Optimization in Chemical Engineering
Prof. Debasis Sarkar
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 36
Introduction to Linear Programming

Welcome to week 8 lecture 36. In this week we will start a new topic and this will be Linear Programming Problems. So, we will talk about linear programming problems over two weeks. In the first week we will give an interaction to linear programming problems, we will revisit the linear programming problem formulation some of the linear programming problem formulation you have seen during first week, during first and second week. Again we will take some examples of linear programming problem formulation and then in this week, we will discuss about graphical solution of linear programming problems.

Then you will to introduce certain definitions and basic concepts about linear programming problems. In the following week we will discuss about more advanced solution techniques for linear programming problems.

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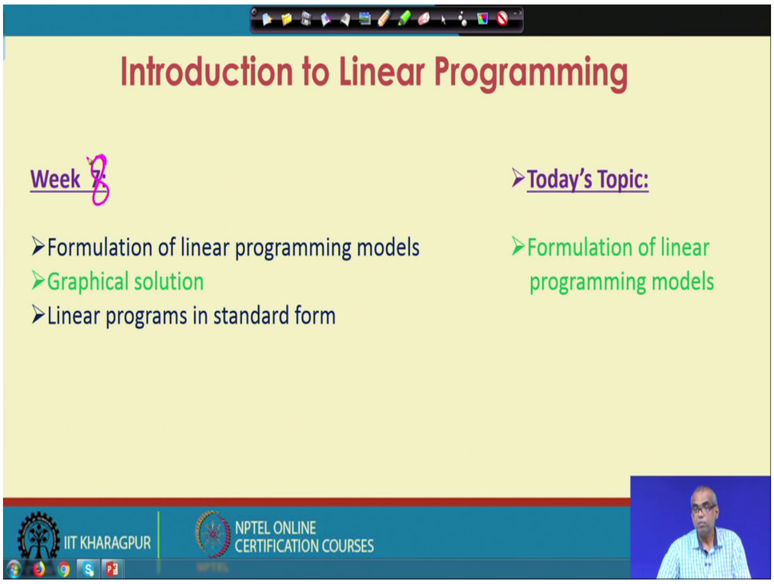

So, today we will talk about problem formulation of linear programming models, graphical solution and linear programs in standard form. So, these are the broad topics that we will cover in week 8.

Specifically in today we will talk about formulation of linear programming models.

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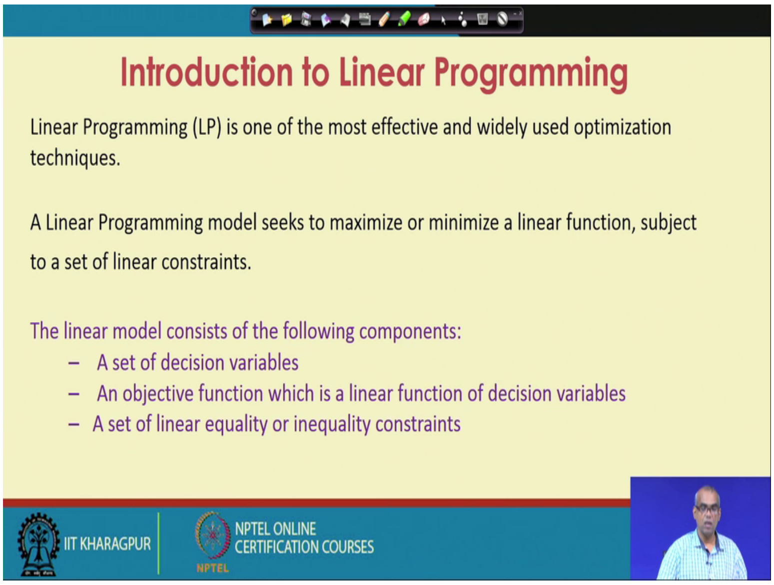

Linear programming is one of the most effective and widely used optimization techniques. A linear programming model seeks to maximize or minimize a linear function subject to a set of linear constants. The linear model or linear programming model or linear programming problem consists of the following components number 1, a set of decision variables, an objective function which is a linear function of decision variables and a set of linear equality or inequality constants.

So, in case of a linear programming problem your objective function will be linear, all the constants will be linear. The constants will be equality or inequality type it can be less or equal to type or it can be greater or equal to type. Later on we will see there is a standard form which we can express the linear programming problems. The decision variables can take on any real values.

So, to summarize a linear programming problem we will have a set of decision variables, an objective function, which is a linear function of decision variables and set of linear equality or inequality constants.

