables, their cost coefficients are all 0 and x 1, x 2, x 3 are the non basic variables. So, you can check this that is in canonical form and x 1, x 2, x 3 are non basic variables and the coefficients of the constants are written. This column represents the right hand side constants which are 100, 150 and 320.

Now, the first step is to compute the relative profit coefficients C j bar we know for basic variables C j bar equal to 0. So, you can directly put 0, 0 and 0 here corresponding to a basic variables S 1, S 2 and S 3. So, C j bar is calculated using this equation. C j bar equal to C j minus sigma C B i a i j. Now, note C B i is basically the cost coefficients of basis S 1, S 2 and S 3. So, they are all 0. So, in the first table C j bar is basically equal to C j here for the non basic variables x 1 and x 2. So, that is why you get 2 and 3.

Now, let us look at which C j bar is most positive because that will enter as basic variable into the basis. So, you see that 3 is most positive. So, x 2 enters basis, x 2 non basic variable will enter as basic variable. So, which basic variable will leave for that we have to compute the ratio b i bar divided by a is bar. So, b i bar a i bar S ratio we have to find out. So, what we do is corresponding to non basic variable column x 2, we divide b j or b j bar by this coefficients.

So, focus your attention here, here and now look at the minimum ratio column. So, this is 100 divided by 1 is 100. This is 150 divided by 2 is 75, 320 divided by 2 is 160. So, this is how you compute minimum ratio. So, once the minimum ratio has been computed, we see that 75 is the minimum corresponding to second row. So, S 2 will leave basis and x 2 will enter. So, x 2 enters, S 2 leaves.

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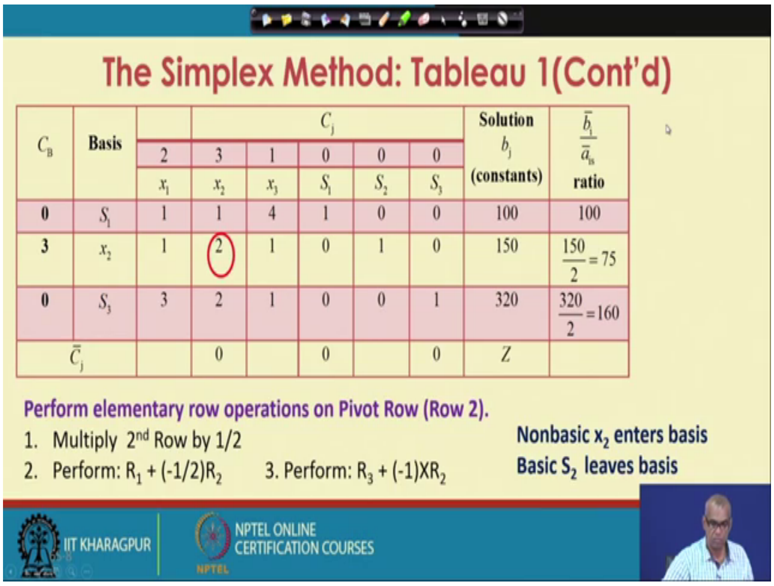

So, the pivot raw is row number 2 and the pivot element is circled which is 2 here. You see that x 2 has replaced S 2. So, now, we have to perform elementary row operations, so that we have one here and 0 elsewhere in this column so that you have the canonical form. So, this can be obtained by multiplying the second row by half. For this first column, you have to perform sorry for this first row you have to perform first row plus minus half into second row and for this, you have to add to this third row, the second row multiplied by minus 1. So, once you do this, you will have one here and 0 in these two places.

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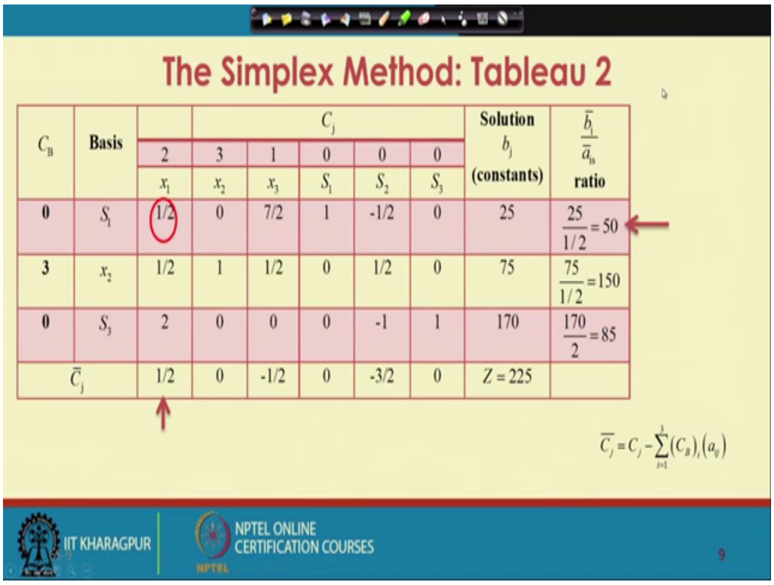

So, this is what we will get in tableau 2. Note, we have 1 here and 0 elsewhere in the column. Next we compute C j bar, note that you are also performing elementary operations on these column for constant. So, you find out C j bar now for each of these non basic variables. So, what are the non basic variables, x 1, x 3 and S 2; C j bar for basic variables x 2, S 1 and S 3 are all set 0. So, using this formula we compute C j bar. So, C j bar can be computed as c j. So, let us find out for non basic variable x 1. So, this is 2 which is c j minus C B i a i j. So, 0 into half plus 3 into half plus 0 into 2; so, this is what this equation represents. So, this is basically 2 minus 3 by 2 which is equal to half. So, this is half. So, this is how you compute C j bar for non basic variable x 3 as well as S 2.

