Optimization In Chemical Engineering

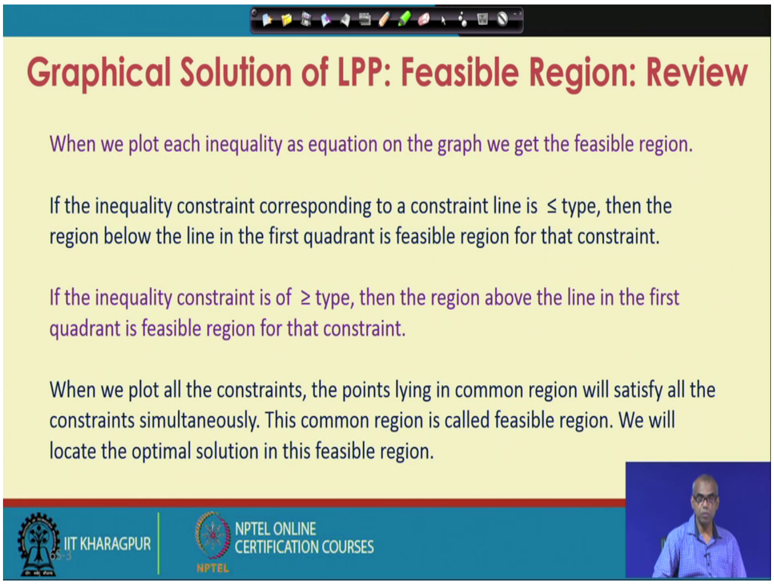
Prof. Debasis Sarkar.
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 38
Introduction to Linear Programming (Contd.)

Welcome to lecture 38, this is week 8 and we are talking about Introduction to Linear Programming. In our previous lecture we have introduced graphical solution of linear programming problems with we took a maximization problem and showed how to solve the problem graphically.

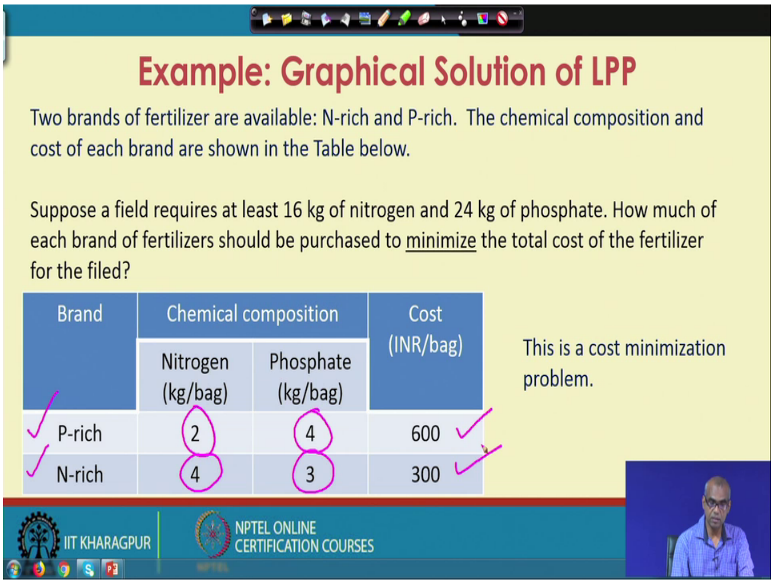
We have discus the 2 variable problems can be very conveniently solved using graphical method of solution. In today's lecture we will again take another 2 variable problem, which is a minimization problem and solve using graphical method and then we will also discuss some more features about graphical solution of linear programming problems.

(Refer Slide Time: 01:04)



Let us just review what we discussed about feasible region that we encounter, when we solve a linear programming problem using graphical method, when you plot each inequality as equation on the graph. We get the feasible region if the inequality constraint corresponding to a constraint line is less or equal to type, then the region below the line in the first quadrant is feasible region for that constraint. If the inequality constraint is of greater or equal to type then the region above the line in the first quadrant is feasible region for that constraint, when we plot all the constraints the points lying in common region will satisfy all the constraint simultaneously. This common region is called feasible region will locate the optimal solution in this feasible region only.