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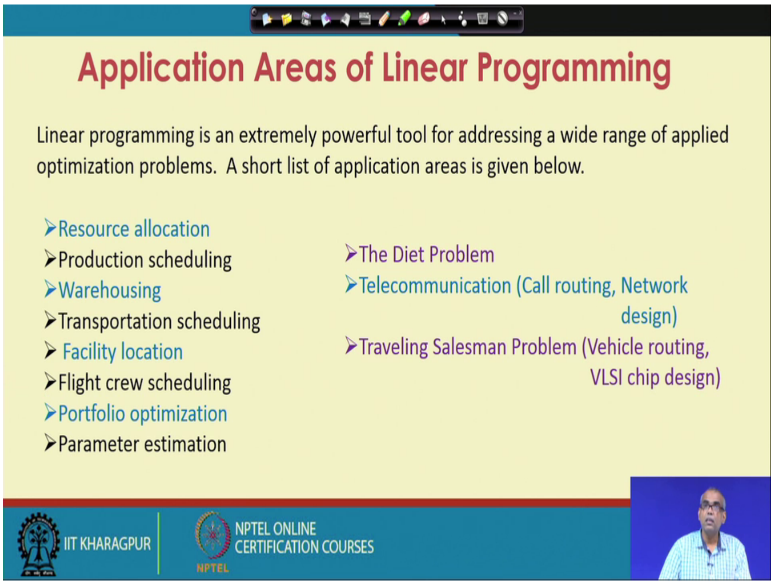

Linear programming is an extremely powerful tool for addressing a wide range of applied optimization problems. Here is a short list of application areas resource allocation, production scheduling, warehousing, transportation scheduling, facility location, flight crew scheduling, portfolio optimization, parameter estimation, some parameter estimation problems can be posed as linear programming problems.

In case of portfolio optimization is basically related to investment, if you have some money you would like to maximize the return on investment. So, what will be the portfolio of your investment can be formulated as a linear programming problem. Here you have some money and you want to distribute that money in different investment schemes so that you can maximize the return on investment.

Flight crew scheduling, imagine you are the manager of an airlines; you have to schedule the crew members such that all the flights have covered. The crew members can operate only a certain portion of the time of 24 hours maybe say 8 hours and in a given time let us say a pilot can operating only one fly. So, this problem can be formulated and solved as a linear programming problem. In case of facility location, you have certain potential sides for opening certain facility that will be servicing certain demands.

Now, you have to select a set of facility location from a set of potential sites. This is also an interesting linear programming problem, which finds applications in business. Similarly transportation scheduling, warehousing production scheduling resource allocation all are important real life linear programming problems. The drive problem or the product mix problem are other types of linear programming problems which are very common. Blending of fuels in refineries is the another linear programming problem of practical interest. Linear programming problem finds good applications in telecommunications in call routing or network design. Call routing is actually a call management systems where a particular call is passed through a given queue.

Traveling salesman problem is another important linear programming problem where a salesman has to visit several cities and how the salesperson will plan his visit such that each city is visited only once, and the total time or the total cost associated with the trip is minimized. Related problems are vehicle routing or VLSI chip design that means, very large scale integration chip design. So, all these are very important linear programming problems, which has real life applications.

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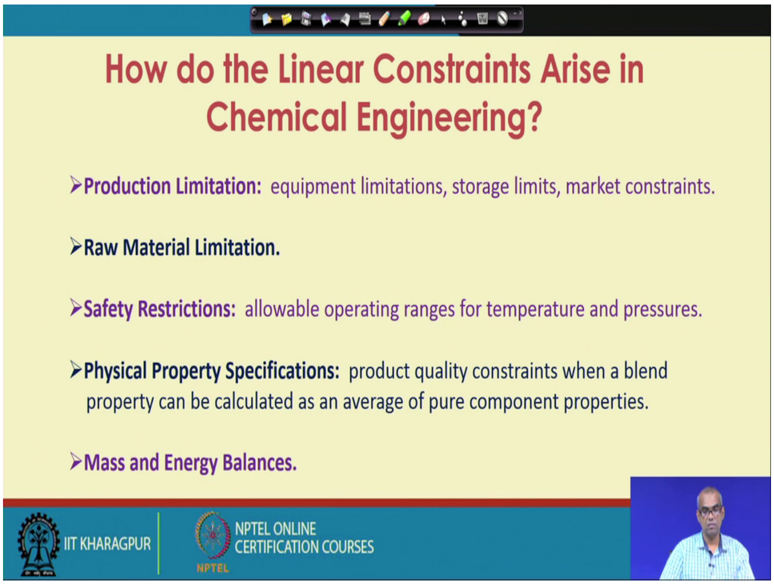

Now, we have talked about that a linear programming problem will have linear objective functions and all the constants will be linear. So, how do the linear constants arise in chemical engineering? They can arise due to production limitation, raw material limitation, safety restriction, physical property specifications and mass and energy balances.

In case of production limitations you will have equipment limitations, you will have storage limits, storage vessel cannot hold more than a certain amount of quantity, you have market constraints. In case of safety restrictions your operating temperature or pressure cannot exceed an allowable value. Physical property specifications imagine you are blending two liquid streams and you know the physical properties of these pure component streams. Now, the physical properties of the blend can be calculated as an average of pure component properties. We often do this and linear constants arise when you do such averaging. Mass and energy balances will appear as linear constants.

