Optimization in Chemical Engineering
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Lecture - 04
Introduction to Optimization (Contd.)

Welcome to lecture 4, in this first week of Optimization in Chemical Engineering course, we are talking about introduction to optimization. So, after giving you a brief introduction to optimization and after talking about problem statement classification of optimization problems and after giving few examples on engineering applications of optimizations, in today's class and in the next class we will briefly review linear algebra that may be required for our course.

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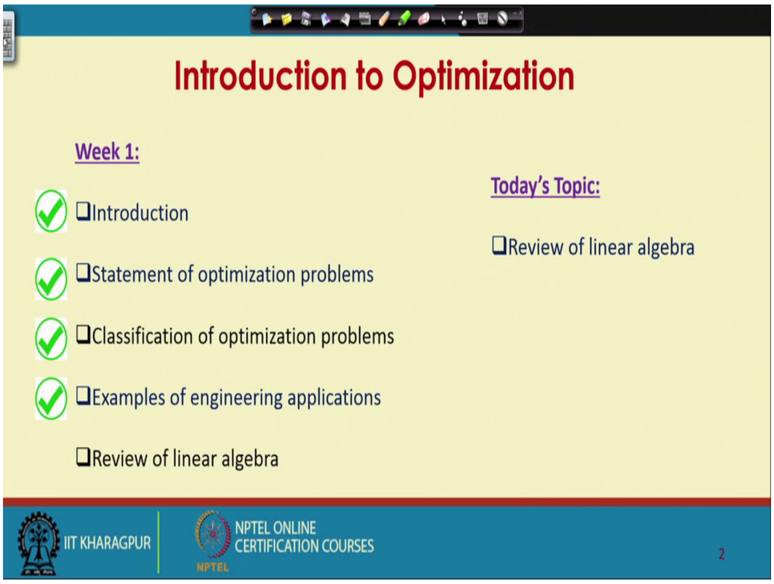

So, today we will start our review on linear algebra.

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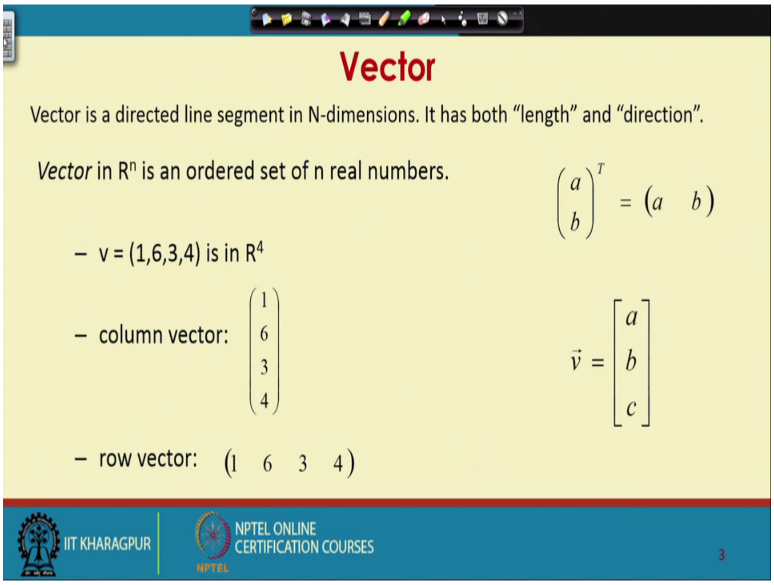

So, dealing our discussion on linear dealing our discussion on review on linear algebra, we will essentially talk about vectors and matrices. So, today we will talk about vectors and various vector operations. Vector is the directed line segment in N dimensions it has both lengths and direction. Vector in N dimensional real space is an ordered set of n real numbers. So, an N dimensional real space is shown like this R to the power n. So, v equal to 1634 is a vector in 4 dimensional real space, why because it has 4 number of elements. The column vector is written like this whereas, row vector is written like this. You know the transpose of the column vector will be a row vector a vector is also represented like this.

