Chemical Process Instrumentation

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Lecture – 09
Performance Characteristics of Instruments and Data Analysis - I (Contd.)

Welcome to week 2 lectures 9; in week 2, we are talking about performance characteristics of instruments and data analysis part one in your previous lecture we have talked about generalized mathematical model for instruments, we have seen what will be the form of the equation for zero order instrument, first order instrument, second order instrument.

We started with an ordinary differential equation and by setting n equal to 0 1 and 2 we obtain the differential equations for zero order instruments, first order instruments and second order instruments, we also learned little bit of Laplace transformation technique for solution sub ordinary differential equation, by taking the Laplace transformation you can find out the transform function of instruments; which basically is y s by x s, where y s represents the output of the instrument as Laplace transform quantity divided by x s which is the Laplace transform quantity of input.

So, transform function is y s by x s which is the Laplace transform quantity of output divided by Laplace transformation of input. So, we have also seen what are the, what are the transform functions? For zero order instruments, first order instrument and second order instruments. So, in this lecture we will take examples of zero order instrument, first order instrument and also briefly talked about step response of first order instrument.

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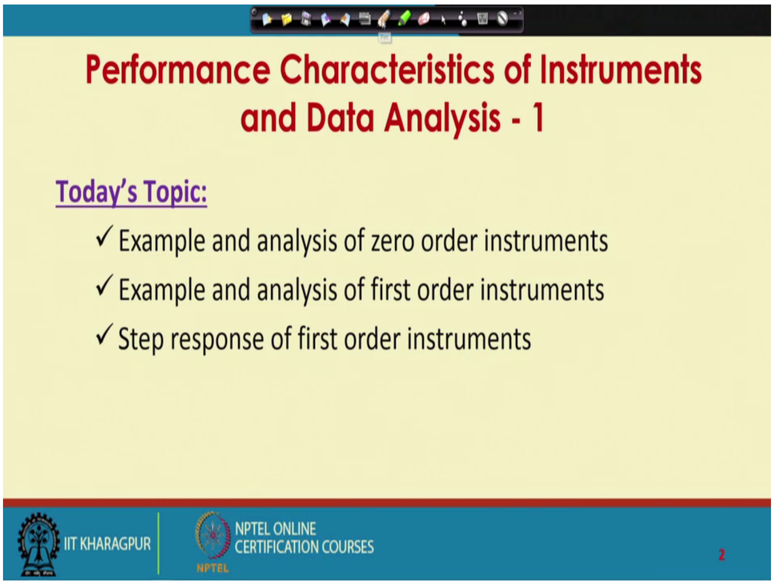

So, today's topic is example analysis of zero order instrument, example and analysis of first order instruments and we will also talk about step response of first order instruments.

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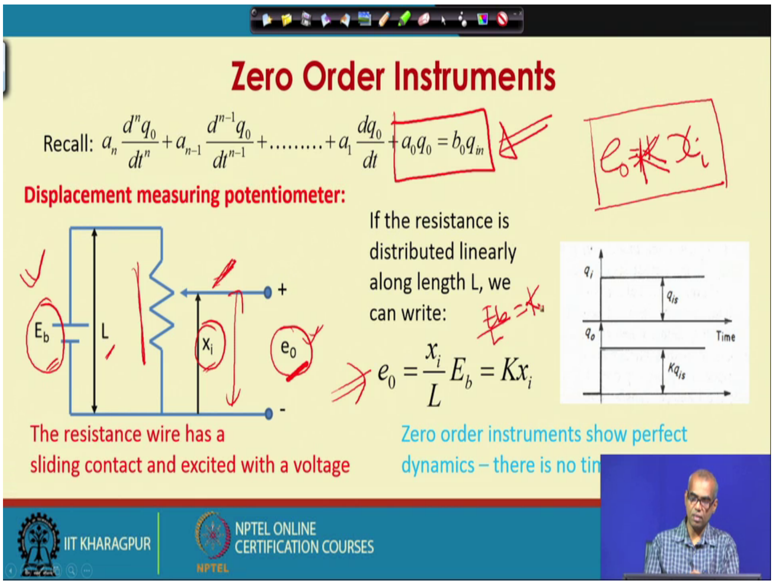

Recall that; the generalized mathematical model for an nth order instrument is represented by this nth order ordinary differential equation. So, for zero order instruments I written only this as the model for the zero order instrument.