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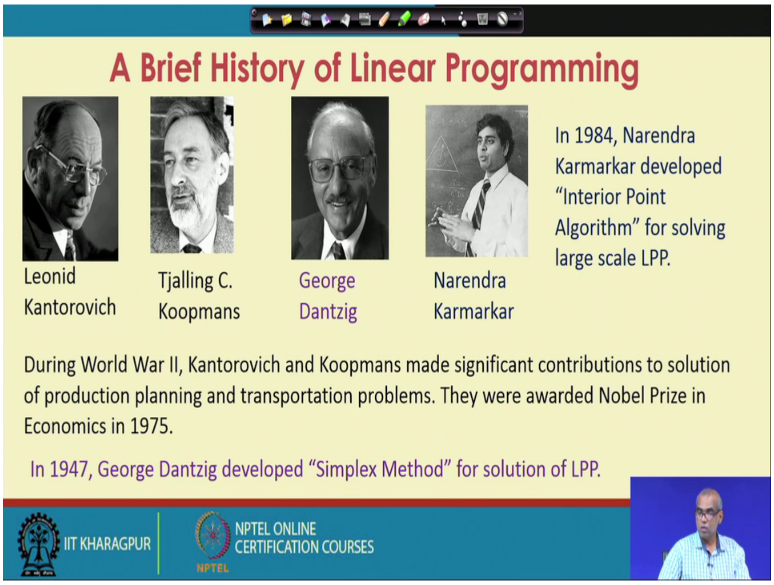

Now, let us talk about the brief history of linear programming problems. During World War II, Kantorovich and Koopmans made significant contributions to solution of production planning and transportation problems. These are all linear programming problems they were awarded Nobel Prize in economics in 1975 in the recognition of their contribution to linear programming problems related to production planning and transportation problems.

In 1947, George Dantzig developed simplex method for solution of linear programming problem. We will talk about simplex method for solution of linear programming problem in next week. In 1984 Narendra Karmarkar developed interior point algorithm for solving large scale linear programming problem. Doctor Narendra Karmarkar graduated from IIT, Bombay and when he developed interior point algorithm he was working with (Refer Time: 11:38) laboratory.

The interior point algorithm proposed from Narendra Karmarkar is a very powerful solution method for large scale linear programming problems.

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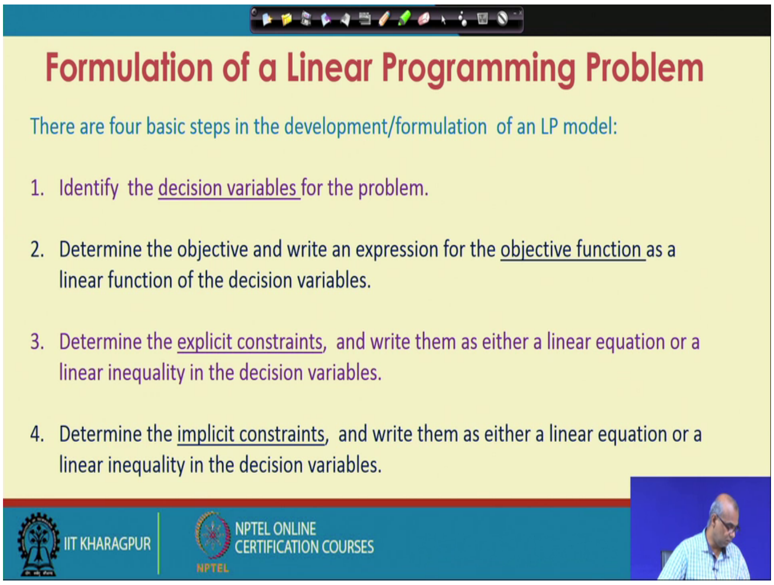

Now, let us see how do I formulate a linear programming problem. There are 4 basic steps involved in the formulation of a linear programming problem or linear programming model. First we need to identify the decision variables for the problem. This is the first step in problem formulation and a very important step. Often times is very obvious what will be the decision variables, sometimes you may face some difficulty and you have to analyze the problem deeply so that you can identify the suitable set of decision variables.

The next step is to identify the objective and write an expression for the objective function as a linear function of the decision variables. The next step is to formulate the constants. We can say there are two types of constants explicit constants and implicit constants. Explicit constants are those which are explicitly stated in the problem implicit constants and those which are not explicitly stated in the problem but from common sense or general understanding of the problem you will be able to formulate such problems for example, non negativity constraints.

So, in the third step we determine the explicit constants and write them as either a linear equation or linear inequality in the decision variables. Next we determine the implicit constants and write them as either a linear equation or linear inequality in the decision variables.