(Refer Slide Time: 02:06)

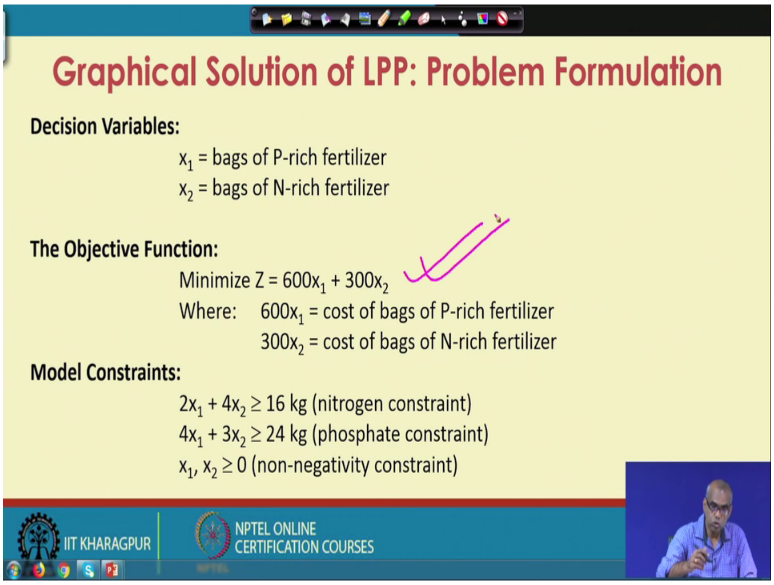


Let us take an example 2 brands of fertilizers are available let us say their nitrogen rich and phosphorus rich, the chemical composition and cost of each brand are shown in the table below. Suppose a field requires at least 16 kg of nitrogen and 24 kg of phosphate how much of each brand of fertilizer should be purchased to minimize the total cost of the fertilizer for the field.

So, there are 2 types of fertilizer available P-rich or phosphorus rich and N-rich or nitrogen rich. Now each bag of phosphorus rich fertilizer contains two kg of nitrogen and 4 kg of phosphate. Similarly each bag of nitrogen rich fertilizer contains 4 kg of nitrogen and 3 kg of phosphate, each bag phosphorus rich fertilizer cost 600 rupees let us assume and each bag of nitrogen rich fertilizer cost 300 rupees.

So, now, we have to decide how many bags of phosphorus rich fertilizer and how many bags of nitrogen rich fertilizers must 1 purchase, so that the total cost for the fertilizer of a given field is minimum and the purchase of fertilizers must satisfy the constraints that the field requires at least 16 kg of nitrogen and 24 kg of phosphate. So, this is a cost minimization problem.

(Refer Slide Time: 04:36).