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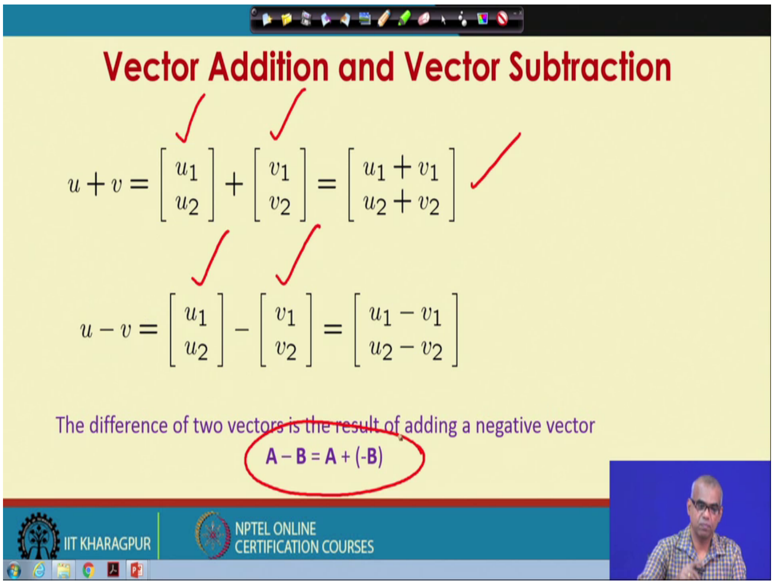

See if you have a vector with elements u 1 and u 2 you have another vectors elements v 1 and v 2, the addition of vectors will be u 1 plus v 1 as first element, and u 2 plus v 2 as second element. So, both the vectors have same number of elements. So, we can perform vector addition. Similarly vector subtraction; you will subtract elements of first vector and elements of second vector. So, u 1 minus v 1 is be the first element, u 2 minus v 2 will be the second element.

The difference of 2 vectors is the result of adding a negative vector. So, it can be written as A minus B equal to A plus minus of B.

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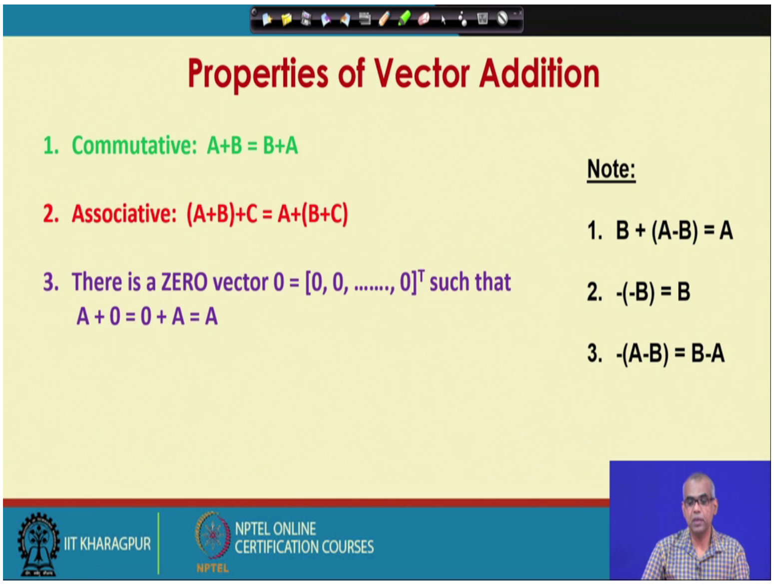

Here are some properties of vector addition. Vector addition follows commutative property. So, A plus B equal to B plus A it follows Associative law.

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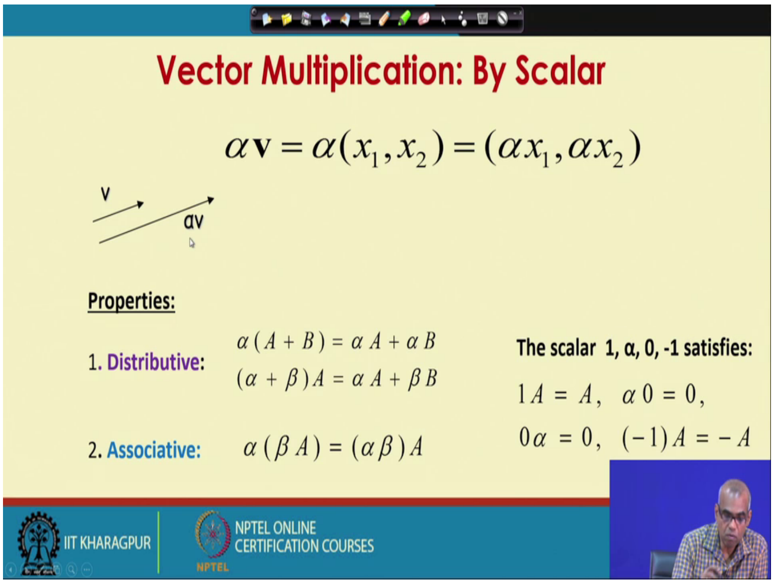

So, if you perform.