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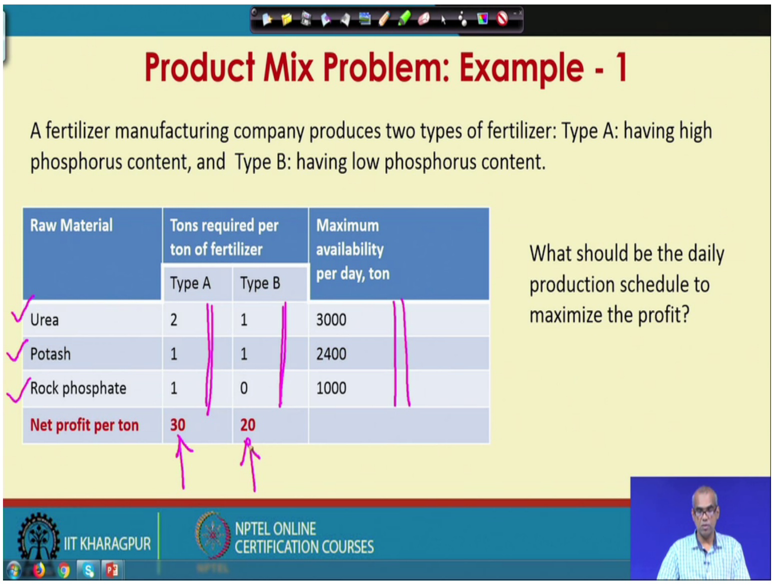

Now, let us take a problem. We have talked about this problem in the early weeks when we took certain examples of different types of optimization problem.

Now, let us take the same problem and go through each step of the problem formulation. A fertilizer manufacturing company produces two types of fertilizer type A having high phosphorous content, and type B having low phosphorous content. So, look at the table the raw materials used are urea potash and rock phosphate. The tones required per ton of fertilizer type A and type B are given. The maximum availability part they for urea, potash and rock phosphate are also given in terms. Net profit per ton for type A and type B fertilizers are also given.

So, the question you ask is what should be the daily production schedule to maximize profit. My production schedule carry mean how many tons of type A and how many terms of type B should be produced so that profit is maximized.

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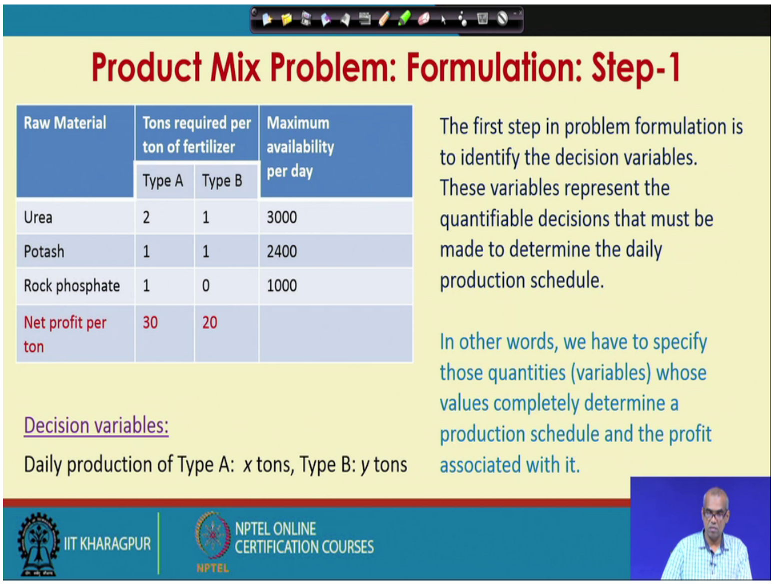

So, in the first day we have to identify the decision variables. Decision variables represent the quantifiable decision that must be made to determine the daily production schedule. In other words we have to specify those quantities or variables whose values completely determine a production schedule and the profit associated with it.

So, the decision variables will determine completely a production schedule as well as it will determine the profit associated with it because the profit associated with it has to be expressed as an linear expression of decision variables. So, here the choice is obvious. We are asking the question how many terms of A and how many terms of B should be produced. So, let the decision variables be daily production of type A x tons and daily production of type B y tones. So, these are decision variables for my problem.

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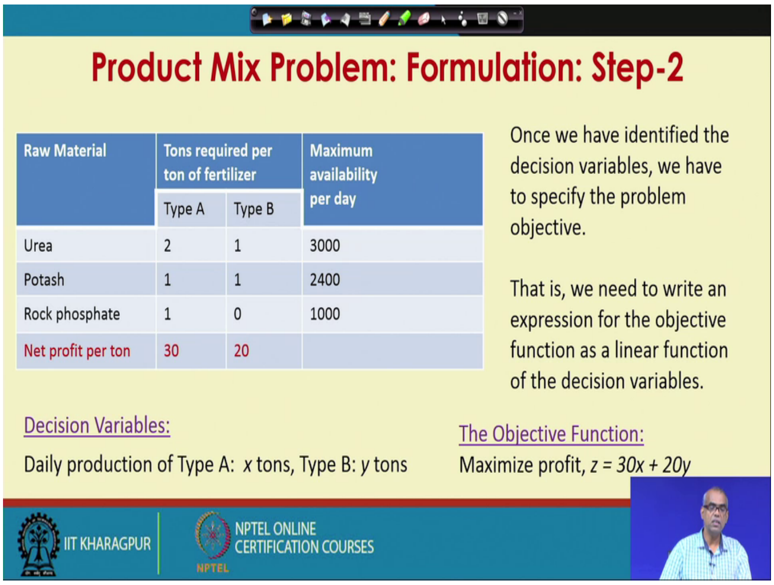

Once you have identified the decision variables we have to specify the problem objective in the next step. That is you need to write an expression for the objective function as a linear function of the decision variables decision variables are x and y, which represent on type A produced in terms of type B to be produced respectively. So, the objective function is related to maximization of profit.