Now, you see that the most positive C j bar is half which corresponds to non basic variable x 1. So, x 1 will enter basis. So, which basic variable you will leave, for that we compute the ratio b i bar divided by a is bar. So, for that we take the divisions 25 by half is 50, 75 by half is 150, 170 divided by 2 is 85. So, minimum is 50. So, S 1 leaves basis. So, S 1 leaves and in place of S 1, x 1 enters.

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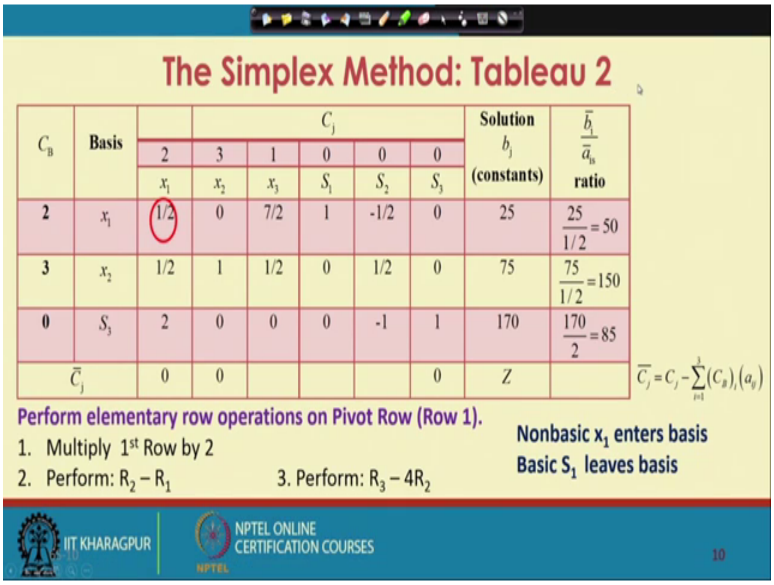

So, this is has been shown now, x 1 has entered basis and S 1 has left. So, the pivot row is row number 1 and the pivot element is half which has been circled in red. So, again we perform elementary row operations so that we get 1 here, 0 here and 0 here. So, this can be obtained by multiplying the first row by 2 by performing R 2 plus R 1 into minus 1; that means, R 2 minus R 1 will get this and by performing R 3 minus 4 R 2, you can make 0 here.

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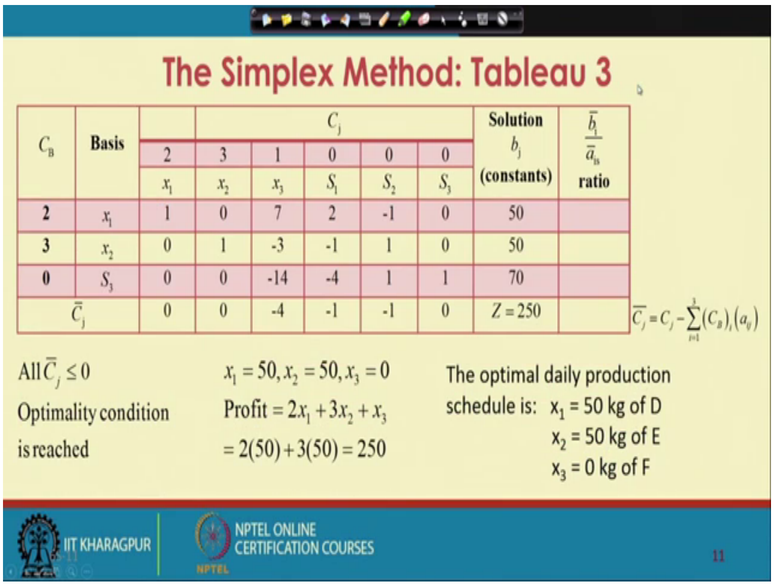

So, once you do this, you will get the cut tableau as shown. Note here 1, 0, 0. So, again we compute C j bar following this equation and we now find that all C j bar are less or equal to 0, there is no c j bar which is positive or greater than 0, all C j bar less or equal to 0. So, we have reached optimality conditions and at optimal condition, the basic variable x 1 is 50, the basic variable x 2 is 50, non basic variable S 3 which is slack variable is 70, but this does not participate in the computation of objective function Z. So, the profit is 2 x 1 plus 3 x 2 plus x 3, x 3 is 0 because x 3 is non basic variable at the optimal final w. So, profit is computed as 250.