A displacement measuring potentiometer can be considered as an example of zero order instruments. This is a displacement measuring potentiometer, the resistance wire has a sliding contact and this is excited with a voltage E b, if the resistance is distributed linearly along the length of the wire we can relate the output with this input as this. So, basically you can write e 0 into L is equal to E b into x i or e 0 equal to x i by L into E b and E b by L is K or the static sensitivity, so the zero order displacement measuring potentiometer assumes the model equation as output equal to static sensitivity K into x i which is the input, this is x i.

Note that; this is the sliding contact so it can go up come down accordingly x i will change, accordingly output voltage will change and output voltage is E b by L into x i and E b by L is static sensitivity K. Now look at this equation; this equation does not have any differential terminal in it, it is a pure algebraic equation we say zero order instrument show perfect dynamics, because they are cannot be any time delay or lag because there is no time term involved in it, so output will immediately follow input.

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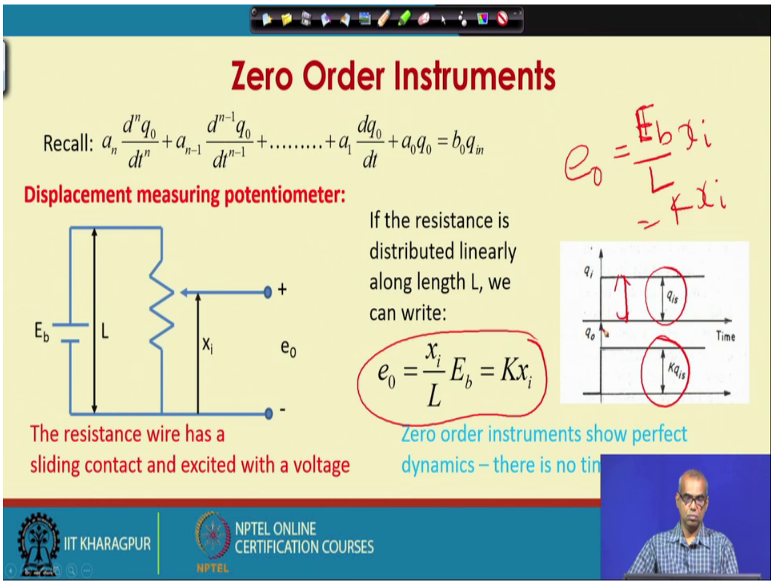

As this equation suggests E b; if I give this output this input sorry if I give this input immediately the output will be static sensitivity multiplied by this quantity, there will not be any time delay or lag, so output becomes the static sensitivity which is constant multiplied by the input.

So, this instrument can be considered as an ideal instrument, because there is no time delay or there is no time lag output immediately follows input and output is a constant multiple of input, that constant multiple is the static sensitivity for displacement measuring potentiometer this becomes; E b by L, where E b is the exciting voltage and L is the length of the wire.

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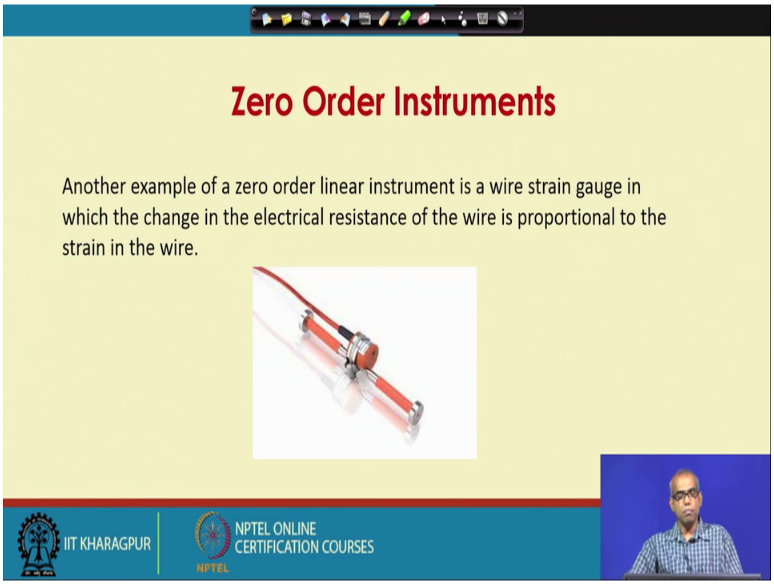

Another example of a zero order linear instrument is a wire strain gauge in which the change in the electrical resistance of the wire is proportional to the strain in the wire.

