Optimization in Chemical Engineering
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Lecture - 42
Linear Programming: The Simplex Method (Contd.)

Welcome to lecture 42. In our previous lecture, we started our discussion on simplex method for solution of linear programming problem. So, we will continue our discussion on simplex methods. And in this lecture, we will take an example of linear programming problem and solve it using Simplex Method.

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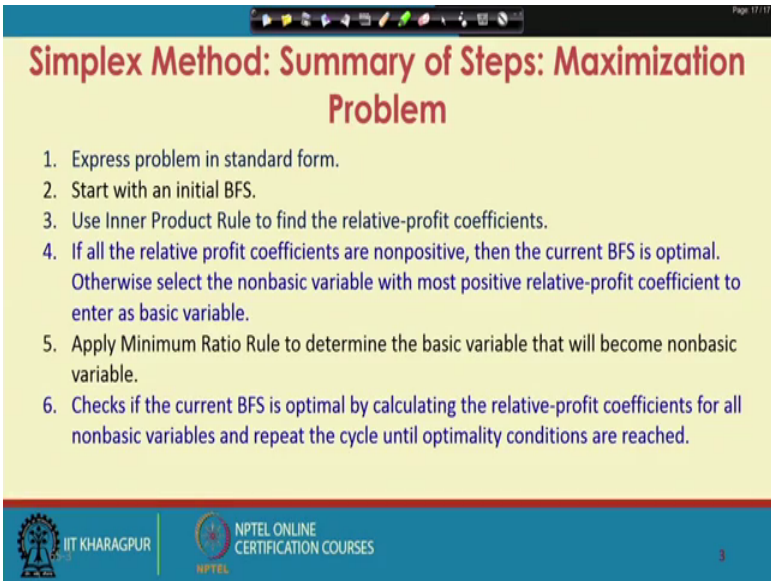

So, to solve a linear programming problem using simplex method, let us first look at the steps that are necessary. And we are talking about a maximization problem. First we express the problem in standard form, then we start with an initial basic feasible solution. So, to start with an initial basic feasible solution, your set of equations have to be converted to canonical form. Now, you use inner product rule to find the relative profit coefficients of all non-basic variables.

For basic variables, these values are 0. If all the relative profit coefficients are non-positive, then the current basic feasible solution is optimal. Otherwise select the non-basic variable with most positive relative-profit coefficient to enter as basic variable. Apply minimum ratio rule to determine the basic variable that will become non-basic variable. Check, if the current basic feasible solution is optimal by calculating the relative profit coefficients for all non-basic variables, and repeat the cycle until optimality conditions are reached.

So, basically you first express the linear programming problem in its standard form, and then you start with an initial basic feasible solution. We have to find out the relative profit coefficients using inner product rule for all non-basic variables. The non-basic variable with most positive relative profit coefficient we will enter as basic variable. And we apply minimum ratio rule to determine, which basic variable, which basic variable will be converted to non-basic variable.

And after this we will check if the current basic feasible solution is optimal by calculating the relative profit coefficients for all non-basic variables. If the relative profit coefficients for all non-basic variables are less or equal to 0, then you have optimal solution; otherwise we will repeat the cycle until optimality conditions are reached.

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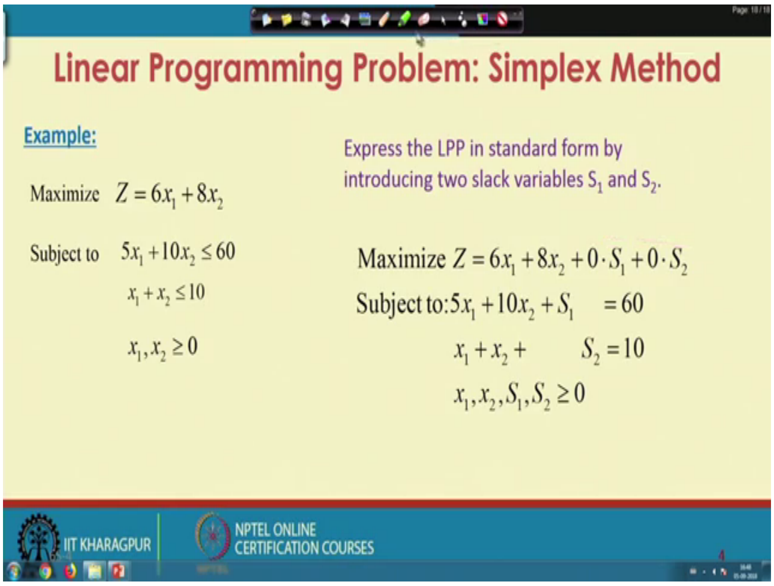