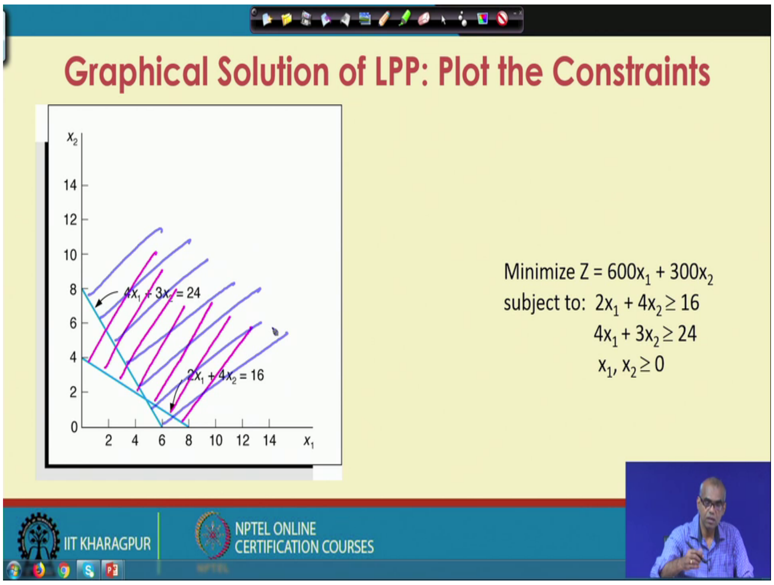


So, let us consider decision variables as follows x 1 equal to bags of phosphorus rich fertilizer and x 2 equal to bags of nitrogen rich fertilizer. Each phosphorus rich fertilizer cost 600 rupees, so the cost associated with x 1 bags of phosphorus fertilizer is 600 x 1, similarly cost of x 2 bags of nitrogen rich fertilizer is 300 x 2. So, the total cost of fertilizer is 600 x 1 plus 300 x 2.

So, we have to minimize this expression. So, this is my objective function. So, what are the constraints, the constraints are there that each the field must have at least 16 kg nitrogen and 24 kg phosphate. So, now, let us look at the table again, each bag of phosphorus rich fertilizer contains 2 kg of nitrogen and each bag of nitrogen rich fertilizer contains 4 kg of nitrogen. So, x 1 bags of phosphorus rich fertilizer will have 2 x 1 kg of nitrogen similarly x 2 bags of nitrogen rich fertilizer will have 4 x 2 kg of nitrogen. So, 2 x 1 plus 4 x 2 must be greater or equal to 16 kg, similarly 4 x 1 plus 3 x 2 must be greater or equal to 24 kg.

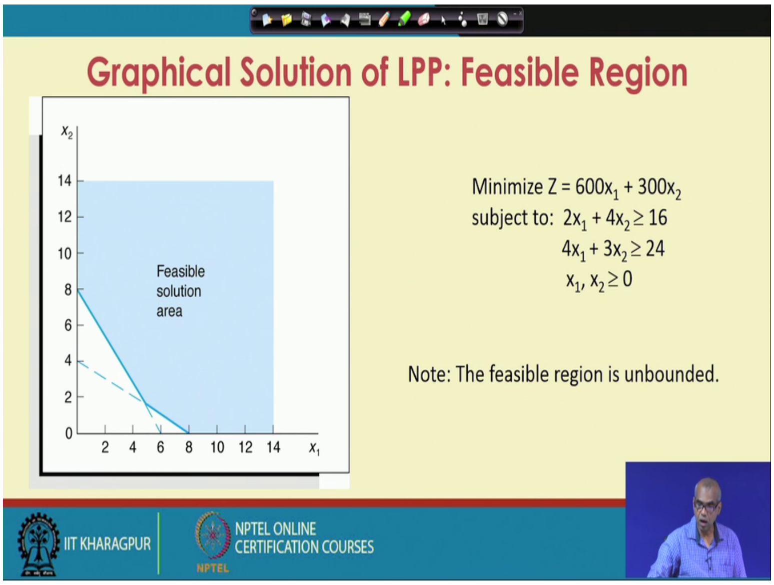
So, that constraint on phosphate is satisfied and of course, there will be non negativity constraints on x 1 and x 2. So, x 1 is greater than or equal to 0, x 2 is greater than or equal 0. So, this is the problem formulation for the fertilizer problem.

(Refer Slide Time: 07:00)



So, we plot the constraints 2 x 1 plus 4 x 2 equal to 16 we have plot the constraints as equations.