So, net profit per ton are given for type A it is 30 and for type B it is in 20, let us say in unit or rupees or any unit of money. So, if I am producing x terms of type A the profit associated with type A is 30 x and the profit associated with type B is 20 y. So, the total profit z equal to 30 x plus 20 y. So, the objective function is z equal to 30 x plus 20 y and we need to maximize this function.

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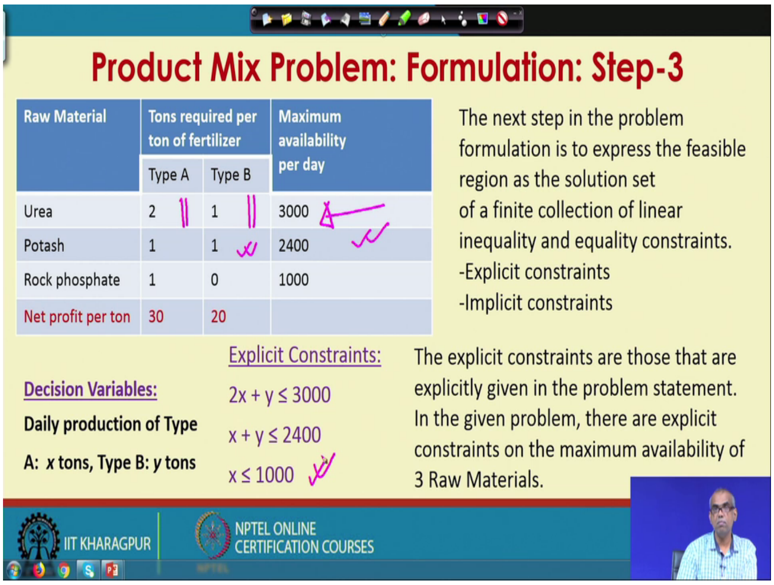

So, you have now decided the decision variables we have also determined the objective function. So, now, you have to formulate the constraints.

So, the next step in the problem formulation is to express the feasible region as the solution set of a finite collection of linear inequality and equality constraints. The feasible region is the region, where if we take a point the point will satisfy all the constraints simultaneously and we have to look for the optimal solution within this feasible region. So, the constants together will determine or define the feasible region.

So, we discuss that we have explicit constants and implicit constants the explicit constants of those that are explicitly given in the problem statement in the given problem. There are explicit constants on the maximum availability of 3 different types of raw materials namely urea potash and rock phosphate. For example, urea the maximum availability per day is 3000 ton for potash the maximum availability per day its 2400 ton and for rock phosphate the maximum availability per day is 1000 ton.

So, to produce 1 ton of fertilizer a we need 2 tons of urea. So, to produce x ton of type A fertilizer we need 2 x ton of urea. Similarly to produce 1 ton of fertilizer B we need 1 ton of urea. So, to produce y ton of fertilizer B we need y ton of urea. So, total amount of urea is 2 x plus y that must not exceed the maximum available value per day.

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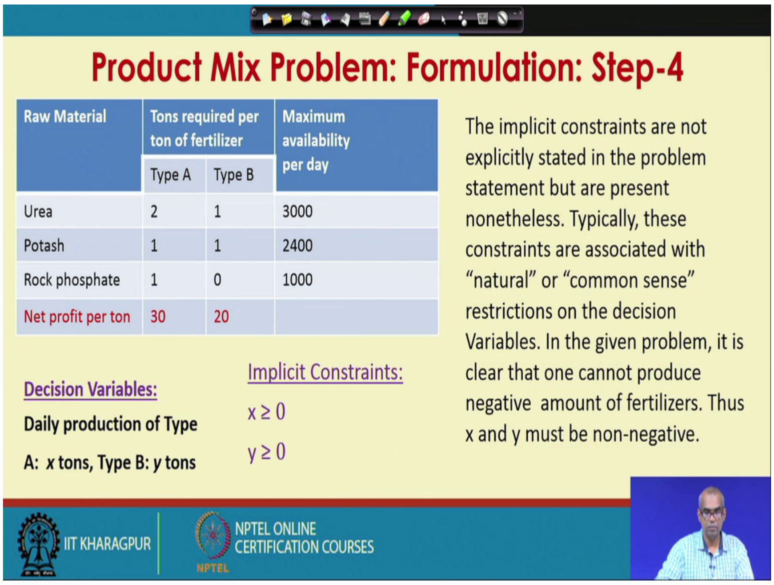

So, 2 x plus y is less or equal to 3000 similarly for potash we can write x plus y is less or equal to 2400 and for rock phosphate it is required only for type A. So, x is less or equal to 1000. So, these are explicit constraints associated with the maximum availability of the raw materials per day. After we have written down the explicit constants let us look at the implicit constraints.

The implicit constants are not explicitly stated in the problem statement, but are present nonetheless typically these constants are associated with natural or common sense restrictions on the decision variables in the given problem it is clear that one cannot produce negative amount of fertilizer type A or type B. So, x and y cannot have negative values. Note that it can geo values you can see you can say that, I may not produce any of this fertilizer type A or type B, I may not produce type A I may produce all type B or vice versa. So, x and y cannot have negative values it must be non-negative. So, the implicit constraints on x and y are x greater or equal to 0 y greater or equal to 0.