So, let us take this example. We are taking a simple example of two variables with two inequality constants. So, maximize Z equal to 6 x 1 plus 8 x 2 subject to 5 x 1 plus 10 x 2 less or equal to 60 x 1 plus x 2 less or equal to 10 and there are non-negativity restrictions on decision variables x 1 and x 2.

Clearly, the problem is not in a standard form. So, you have to express it in the standard form. So, you have to express the linear programming problem in standard form by introducing two slack variables S 1 and S 2, because we have to less or equal to type inequality constants. So, we have to add two slack variables S 1 and S 2, which are greater or equal to 0 to make them equal to right hand side.

So, we add S 1 and S 2. So, and rewrite the constant as 5 x 1 plus 10 x 2 plus 1 equal to 60, x 1 plus x 2 plus S 2 equal to 10. Note that the slack variables S 1 and S 2 are also restricted to be non- negative that means, S 1 and S 2 will also be greater or equal to 0 similar to x 1 and x 2. Also the slack variables S 1 and S 2 do not participate in the objective function. So, their cost coefficients are 0.

You also note from the set of constant equations that you have a canonical form here. So, S 1 and S 2 can be chosen as basic variables, and then x 1 and x 2 will be non-basic variables which can be set to equal to 0.

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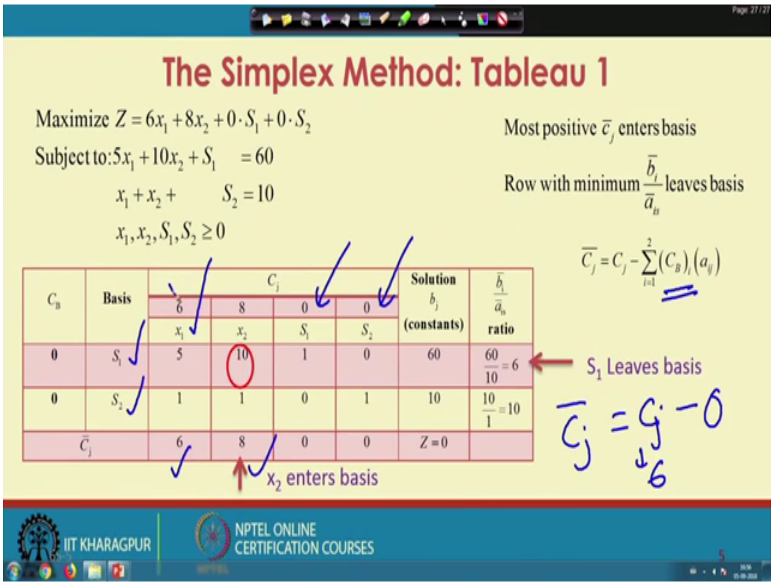

So, this is my linear programming problem in its standard form, which clearly shows the canonical form where S 1 and S 2 are basic variables; non-basic variables are x 1 and x 2.

Now, to solve the linear programming problem using simplex method, we compute the steps in a tabular form and this is the initial table known as tableau 1. So, look at here this C B means cost coefficients for the basic variables, and here are the basic variables or basis.

So, this is C j the cost coefficients for all x 1, x 2, S 1 and S 2. These part are basically the coefficients of these constants. So, note 5 10 1 and 0 for S 2. So, for 5 10 1 and 0.

Similarly, if you look at the next constraint, the coefficients are 1, 1, 0 for S 1 and 1 for S 2. This column is for the solution or the right hand side vector so 60 and 10. This column is for the ratio b i bar a i s bar.