So, the production schedule is that we produce 50 kg of D per day, 50 kg of E per day but do not produce any F. According to the data shown or whatever data we have considered, the optimal daily production schedule is 50 kg of D and 50 kg of E no F that will maximize the profit. Note that this optimal production schedule may change if we change the cost coefficients in the objective function.

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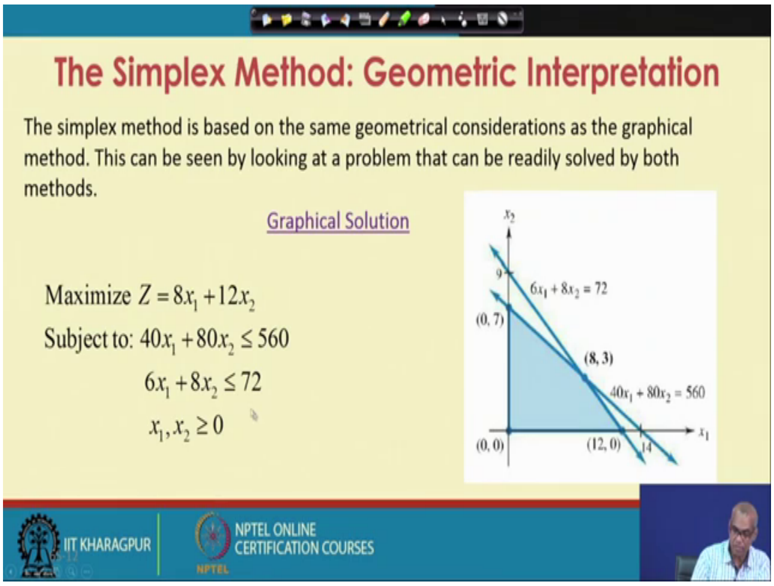

So, now let us get geometric interpretation to this simplex method. The simplex method is based on the same geometrical consideration as the graphical method. This can be seen by looking at a problem that can be readily solved by both the methods. So, if I take a two variable problem in two equations, this can be easily solved using graphical solution method, we can also solve that using simplex method. Then, we will be able to demonstrate that the simplex method is based on the same geometrical considerations as the graphical method.

So, consider the problem shown a simple linear programming problem in two variable and two constants. So, we can obtain the solution using graphical method and the solution is obtained as x 1 equal to 8, y equal to x 2 equal to 3, x 1 equal to 8 x 2 equal to 3. So, look at the corner points 0, 0 which is the origin one corner point of the feasible space. Feasible space is shaded in blue; 0, 7 is another corner point; 12, 0 is another corner point and 8, 3 is another corner point and 8, 3 happens to be optimal solution.

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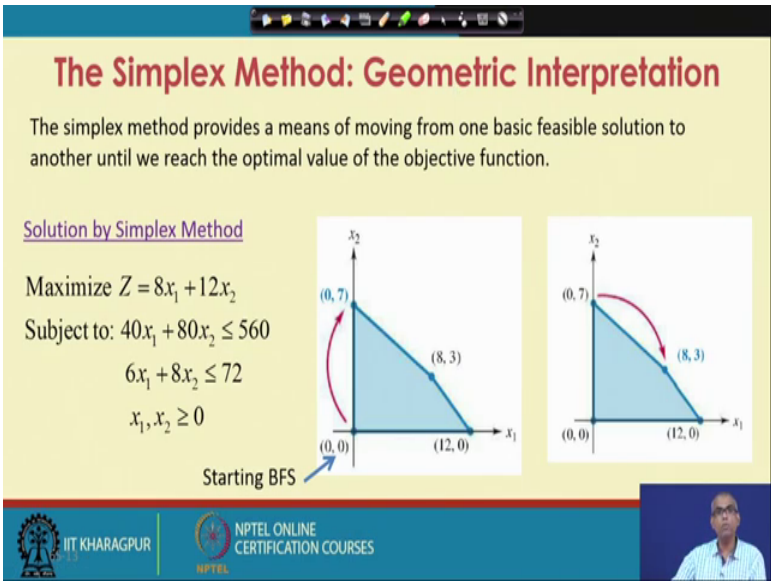

Now, if I solve the same problem using simplex method, I will introduce two slack variables here; S 1 and S 2. So, that the inequality constants become equality constants and S 1 and S 2 will be considered as basic variable x 1 and x 2 will be non basic variables. So, starting basic feasible solution will be x 1 equal to 0, x 2 equal to 0 which is origin.

So, from there, you will go to this corner point and from this corner point you will come to this; that means, the initial tableau you will have this as basic feasible solution. In the next tableau, you will have this basic feasible solution and in the final tableau or the third tableau, you will have 8, 3 as a basic feasible solution. So, basically the simplex method provides a means of moving from one basic feasible solution to another until we reach the optimal value of the objective function.

So, simplex method is an efficient way or moving from one basic feasible solution to another until we reach the optimal value of the objective function and the basic feasible solutions are always corner points of the feasible region.

So, with this we stop our lecture 43 here.