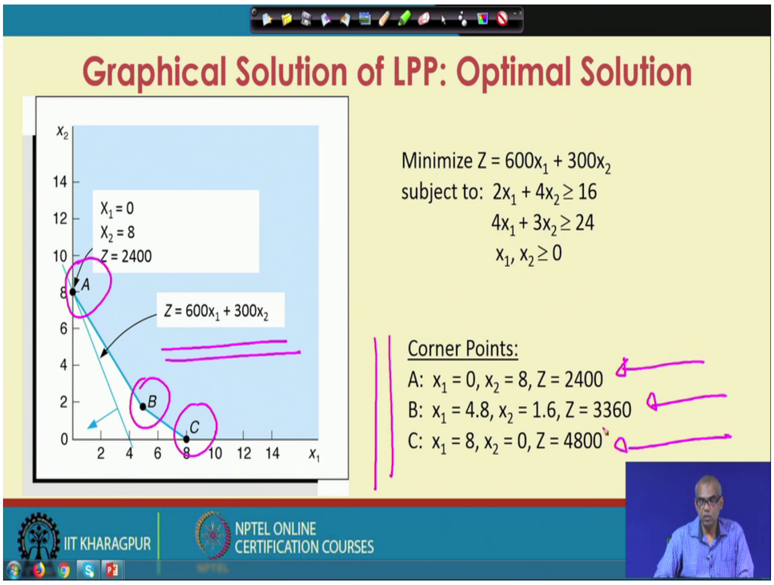
(Refer Slide Time: 07:31)



So, both the constraints have been plotted as equations. So, next we identify the feasible region.

Both the constraints are of greater or equal to type. So, the feasible region for this constraints is this and for the other constraint its this. So, now, identify the common region. So, that is what has been done here. So, the feasible solution area is shown. Note that the feasible region is unbounded.

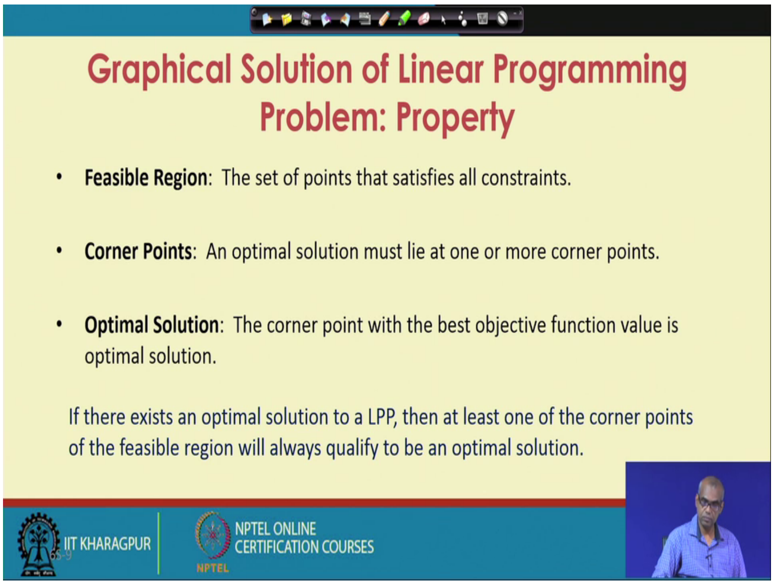
(Refer Slide Time: 08:39)



Now, to identify the optimum plot the objective function. So, 600 x 1 plus 300 x 2 equal to Z has been plotted. We can plot it for various values of Z and now by moving the constraint line you identify that A has minimum objective function value. There are 3 corner points A, B and C.