Here in this row you write C j bar which is the reduced or relative profit coefficients. So, now let us try to understand how the computations will be performed. So, we learn from here that we can start with the initial basic feasible solution S 1 and S 2.

And then we write the coefficients of these constants. These two rows correspond to coefficients of these constants. Here I have put the right hand side vectors. Now, let us find out the relative profit coefficients. Note that here the cost coefficients for all x 1, x 2, S 1, S 2 are given. For x 1, it is 6; for x 2, it is 8; and 0 for S 1 and 0 for S 2.

So, now I can make use of the inner product rule to find out the C j bar the relative profit coefficients. For the initial basic feasible solution, I have S 1 and S 2 as basis. And for S 1 and S 2 the C j terms are 0.

So, the cost coefficient for the basis S 1 and S 2 are both 0. So, C j bar for x 1 is basically will be C j for x 1 which is 6 minus 0, because C B is 0 for S 1 S 2 both. So, basically the C j bar or the relative profit coefficients for both x 1 and x 2 will be same as C j for x 1 and x 2 which is 6 and 8. Now, we have found out the C j bar.

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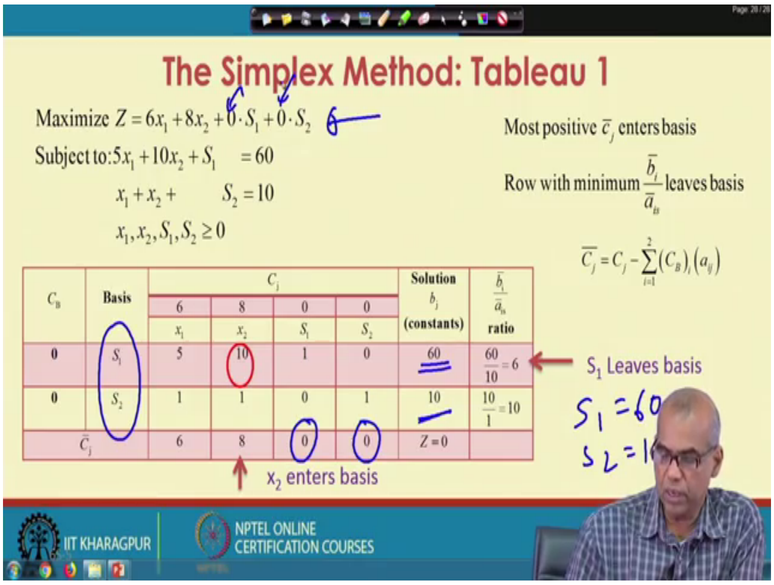

Note that for the basic variables the C j bar are 0. So, you know the right hand side constant 60 and 10. So, S 1 equal to 60 S 1 equal to 60 S 2 equal to 10. And x 1 equal to 0 x 2 equal to 0, this is the solution corresponding to this tableau. So, this becomes Z becomes 0, because x 1 equal to 0, x 2 equal to 0. So, irrespective of the value of S 1 equal to 60 and S 2 equal to 10, since their coefficients are 0 we have Z equal to 0.

Now, let us see how do we calculate the ratio b i bar divided by a i s bar. So, b i bar 60 divided by 10, why 10, because we have to now choose focus on this column. We are focusing on this column, because after we have found out the reduced or the relative profit coefficients we see that relative profits coefficients for x 2 is most positive.

So, most positive C j bar enters basis. So, x 2 enters basis. So, i which stands for x 2 here. So, C j bar tells me to find out which non-basic variable will enter the basis, and we find that from the most positive value of C j bar.

So, this b i bar by a i s bar have to be found out corresponding to this column. So, i b i bar divided by a i s bar are calculated as 60 by 10 is 66 and 10 by 1 as 10. The minimum ratio rule tells me that the minimum of b i bar divided by a i s bar will leave the basis. So, S 1 leaves the basis. So, S 1 leaves the basis, and x 2 enters the basis.