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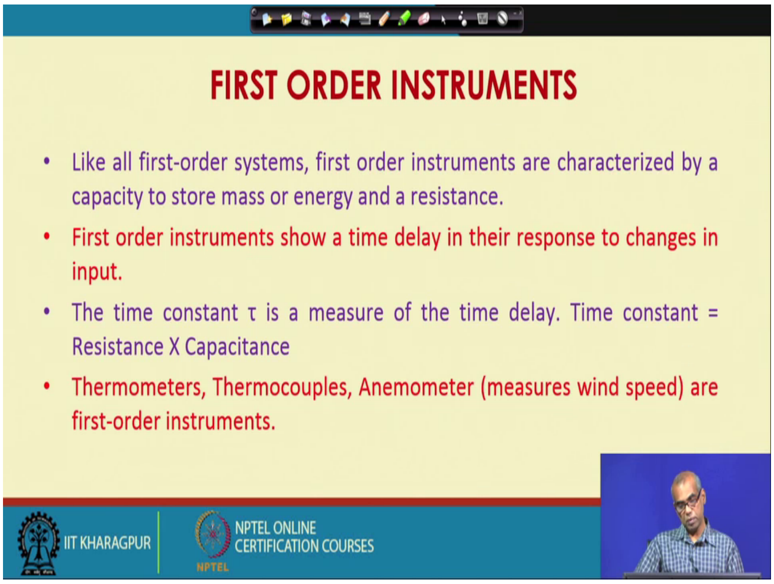

Now, let us talk about first order instruments; like all first order systems, first order instruments are characterized by capacity to store mass or energy and a resistance. So, in all first order systems or all first order instruments you will be able to identify two terms; one is capacitance, another is resistance, so capacitance represents the instruments capacity to store mass or energy and there will also be a resistance associated with this flow of mass or flow of energy.

So, these two terms; capacity to store mass or energy and resistance will always the presenting any first order instrument, now what will represent capacity? And what will represent resistance? Will depend on the particular instrument we are talking about, but in all first order instruments there will be these two terms. The first order instruments show a time delay in their response to changes in input, so first order instruments will have a time constant, you have seen in the mathematical model for a first order instrument there was two terms; one was, case static sensitivity and another was time constant tau which has unit of time, so this time constant is a measure of delay in response this indicates in some sense speed of response.

The time constant tau is a measure of the time delay and the time constant is product of the resistance in the capacitance, as I told you that all first order instruments will have a capacitance and will have a resistance and the time constant of this first order instrument will be the product of this capacitance and resistance. Thermometers thermocouples anemometer that measures wind speed are all examples of first order instruments, so when I talk about thermometer and thermocouples; I consider bear thermometers and bear thermocouples, what it means that? Sometimes thermometers and thermocouples are put inside a protective cover, to protect it from the effect of measuring environment.

So, in that case basically the combined system becomes two first order systems connected in series, because thermometer itself is a first order systems and then you have a protective cover around it which is like a say; sealed tubes sealed at one in and inside the tube you have put the thermometer or thermocouple, so this tube or this protective tube or sealed itself works like a first order system, because it also has capacity to store say thermal energy in case of thermometer and there will also be a resistance associated with this flow of the energy.

So, a bear thermometer or bear thermocouple; that means, the thermometer and thermo couple without the protective cover is the first order instrument, but if I consider the thermocouple or the thermometer is put inside the protective tube then together it becomes to first order systems connected in series. So, the overall responsible will then be second order in nature.

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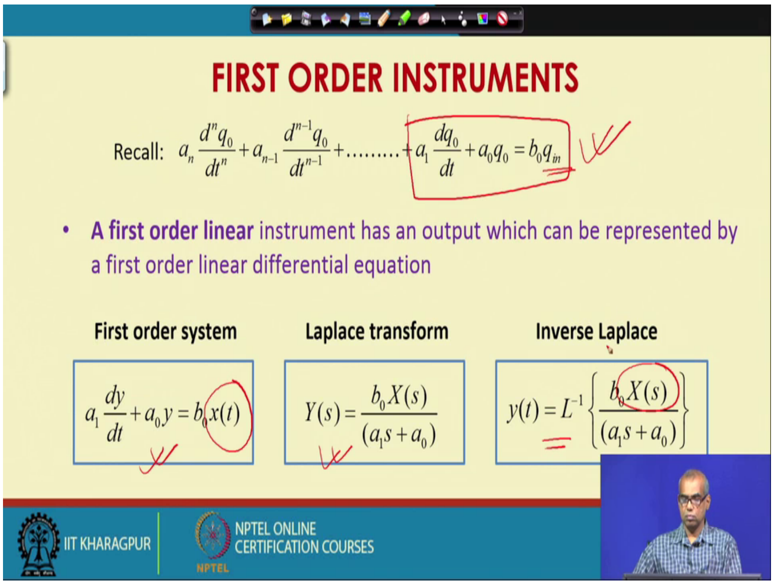