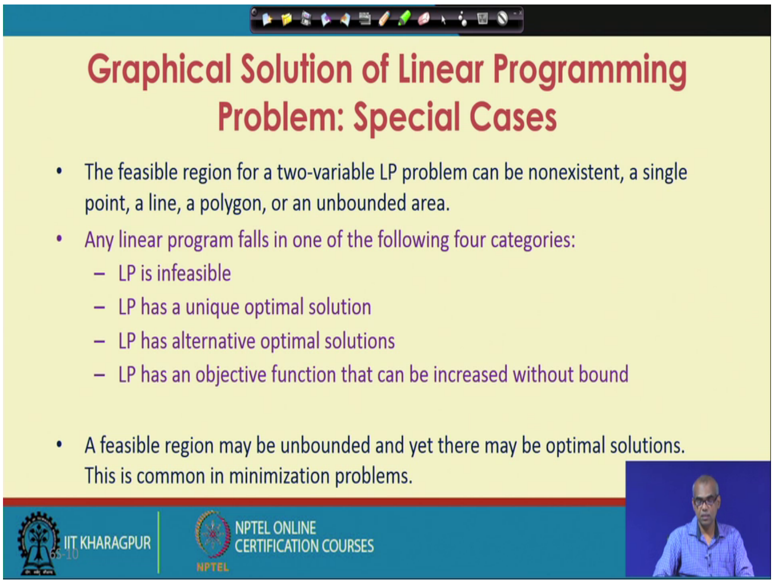
You can find out what is the value of x 1 and x 2 at each of these corner points and can find out the objective function value associated with these 3 corner points. You can see that the corner point A has a minimum objective function value and there is the optimal solution.

(Refer Slide Time: 10:05)



So, these following property, we learned about linear programming problems. The set of points that satisfies all constraints are called feasible region an optimal solution must lie at one or more corner points. The corner point with the based objective function value is optimal solution. If they are exists an optimal solution to a linear programming problem then at least one of the corner points of the feasible region will always qualify to be an optimal solution.

(Refer Slide Time: 10:56)

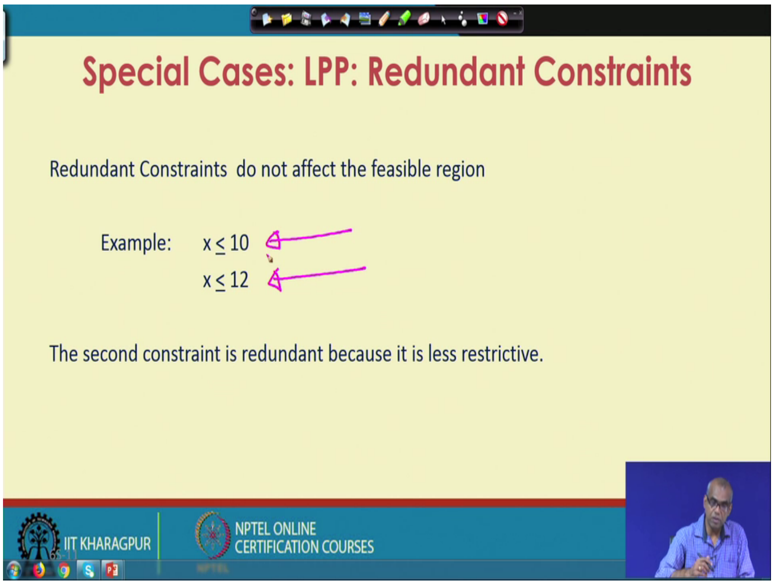


Now, we will talk about some special cases about graphical solution of linear programming problems. The feasible region for a two variable linear programming problem can be nonexistent a single point align a polygon or an unbounded area, I repeat the feasible region for the 2 variable linear programming problem can be nonexistent a single point align, a polygon or an unbounded area.

Any linear program falls in one of the following 4 categories, linear programming problem is infeasible, linear programming problem has a unique optimal solution, linear programming problem has alternative optimal solutions, linear programming problem has an objective function that can be increased without any bound; that means, it has unbounded optimal solution. Note that a feasible region may be unbounded and yet there may be optimal solutions.