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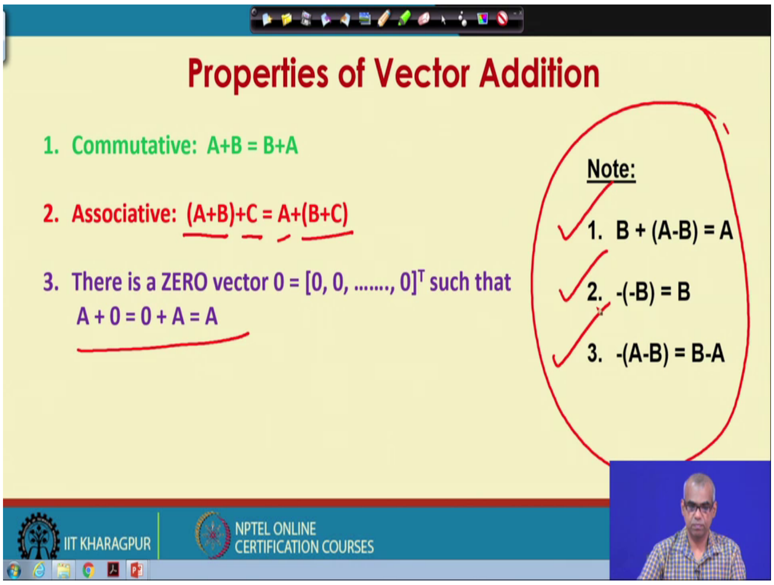

A plus b first and then add C to it, the result will be same as if you perform B plus C and then add A to it. There is a zero vector for which all the elements are 0. So, if I add a zero vector to another vector a the result will be A. So, A plus 0 equal to 0 plus A equal to A. Note this properties B plus A minus B equal to A minus of minus B equal to B minus of A minus B equal to B minus A. So, vector follows these rules.

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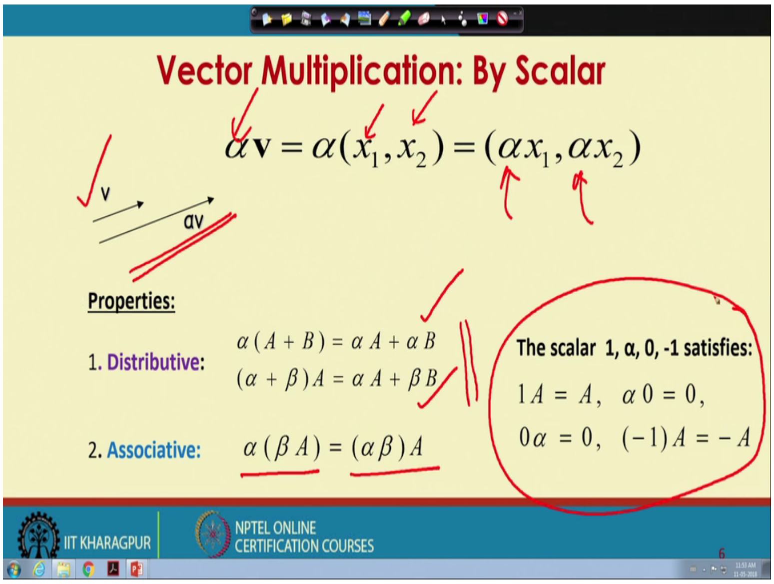

What happens if I multiply a vector by a scalar consider v a vector which has elements let us say x 1 and x 2. You multiply the vector let say by a scalar alpha. What happens is, each element of the vector gets multiplied by alpha. So, the direction of the vector remain same, but it magnitude changes, magnitude becomes alpha times the vector. So, it becomes scaling. So, vector multiplication by scalar follows distributive property. So, alpha into A plus B is alpha A plus alpha B, alpha plus beta into A equal to alpha A plus beta B where both alpha and beta are scalars.

Vector multiplication by scalar also follows associative rule. So, alpha into beta into A is alpha beta into A. If you take scalars 1 alpha 0 or minus 1, they will satisfy 1 into A equal to A, alpha into 0 equal to 0,0 into alpha equal to 0 minus 1 into A equal to minus A.