So now; recall this equation representing, the generalize model for any nth order instrument. So, for first order instrument I will written only the first order terms so this represents the model for first order instrument. So, first order linear instrument has an output which can be represented by a first order linear differential equation.

So this; a 1, a 0 are the combination of systems parameters and their constant coefficients. So, if we consider input as say x t, so I put x t in place of u n this becomes the first order systems mathematically equations a Laplace transformations and then if you take inverse Laplace transformation, we will get the output corresponding to this input.

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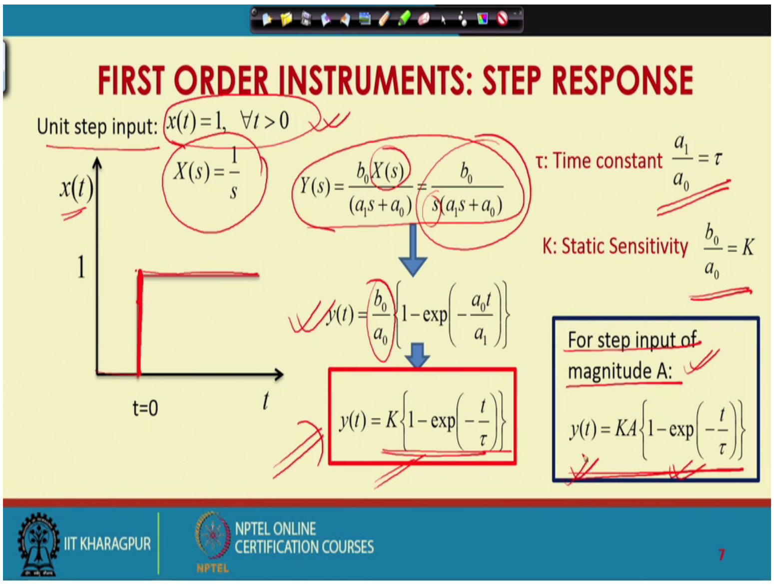

So, let us consider; I am giving a unit step input to my instrument. So, my instrument a first order instrument you can consider it to thermometer at t equal to 0 I give a step input of magnitude 1, so I called it a unit step input. So, the input to the instrument up to this was let say 0 and at time t equal to 0 this is my start of the experiment, so at time t equal to 0; I suddenly make the input as 1 and keep it there, this is the step input so this can be represented as x t equal to 1 for all time t greater than 0 you can also write x equal to 0 for all time t less than 0, this becomes sufficient, so x t equal to 1 for all time t greater than 0.

You have seen that Laplace transformation of this is; 1 by s, so put 1 by s in place of X s I have done it and then if I take inverse of this I get this, which is the output of the instrument for unit step input. Remember b 0 by a 0 represents static sensitivity and a 1 by a 0 represents time constant, so this equation becomes this; y t equal to K into 1 minus e to the power minus t by tau, so this equation represents the output of an instrument for a step input. So, this is the output of a first order instrument for a step input, what kind of step input? Step input of magnitude 1. So, if I give a step input of magnitude a; this will be multiplied by the magnitude of the step input.

So, y t equal to K into a is 1 minus e to the power minus t by tau represents the output of a first order instrument, when I give a step input of magnitude a to this input. So, given a step input of magnitude a; to the instrument this equations represents the output of the instrument, so how the output of the instrument changes with time? Can be obtain from this equations, so if you know the time constant of an instrument if you know the sensitivity, we will be able to find out the response of the instrument from this given equation.