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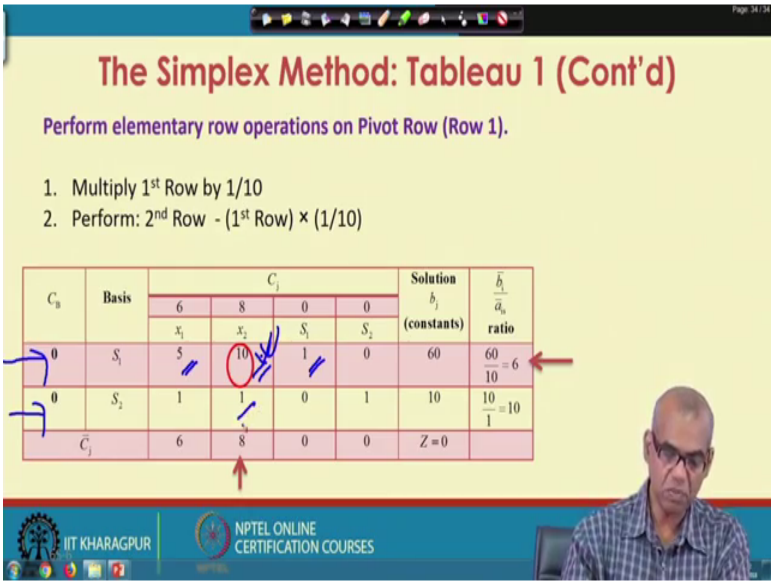

So, the pivot variable is 10. So, now, what we will do is we perform elementary row operations on the pivot row that is row 1, and we will make the coefficient of x 2 here as 1 and everywhere else 0 in this column.

So, to do that first what we do is we multiply the first row by 1 by 10. So, this becomes half, this becomes 1 that is what we want; this becomes 1 by 10 so on and so forth. Then what we do is we take second row and subtract from it first row multiplied by 1 10 then this becomes 0.

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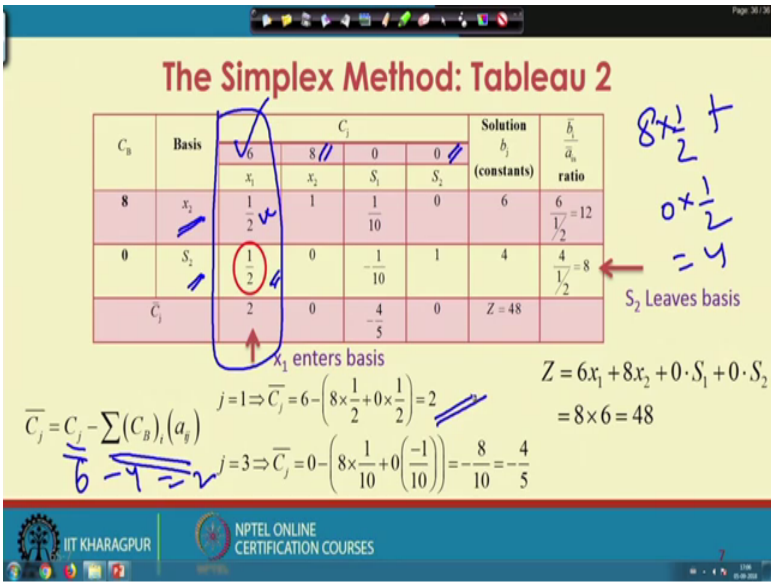

So, if you do this, you will get this tableau. So, you go to tableau 2, note x 2 is coefficient of x 2 is 1 here in the first row; in the second row, this is 0. So, in the previous tableau, we have seen x 2 enters basis and S 1 leaves basis.

So, in this case, I have x 2 and S 2 as basis and x 1 and S 1 as non-basic variables. So, for basic variables, x 2 and S 2 the relative profit coefficients are 0. So, you now have to find out the relative profit coefficients for non-basic variables x 1 and S 1.

So, let us first find out for x 1, so that is column 1, so j equal to 1. So, C j C j bar equal to C j minus sigma c b i a i j. So, let us look at for column 1. So, C j bar will be computed as 6 that is the C j for x 1. And then you have to take the product C B i a i j.

So, basis is x 2. And for x 2 cost coefficient is 8; and a i j for x 2 is half so 8 into half plus you have the other this is S 2 whose cost coefficient is 0 and a i j is half. So, 0 into half which is basically 4. So, basically 6 minus 4 is equal to 2, so 2. Similarly, you can also calculate for the other non-basic variable S 1.