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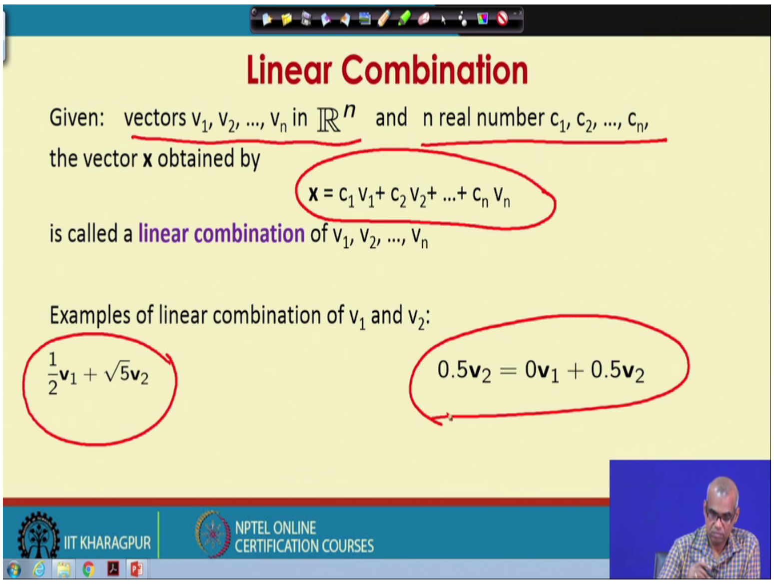

What do you understand by linear combination of vectors? Let us consider vectors v 1, v 2, v 3, v 4 up to v n in n dimensional real space. And also consider n real numbers as c 1, c 2, c 3, c 4, c 5 up to c n. So, the vector that I obtained by linear combination of v 1, v 2, v 3, v 4, v n is called a linear combination of vectors v 1, v 2, v 3 up to v n; that means, the vector x obtained as c 1 into v 1, plus c 2 into v 2, plus c 3 into v 3 up to c n into v n is called a linear combination of vectors v 1, v 2, v 3 up to v n. Examples of linear combinations of v 1 and v 2 may be 0.5 v 1 plus say square root 5 v 2; 0.5 v 2 equal to 0 into v 1 plus 0.5 into v 2.

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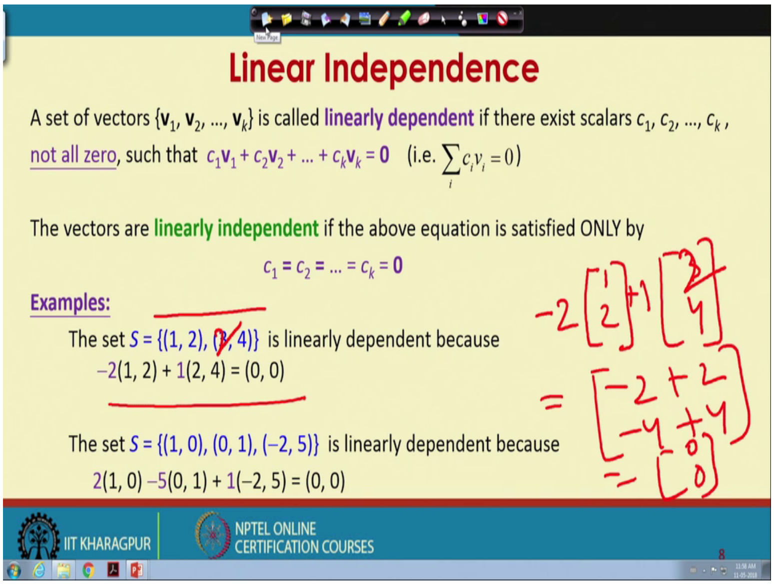

Linear independence of vectors; this is important. We need to understand what you mean by linearly independent vectors. A set of vectors v 1, v 2 up to v k is called linearly dependent if there exist scalars c 1, c 2 up to c k not all 0 such that, c 1 v 1 plus c 2 v 2 up to c k v k equal to 0. So, if you have vectors v 1, v 2 up to v k, and there exist scalars c 1, c 2 up to c k which are not all zeroes then if c i v i equal to 0 meaning c 1 v 1 plus c 2 v 2 up to c k v k equal to 0, then the set of vectors v 1, v 2 up to v k is called linearly dependent.

The vectors v 1, v 2 up to v k will be linearly independent if sigma c i v i equal to 0 or c 1 v 1 plus c 2 v 2 up to c k v k equal to 0, only when c 1, c 2 all c k equal to 0. So, for vectors that are linearly independent sigma c i v i equal to 0 only when c 1 c 2 up to c k equal to 0. So, here are some examples of linearly dependent vectors, consider I have a set of vector 1 2 and 3 4. So, you have 1 2 you have 3 4.