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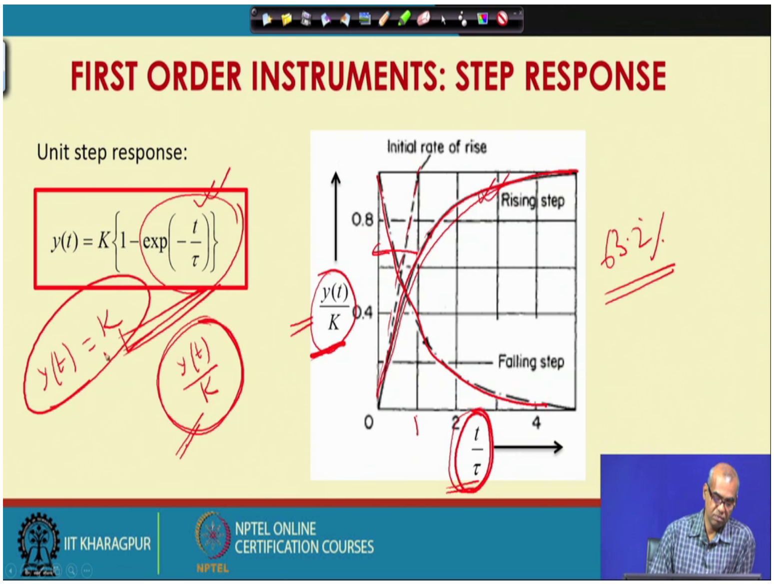

Once again; this is the response of a first order instrument for unit step input, now if I make a plot of y versus K; so y versus K and again it versus t by tau, so I am plotting y t by K for t by tau. Note that: if I am given a unit step input this is the response, what will be the ultimate or the final response? That can be obtained for very large t. So let us put t equal to infinity, if I do that this terms becomes 0, so the final response becomes K for unit step input.

So, y t by K becomes the dimensionless; measure of the input, similarly t by tau is the dimensionless time because time tau as time constant. So, if I make a plot of y t by K versus t by tau I will get a response like this for a unit step input. Note that: the initial rate of rise was this, but this initial rate of rise is not maintained throughout. The output of the instrument increases and asymptotically matches with this value, also at t by tau equal to 1; that means, at t equal to tau the output that you get; will be 63.2 percent of the final response, so this is the characteristic of a first orders instruments, that at time t equal to tau you will achieve the 63.2 percent of the final response.

So, by plotting the output of a first order instrument let us say thermometer, i have my thermometer versus radiator particular temperature then I suddenly give a step input to the thermometer or in or any input of a any magnitude a, I give to the instrument and then record that thermometers reading so but looking at the 63.2 percent of the response final response because I know what will be the final response, I will be able to measure or estimate the time constant. So, this method gives me a way to measure the time constant of any first order instrument by looking at it is response, so just look at the 63.2 percent of the final response and the corresponding time is the value of the time constant tau.

So, this was the response of a positive step input. So, my x ray thermometer was a 20 degree Celsius and suddenly put it to boiling water which is 100 degree Celsius. So, this is a positive step input of magnitude 100 minus 20 equal to 80, I can also bring back the thermometer from 100 degree Celsius boiling water 220 degree Celsius temperature. So, in that case it will be a negative step input of the same magnitude, because 20 minus 100 is minus 80 so in that case the response will be this. So, this is the falling step input response of response of falling step input or negative step input and this is the response for the positive step input or rising step input. The characteristics will be same; at time constant tau, you will achieve the 63.2 percent of the final response and you can also find out the final response from here.

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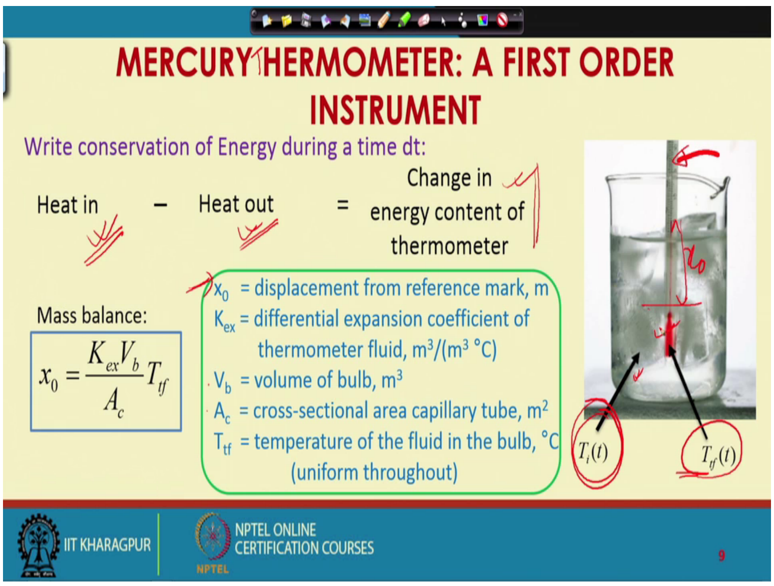