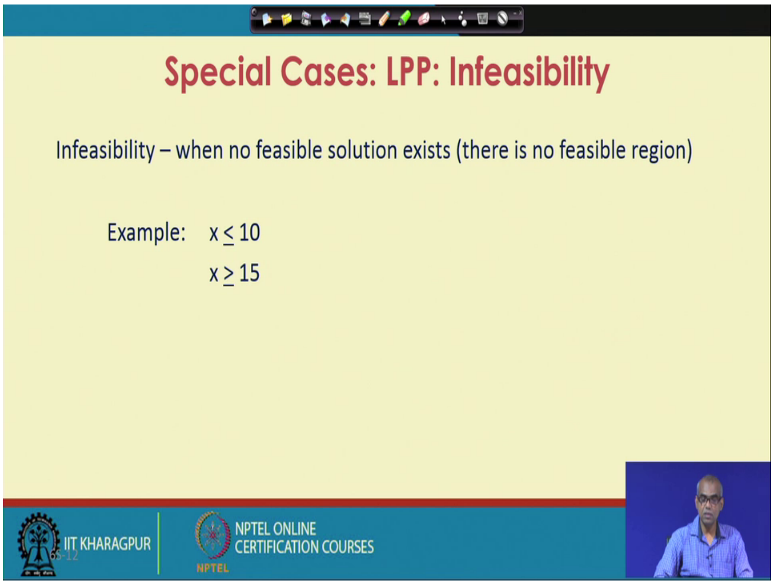
This is common in minimization problems and you have seen this today in association with the fertilizer problem where the feasible region was unbounded, but we could find the optimal solution. So, there arise many special cases, linear programming problem may be infeasible, linear programming problem may have unique optimal solution, linear programming problem may have alternative optimal solutions, a linear programming problem may have objective function that can be increased without any bound.

(Refer Slide Time: 13:12).



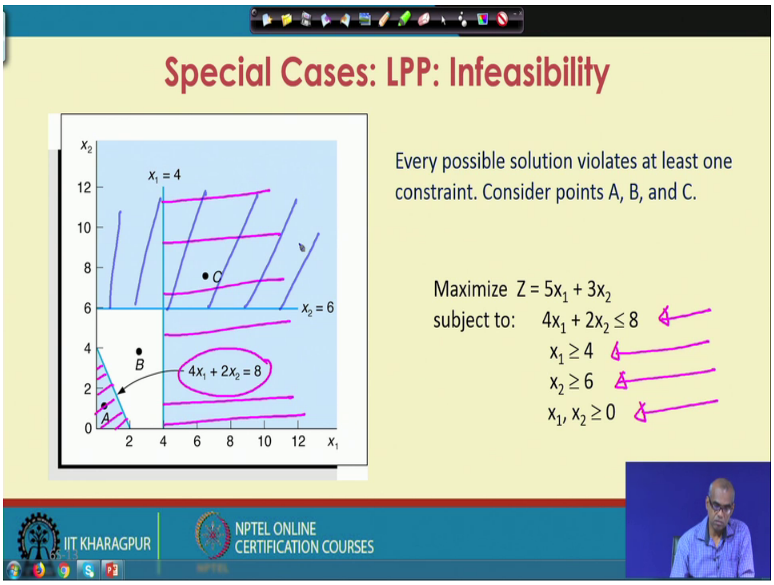
Now, let us consider the special cases one by one. Suppose we have a linear programming problem where we have 2 constraint as x less or equal to 10 and then x less or equal to 12. Note that the constraint x is less or equal to 12 is redundant because it is less restrictive compared to x less or equal to 10. So, a non negative variable such as x which is less or equal to 10 must be less than 12. So, redundant constraints do not affect the feasible region.

(Refer Slide Time: 14:30)



Let us now consider about infeasibility, a linear programming problem will be infeasible when no feasible solution exists, in other words there will be no feasible region associated with the linear programming problem, for example, if we have constraint such as x is less or equal to 10 and then x is greater or equal to 15, so clearly it is not possible to have any feasible solution and the linear programming problem will be infeasible.