The question is these vectors linearly dependent or independent. They are linearly dependent because if we do this; that means, if I consider c 1 as minus 2 and c 2 as 1 and take the sum c 1 v 1 plus c 2 v 2, I get 0 why. So, this is minus 2 this is 2 4; so minus 2 plus 2 minus 4 plus 4 equal to 0 0.

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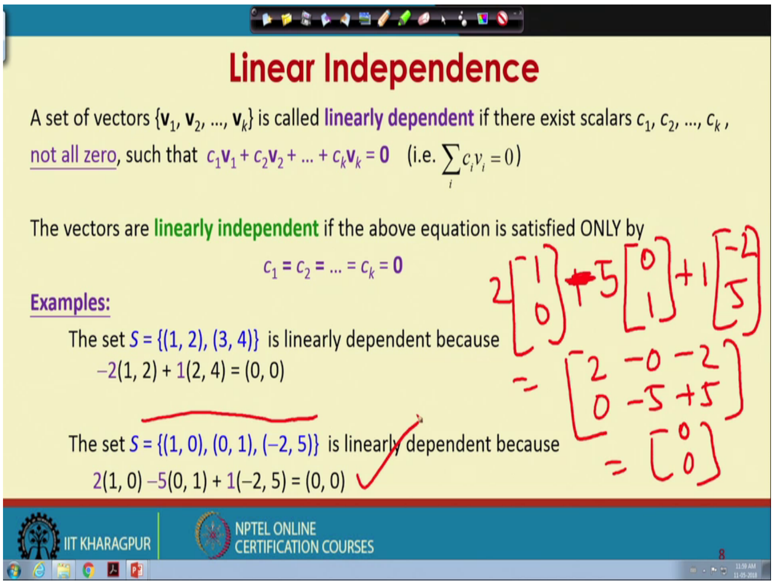

Similarly, the vectors 1 0 0 1 minus 2 5 is also linearly dependent, because you can also show that 2 1 0 plus its minus 5 0 1 plus 1 minus 2 5 equal to 2 minus 2 minus 0 minus 2 0 minus 5 plus 5 equal to 0 0. So, this set of vectors are also linearly dependent.

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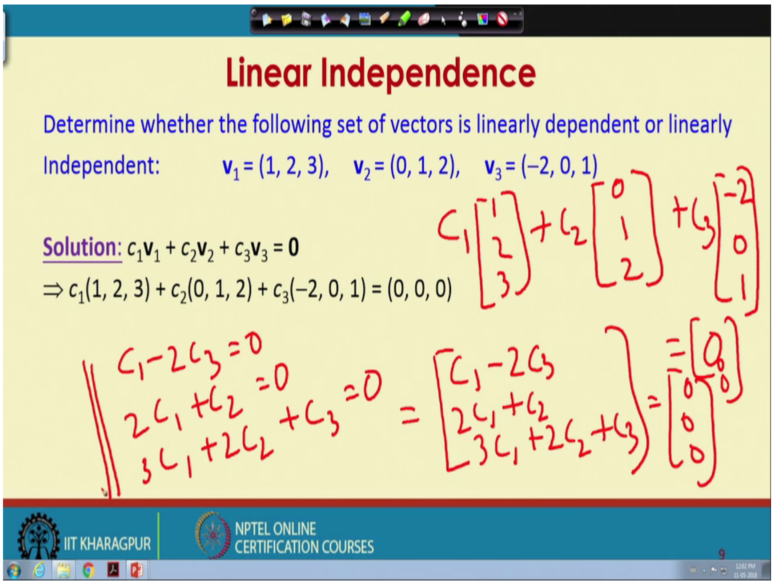

Now, let us consider another example. We have the following set of vectors v 1 equal to 1 2 3, v 2 equal to 0 1 2 and v 3 equal to minus 2 0 1 are these vectors linearly dependent or linearly independent let us follow the same procedure. If the set of vectors has a linearly independent so, the sum c 1 v 1 plus c 2 v 2 plus c 3 v 3 will be equal to 0, only when c 1 equal to 0 c 2 equal to 0 and c 3 equal to 0.