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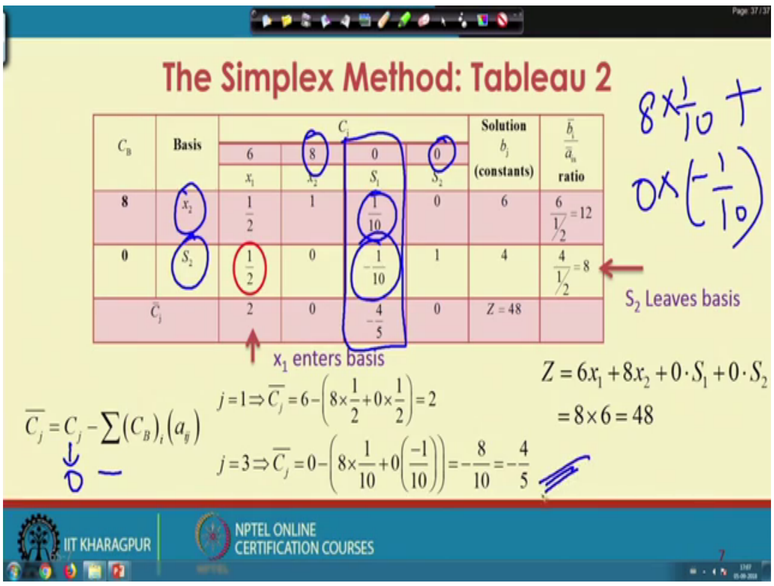

And you will find it the value as minus 4 by 5. How do you get it? For this S 1 C j equal to 0 minus sum of those two terms 8 into 1 by 10 plus 0 into minus 1 by 10. So, if you simplify, it will be minus 4 by 5.

So, the relative profit coefficients are obtained as 2 minus 4 by 5. So, most positive is 2, so that corresponds to non-basic variable x 1, so x 1 will enter basis. So, once we determine x 1 enters basis, let us find out b i bar divided by a i s bar. So, 6 divided by half is 12, and 4 divided by half is 8.

So, minimum ratio rule now says that S 2 will leave the basis. So, x 1 enters S 2 leaves; and this is the pivot variable and this row is the pivot row. So, again we have to do elementary row operations such that this coefficient becomes 1, and here it becomes 0.

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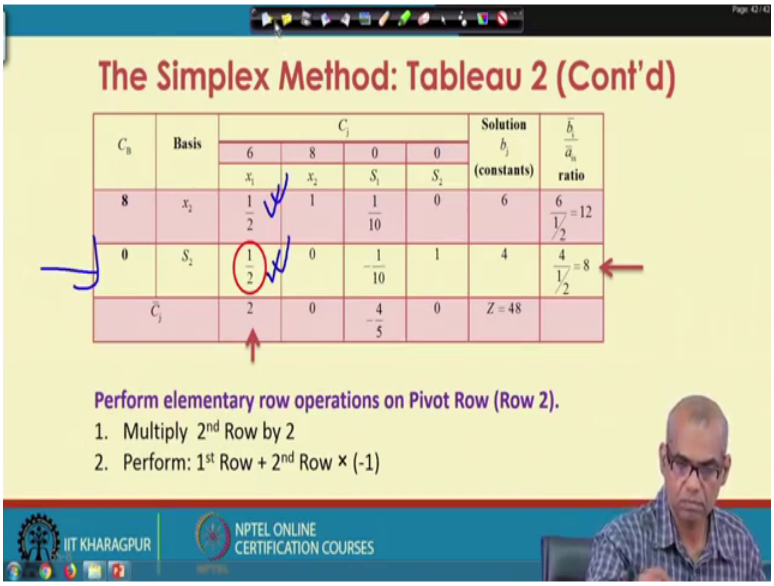

So, this is can be obtained by performing elementary row operations on private row 2. So, the private row is the row number 2. So, to convert this half to 1, we multiply second row by 2, so that half becomes 1. And then to make this 0 in the first row, we perform first row plus second row into minus 1.