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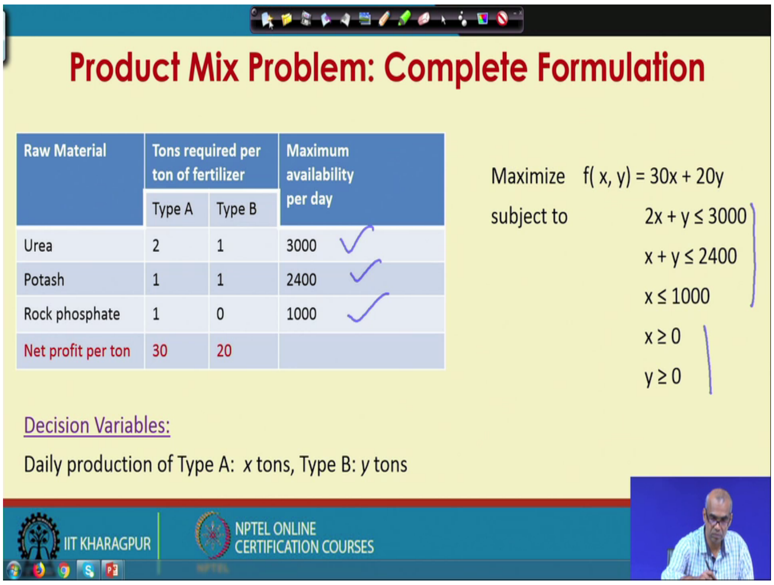

So, the complete formulation is now presented here. So, maximize f equal to 30 x plus 20 y subject to 2 x plus y less or equal to 3000 which is constant associated with availability of urea per day x plus y less or equal to 2400, which is constant associated with potash and then x is less or equal to 1000 which is constant associated with availability of rock phosphate. So, these are explicit constants and then to non-negativity or implicit constants which are x 0 equal to 0 y greater or equal to 0. So, this represents the complete formulation of the product mix problem.

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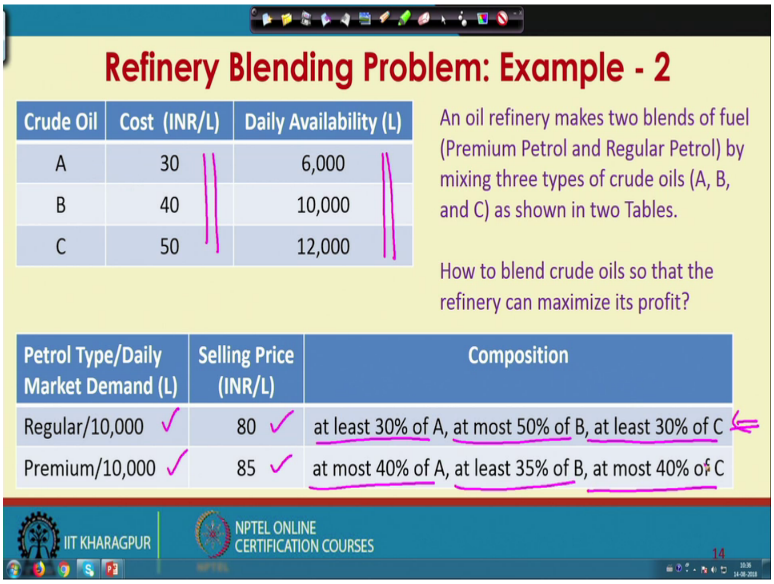

So, let us take another problem which is a refinery blending problem. An oil refinery makes two blends of fuel premium petrol and regular petrol by mixing 3 types of crude oils A, B and C, look at the two tables the details are given in the tables. So, the cost of crude oil say ABC are given the daily availability in liters are also given for the crude oil ABC I also given for the crude oil ABC.

The market demand daily market demand for regular petrol is 10,000 liter and for premium petrol is also 10,000 liter the selling price for regular patrol let us say 80 rupees per liter and for premium petrol it is 85 rupees per liter. There is a restriction and composition of regular petrol and premium petrol the composition on the composition constraint or regular patrol is this that it must contain at least 30 percent of A, at most 50 percent of B and at least 30 percent of C. You have to take care of the language here at least 30 percent of pay that means, it is greater or equal to 30 percent at most 50 percent of B meaning less or equal to 50 percent of B.

Similarly, the constant composition on premium petrol is at most 40 percent of A that means, less or equal to 40 percent of A at least 35 percent of B meaning greater or equal to 30 5 percent of B and at most 40 percent of C that means, less or equal to 40 percent of C. So, these are composition constants.

Now, the question you ask is how to blend crude oils ABC show that the refinery can maximize its profit.