So, let us get the sum C 1 1 2 3 plus C 2 0 1 and 2 plus C 3 minus 2 0 and 1. So, this will be equal to 0. So, do the sum which will be C 1 plus 0 minus 2 C 3, 2 C 1 plus c 2 plus 0, 3 C 1 plus 2 C 2 plus C 3 equal to 0 0 and 0. So, this is also 0 0 and 0. So, basically you have 3 equations C 1 minus 2 C 3 equal to 0, 2 C 1 plus C 2 equal to 0, 3 C 1 plus 2 C 2 plus C 3 equal to 0. So, you have 3 equations in C 1, C 2 and C 3 you can solve this 3 equations and find the values of C 1 C 2 and C 3. You will see that the values are coming as C 1 equal to 0, C 2 equal to 0 and C 3 equal to 0; that means, this set of vectors are linearly independent.

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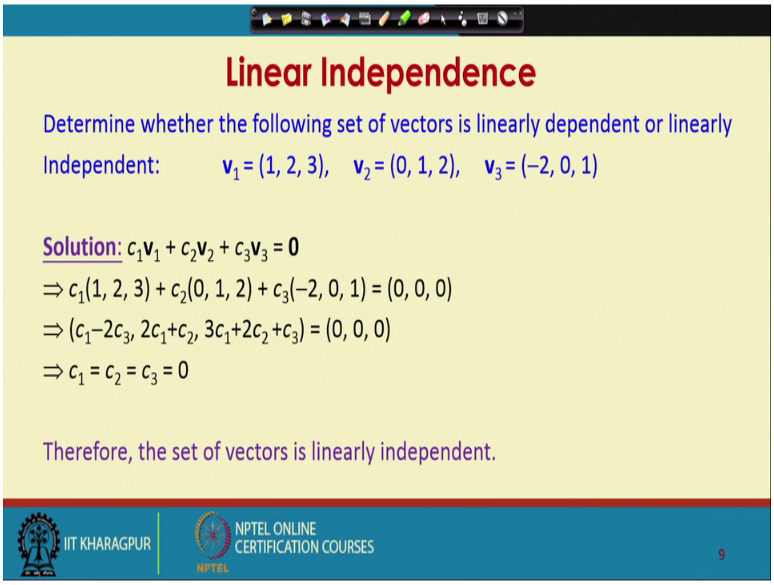

So, that is why you can see here.

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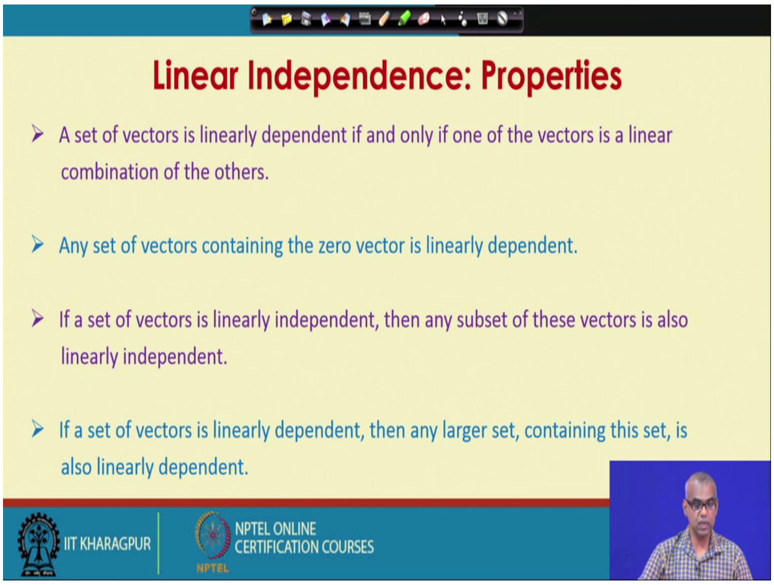

A set of vectors is linearly dependent if and only if one of the vectors is a linear combination of the others. So, we are talking about some properties of linear independence. A set of vectors is linearly dependent if and only if 1 of the vectors is a linear combination of the others.

Any set of vectors containing the zero vector is linearly dependent. So, any set of vectors containing the zero vector is linearly dependent. If a set of vectors is linearly independent then any subset of these vectors is also linearly independent. So, if a set of vectors is linearly independent, then any subset of these vectors is also linearly independent if a set of if a set of vectors is linearly dependent then any larger set containing this set is also linearly dependent I repeat if a set of vectors is linearly dependent then any larger set containing this set is also linearly dependent.