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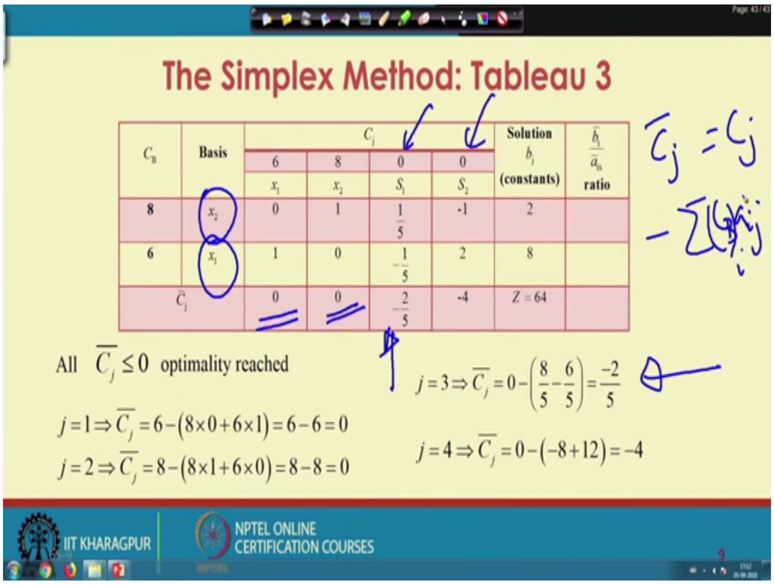

So, now we get tableau 3. In the tableau 3, we have x 1 and x 2 as basic variables and S 1 and S 2 as non-basic variables. So, for basic variables we can straight away write the relative profit coefficients as 0 and we have to find out the relative profit coefficients for the non-basic variables S 1 and S 2.

We find that for S 1 which is column 3 j equal to 3, you find the relative profit coefficients as C j bar equal to C j minus sigma C B i a i j.

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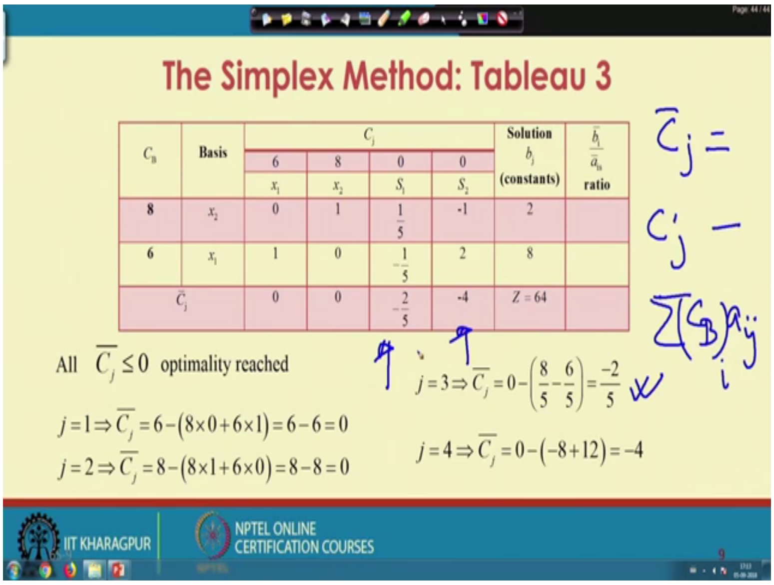

So, C j bar equal to C j minus sigma C B i a i j. So, following these expressions you can find for S 1 as well as S 2 and in gate the values as minus 2 by 5 and minus 4.