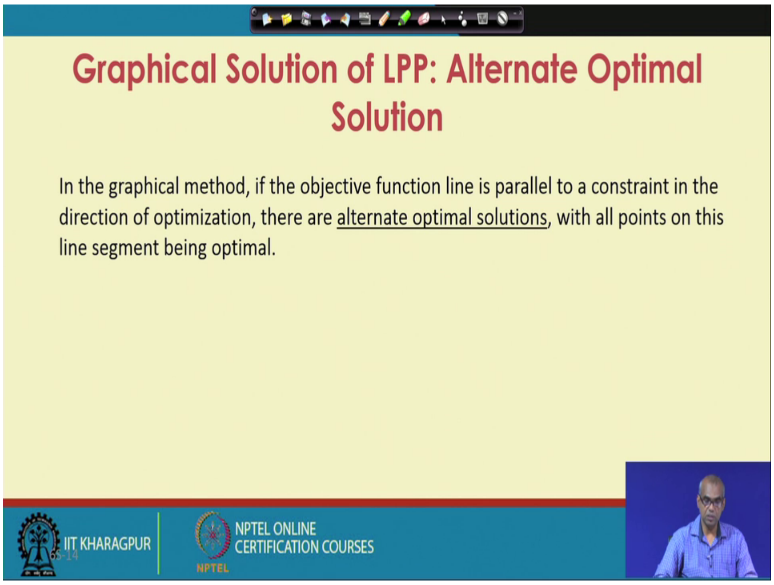
(Refer Slide Time: 15:16).



Considered the problem as shown maximize Z equal to 5 x 1 plus 3 x 2. Subject to 4 x 1 plus 2 x 2 less or equal to 8, x 1 greater or equal to 4, x 2 greater or equal to 6 and x 1 x 2 greater or equal to 0 the non negativity constraints. So, let us plot all these constraints as equations. So, 4 x 1 plus 2 x 2 equal to 8. So, this is plotted. It is less or equal to type. So, my feasible region is this. Now x 1 is greater or equal to 4. So, the feasible region for this constraint is this and x 2 is greater or equal to 6.

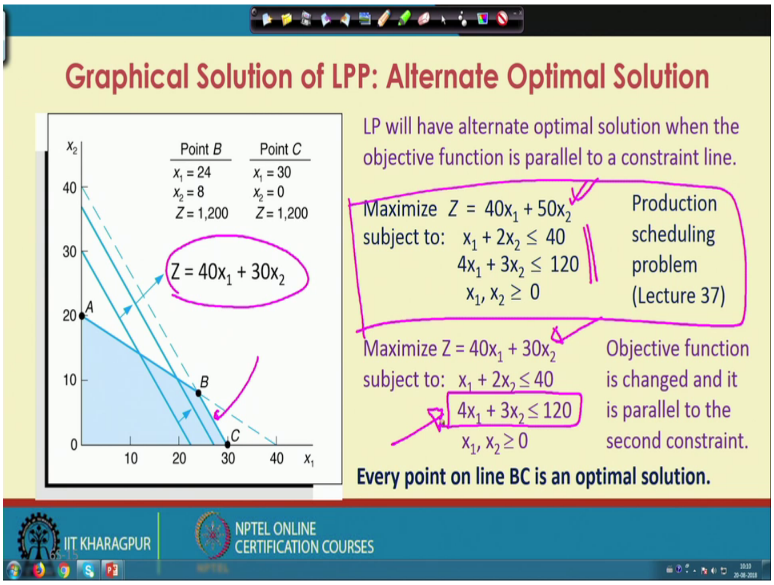
The feasible region for this constraint is this. Now if you consider point A, B, and say point C from different regions you will see that each of these points violet at least one constraint. There exists no feasible solution and the linear programming problem is infeasible because we are unable to find any common region where a point will satisfy simultaneously all the constraints. So, this is a case of infeasible linear programming problem.

(Refer Slide Time: 17:50)



Let us now talk about alternate optimal solutions, in the graphical method if the objective function is parallel to a constraint in the direction of optimization, there are alternate optimal solutions with all points on these lines segment being optimal. So, in a linear programming problem alternate optimal solutions will arise when the objective function line is parallel to any constraint in the direction of optimization.