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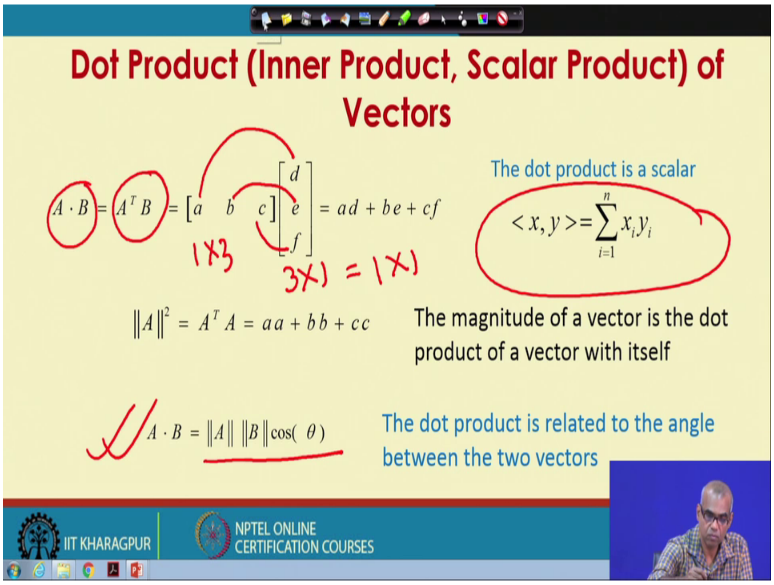

Now, let us talk about dot product of vectors. The dot product of vectors are also known as inner product or scalar product. You can consider this dot product as something similar to matrix multiplication. So, the dot product between 2 vectors A and B, which is also represented as A transpose B is the product like this. So, a b c a and b are both column vectors you have taken A transpose B. So, A becomes row vector and B becomes column vector.

So, if I say if tells of matrix notation this is 1 row 3 column, this is 3 rows 1 column. So, the multiplication is defined because column of the first matrix a is equal to rows of second matrix b you also can immediately see then the product will have dimension 1 by one; that means, is going to be a scalar. So, the dot product is a scalar quantity. So, if the elements of matrix a is a b c and the elements of matrix b are d e f. So, the dot product between a and b is, a d plus b e plus c f which is a scalar quantity. So, the dot product is also represented like this 2 vectors x and y, the dot product is sigma x i y i for all values of i equal to 1 to n where n at the number of components in vectors x and y.

The magnitude of a vector is that dot product of a vector with itself. So, A transpose A if a has elements a b and c, A transpose A becomes a a plus b b plus c c this becomes the magnitude of the vector a. The dot product is related to the angle between the 2 vectors. So, 2 vectors A and B, the dot product between 2 vectors A and B can be written as a to b into cos theta where theta is the angle between these 2 vectors a and b.

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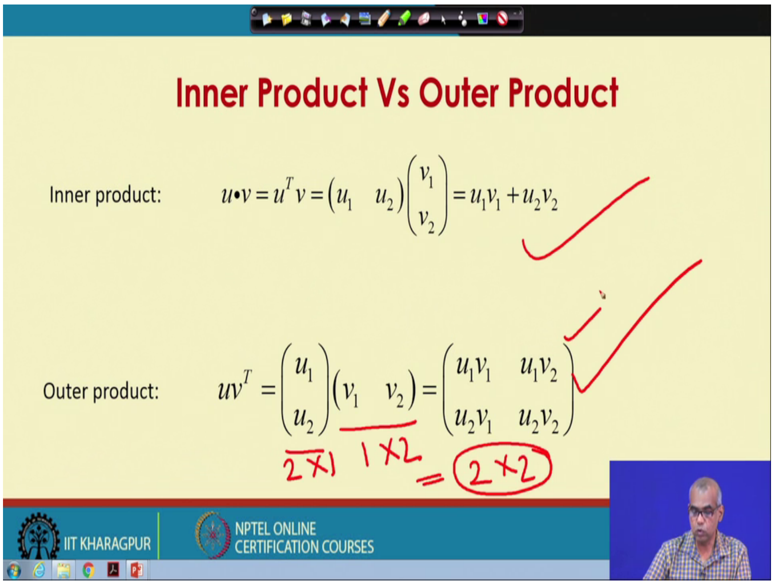

Corresponding to inner product, we have also another term called outer product. So, the inner product or the dot product between u and v let us say I have 2 vectors u and v and the elements of u vector is u 1 and u 2, and the elements of v vector is v 1 and v 2. So, the inner product or the dot product between u and v is u dot v equal to u transpose v is u 1 v 1 plus u 2 v 2.