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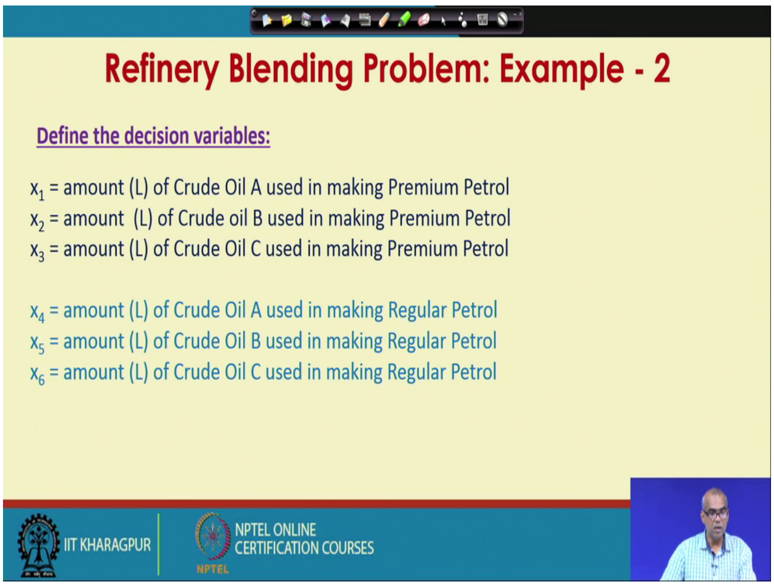

So, let us define the decision variables. The decision variables we choose are amount of crude oil a used in premium petrol x 1, x 2 as amount of crude oil B used in making premium petrol an amount of crude oil C used in making premium petrol we call it x 3. So, x 1, x 2, x 3 are amount of crude oil ABC respectively for making premium petrol similarly x 4 is amount of crude oil A used in making regular petrol x 5 is amount of crude oil B used in making regular petrol an x is amount of crude oil C used in making regular patrol.

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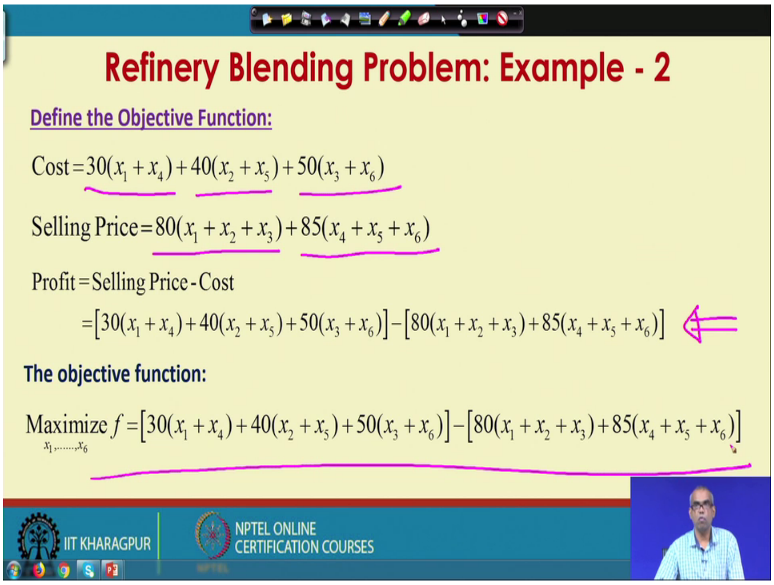

So, now, define the objective function. So, what is the cost? Cost are given as let us take another example on linear programming problem. Here we have a refinery blending problem.

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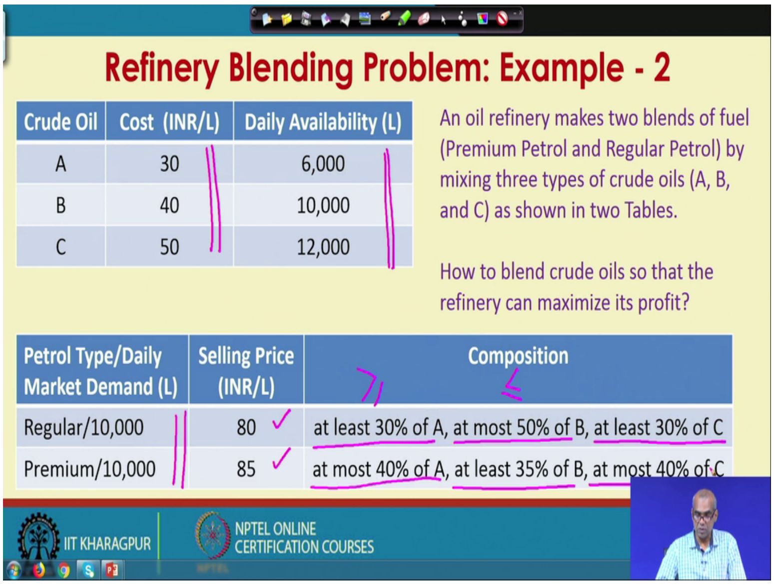

An oil refinery makes two blends of fuel premium petrol and regular petrol by mixing 3 types of crude oils A, B and C. Look at the tables for details. The cost of 3 different types of crude oils are given in terms of rupees per liter, the daily availability of the crude oils are given in liters.