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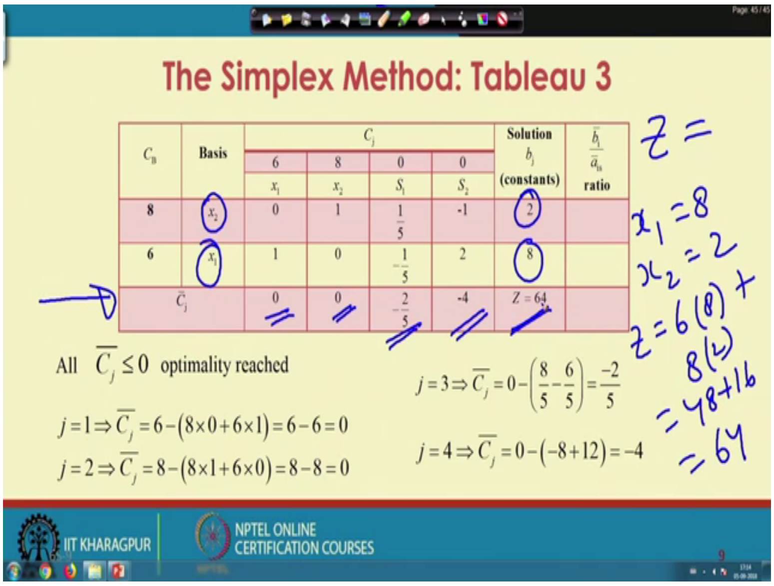

So, now you note the values for the C j bar. For the basic variables, these are 0s; for the non-basic variables, S 1 and S 2 I have minus 2 by 5 and minus 4. So, all the relative profit coefficients C j bar is less or equal to 0, so that is the condition of optimality, we have reached optimality conditions.

So, what is the value of the Z at optimality, x 1 equal to 8, x 2 equal to 2 is the solution. So, Z equal to 6 x 1 so 6 into 8 plus 8 x 2 8 into 2, so 48 plus 16 is equal to 64.

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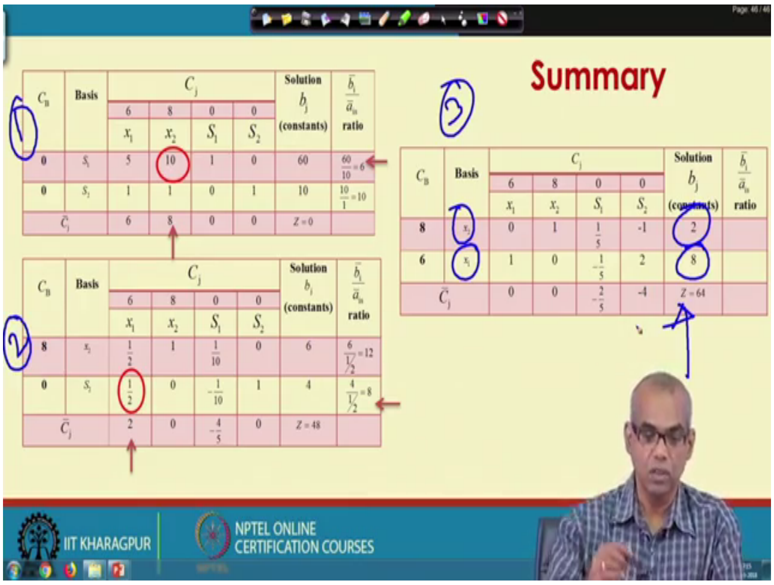

So, now there is no need to compute the minimum ratio, because they have already reached optimality. So, the optimal solution is x 1 equal to 8, x 2 equal to 2 and Z equal to 64.

So, this is the summary of all three tableau that you have performed. So, this was the initial tableau, tableau number 1, where S 1 and S 2 was basis, and x 1 and x 2 was non-basic variable. In the next we got x 2 as basic variables and S 1 became non-basic variable and finally, we got both x 1 and x 2 as basic variables and S 1 and S 2 as non-basic variable.

So, the final solution is the x 1 is equal to 8, x 2 is equal to 2 and Z is equal to 64. So, this is how you can do the computations for solving linear programming problem using simplex method. This will require some practice and you have to take care that you do the multiplication and additions or elementary row operations correctly.

So, there are three main things you must perform the elementary row elementary row operations correctly. And then you must compute the relative profit coefficients correctly, the most positive coefficients corresponding to that non-basic variables enters as basic variable. And then compute the minimum ratio rule; from the minimum ratio rule, you will be able to determine which basic variable leaves. You go on doing these computations in tableaus as shown until you get all C j bar less or equal to 0 that is the condition of optimality. With this we will stop here.