But the outer product is outer product between u and v is, u v transpose u v transpose inner product is u transpose v outer product is u v transpose. So, u and v are both column vectors. So, u is column vectors now v transpose that becomes a row vector. So, u 1, u 2, v 1, v 2; V 1 v 2 is the column vector. So, you have 2 rows 1 column; v 1 v 2 now has become row vector after taking transpose and it has 1 row 2 column. So, the matrix multiplication is defined and the product is going to be 2 by 2 matrix. So, this is what you get u 1 v 1, u 1 v 2 elements of first row, u 2 v 1, u 2 v 2 are the elements of second row.

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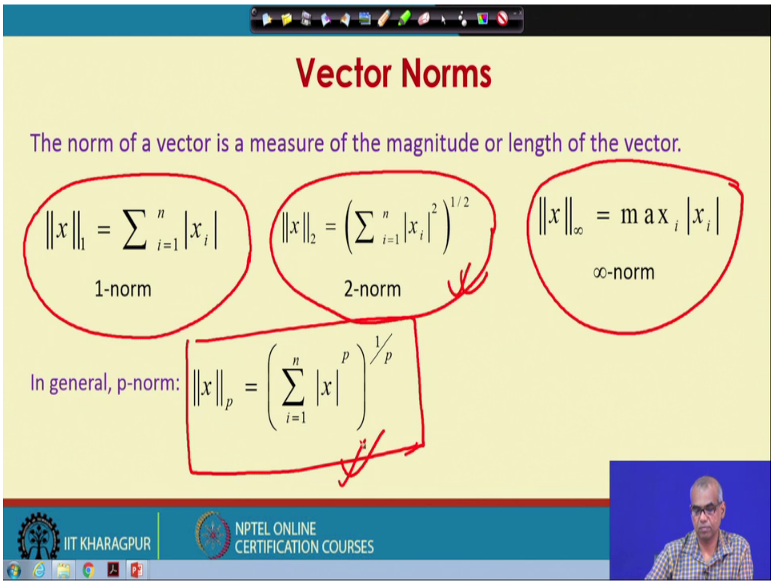

Vector norms: the norm of a vector is a measure of the magnitude or length of the vector. There are different types of norms common norms are 1 norm, 2 norm or infinity norm. 1 norm is defined as this. So, you sum up the absolute values of all the elements in the vector, 2 norm is defined like this. You square all the elements in the vector sum them up and take the square root. Infinity norms is defined as this where you take the maximum value of the elements in the vector. So, in general the p norm is defined as this. Look at the similarity between the 2 norm and the p norm. So, put p equal to 2 it becomes 2 norms. So, mod x to the power p sum them up and then rest to the power 1 by p.

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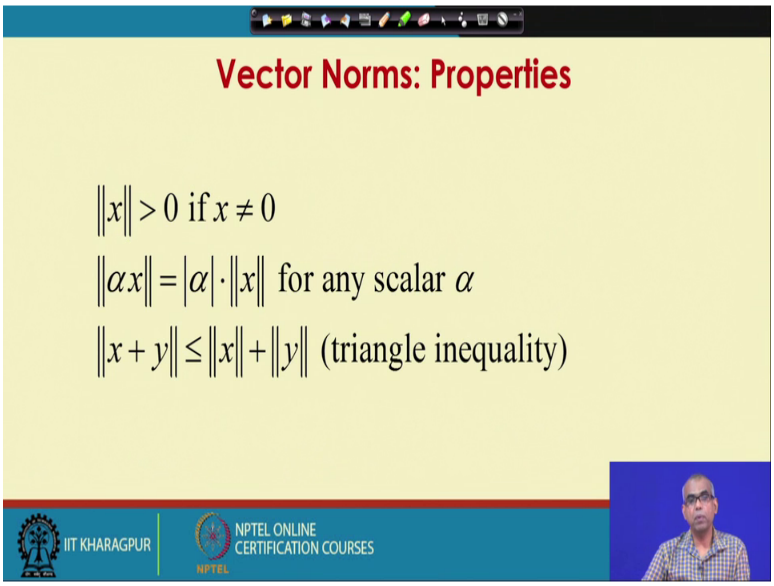

Here are some properties of vector norms. The norm of a vector is greater than 0 if x is not equal to 0. So, if x is not equal to 0 the norm of a vector which represents the length of magnitude of the vector is always greater than 0. Consider vector x consider a scalar alpha. So, the norm of alpha x is equal to absolute value of alpha into norm of x.

And finally, norm of sum of 2 vectors is less than sum of norm of those 2 vectors this is known as triangle inequality. So, if you consider vector x, if you consider vector y you sum the vector x plus y, take the norm of x plus y that will be less or equal to sum of norm of x plus norm of y. So, that is known as norm of triangle inequality.

With this we stop our lecture 4 here.