(Refer Slide Time: 18:31)



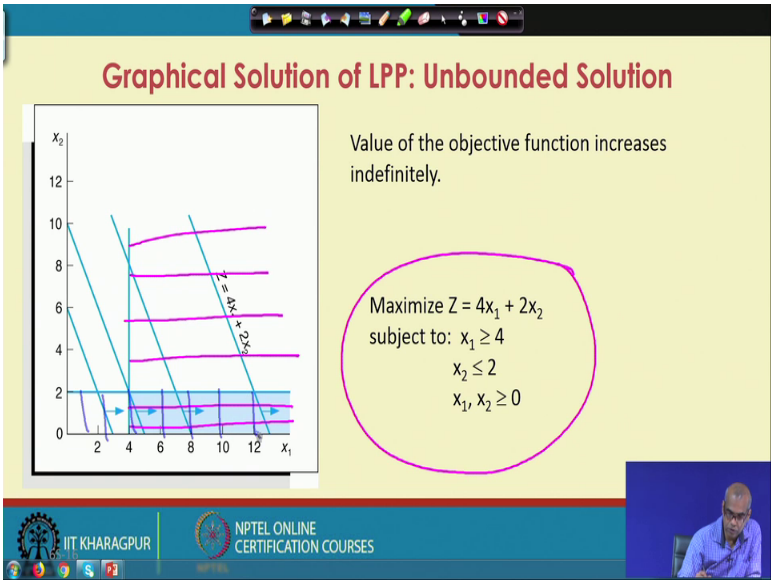
Consider the problem that we discussed in lecture 37, it was a production scheduling problem, the problem formulation was maximize z equal to 4 x 1 plus 50 x 2 subject to x 1 plus 2 x 2 less or equal to 44 x 1 plus 3 x 2 less or equal to 120 and of course, there are non negativity constrains x 1 greater or equal to 0, x 2 greater or equal to 0. So, the solution space is described by the 2 constraints. Now let us change the objective function slightly, so that the objective function becomes parallel to one of the constraints. Note that the production scheduling problem in lecture 37 that we have discussed has unique optimal solution. So, now, we want to change the objective function.

So, that it becomes parallel to one of these constraints. So, in order to do that let us make the objective function as maximize z equal to 40 x 1 plus 30 x 2. Note that I have changed 50 x 2 to 30 x 2. So, now, the objective function is parallel to the second constraint 4 x 1 plus 3 x 2 is less or equal to 120. So, now, let us plot all the constraints as equations, let us also plot the objective function line and now you see the objective function line is parallel to the line segment BC which is part of constraint 4 x 1 plus 3 x 2 is equal to 120.

Now as the objective function line is parallel to BC each and every point on line BC is an optimal solution. You can find the objective function value both at point B and point C, point C is easy to find, point C corresponds to x 1 equal to 30 x 2 equal to 0. So, the objective function Z equal to 30 into 40 is 1200, similarly point B is x 1 equal to 24 and x 2 equal to 8. So, this also gives the objective function value 1200. So, both point B and point C has the same objective function value, not only that any point on the line segment BC will have the same objective function value.

So, this linear programming problem has alternate optimal solution and there will always be alternate optimal solutions when the objective function line is parallel to any of the constraints.

(Refer Slide Time: 22:55)



Now, let us see unbounded solution, when the linear programming problem we will have unbounded solution the value of the objective function can be increased indefinitely consider the problem shown maximize Z equal to 4 x 1 plus 2 x 2 subject to x 1 greater or equal to 4 and x 2 less or equal to 2 x 1 greater or equal to 0 x 2 greater or equal to 0. So, it x 1 greater or equal to 4 gives you this feasible region, similarly x 2 less or equal to 2 gives you this feasible region.

So, there is a common region and the common region is this, which as you can see is unbounded. So, these objective function lines can be moved independent indefinitely increasing the value of the objective function. So, the linear programming problem has unbounded optimal solution. With this we stop our discussion on graphical solution of linear programming problem in lecture 38 here.