The daily market demand for regular petrol and premium petrol are both 10,000 liters the selling price for regular petrol is 80 rupees per liter and selling price for premium petrol is 85 rupees per liter. Now, there are constants and composition of a regular petrol and premium petrol. The composition constants have stated as a regular petrol must contain at least 30 percent of A, at most 50 percent of B and at least 30 percent of C. So, when it is stated at least 30 percent of A it means is greater or equal to 30 percent of A. When it is stated at most 50 percent of B it means less or equal to 50 percent of B. So, these needs to be taken care of.

Similarly the composition constants on premium petroller at most 40 percent of A means less or equal to 40 percent of A at least 35 percent of B means greater or equal to 35 percent of B and at most 40 percent of C meaningless or equal to 40 percent of C.

Now, to formulate the problem let us first define the decision variables the question you ask is how to blend crude oil A, B and C. So, we define the suitable decision variables as amount of crude oil A, amount of crude oil B, an amount of crude oil C that needs to be blended for making regular petrol. So, let x 1 equal to amount of crude oil A, x 2 equal to amount of crude oil B and x 3 equal to amount of crude oil C that will mix together to make regular petrol. Similarly x 4 is amount of crude oil A, x 5 is a amount of crude oil B and x 6 equal to amount of crude oil C that will blend in making premium petrol.

Now, let us define the objective function. So, define the objective function we have to here get an expression for profit. So, profit is selling price minus cost. So, cost is 30 into x plus x 1 plus x 4. So, this is the cost associated with crude oil A, this is for crude oil B and this is for crude oil C. So, you sum them up and you get the total cost.

Similarly, selling price x 1 plus x 2 plus x 3 is the liter regular petrol. So, 80 into x 1 plus x 2 plus x 3 is the selling price for a regular petrol, similarly 85 into x 4 plus x 5 plus x 6 is the amount you get by selling the premium petrol. So, property is selling price minus cost price and this is the expression for depth, and you see that this is a linear expression. So, objective function is this expression for the profit which is a linear function in decision variables x 1 to x 6.

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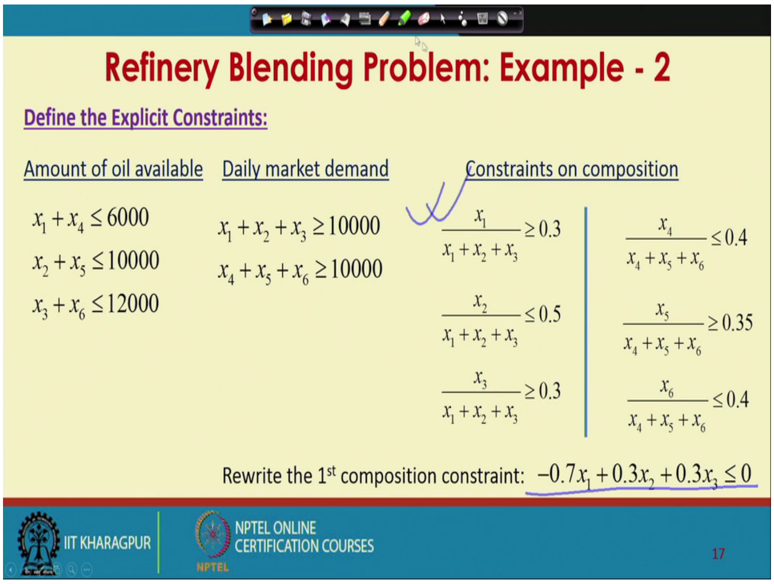

Now, define the explicit constants first constants are amount of oil x 1 plus x 4 is the crude oil A. So, that must be less or equal to 6000 that is daily available limit. Similarly x 2 plus x 5 is less or equal to 10,000 and x 3 plus x 6 is less or equal to 12,000 daily market demand must be met and market demand for both regular petrol and premium petrol are 10 thousand liters you produce x 1 plus x 2 plus x 3 liters of regular petrol and x 4 plus x 5 plus x 6 liters of premium petrol. So, x 1 plus x 2 plus x 3 must be greater or equal to 10,000 and x 4 plus x 5 plus x 6 must be greater or equal to 10,000.

Similarly, constant compositions note that x 1 by x 1 x 1 by x 1 plus x 2 plus x 3 is greater equal to 0.3 that means, at least 30 percent A. Similarly the next one is x 2 by x 1 plus x 2 plus x 3 is less or equal to 0.5 that means, at most 50 percent of B, similarly the other constants. So, we have the constants on amount of oil available daily market demand and constraints on composition.

So, you have to rewrite these composition constants, you have 6 composition constants. So, you can write as if you look at this equation first equation x 1 by x 1 plus x 2 plus x 3 is greater equal to 0.3, you can write as x 1 is greater or equal to 0.3 into x 1 plus x 2 plus x 3 which can be written as minus 0.7 x 1 plus point 3 x 2 plus 0.3 x is less or equal to 0. So, is there any implicit constants? Implicit constants are there because x 1 to x 6 has to be non-negative. So, this represents the complete formulation. With this we stop lecture 36 here.