So now, will take a physical example of a first order system. So, let us consider a mercury thermometer; T is missing here, a mercury thermometer is a first order system of mercury thermometer is a first order instrument what we should do now is; will write down or will develop the mathematically equation for the mercury thermometer and will see that we obtain a first order ordinary differential equation. So, mercury thermometer is a temperature measuring instrument. So for; obviously, to develop the mathematical model for the thermometer you have to write energy balance equation. So, let us write conservation of energy during any time T, for this thermometer.

So, you have this thermometer ordinary mercury in glass thermometer, put into a beaker; let us say, there is water or some fluid in the beaker whose temperature is represented by T i which is a function of time. So, thermometer bulb receives this thermal energy, so let us there is a mercury inside it undergoes restricted thermal expansion, so it goes up mercury goes up through this capillary and from the scale attached to the or graduated on the thermometer you will be able to find out the temperature. So, let us say the temperature of the fluid is T i and the temperature of the thermometer or the thermometer bulb is T t m.

So, now the energy balance equation is; heat in, to the thermometer minus heat out from the thermometer equal to change in energy content of the thermometer. So, let us look at the bulb of the thermometer because that is where this energy exchange is taking place, so heat in to the mercury ball heat out from the bulb and change in energy content of the bulb of the thermometer. Let us makes certain assumption that there is no heat loss, let us also assume; that the physical properties of the thermometer fluid or mercury does not change with time, so density of the mercury does not change with time specific it is capacity does not change with time so on and so forth all physical properties remain constant.

Then there is one important assumption; which you make is as follows, that all the capacity to store thermal energy recites in the bulb and all the resistance to flow this thermal energy also recites here. So, I am doing lumping of capacitance and resistance, so this is not as lumping of parameter, I do not consider that this capacitance and resistance varies in space.

So, the capacitance or the capacity to store the entire thermal energy is located in one place in the bulb, the same thing about the resistance so all the resistance to flow to this thermal energy recites in the bulb, so this is known as lumping of the parameters it leads to lumped parameter models. If I consider that there is a variation of capacitance and resistance in space, then we will not get an ordinary differential equation, but we will get a partial differential equation, so our mathematical model will be more complex but this lumping of this parameter is a reasonably good approximation here.

Now, let us use this notations; x 0 equal to displacement from the reference mark, let say you have a reference mark 0 here right so this becomes x 0, K e x is the differential expansion coefficient of the thermometer fluid, V b is the volume of bulb, A c is the cross sectional area of the capillary tube, T t f is the temperature of the fluid in the bulb, we discussed it previously we consider it uniform throughout. Now as I told you; the bulb in the mercury receives thermal energy and undergoes restricted thermal expansion, because of that a pressure is developed and the mercury moves up through this capillary.

So, what moves up; through this capillary, that amount of balloon comes from the expansion of the mercury due to change in temperature, so I can write down of mass balance equation. So, the amount of mercury in the capillary initially it was below, the mercury was at this reference level now I have mercury for this x 0 length of capillary. So, the amount of mercury that is represent there is x 0 into A c that volume, A c represents cross sectional area of the capillary tube, so A c multiplied by x 0 represents the volume of the mercury in this capillary, so that will be equal to K e x into volume of the bulb into T t f, why T t f? Basically T t f minus 0, initially it was a 0 let say it was not temperatures showing 0 temperatures. So, the change in temperature is T t f.

So, the x 0 into A c has to be equated to volume expansion coefficient time's volume, time's temperature change look at the unit here. So, x 0 into A c which is volume; becomes K e x, V b into here or I can rearrange in and write as x 0 equal to k e x into V b into T t f by A c, we will make use of this equation later.

Now, the heat input can be written as this; which represents heat transfer coefficient, area of heat transfer this is the area of the bulb and the temperature difference thermal energy goes from the liquid in the bigger to the bulb, so liquid temperature is t i bulb temperature is T t f, so t i minus T t f represents the temperature difference, there is no heat loss so there is not heat out, so heat in minus heat out is represented by this term, change in energy of thermometer comes from m c p d t; that means, mass of the mercury into specific heat of the mercury into the temperature difference.

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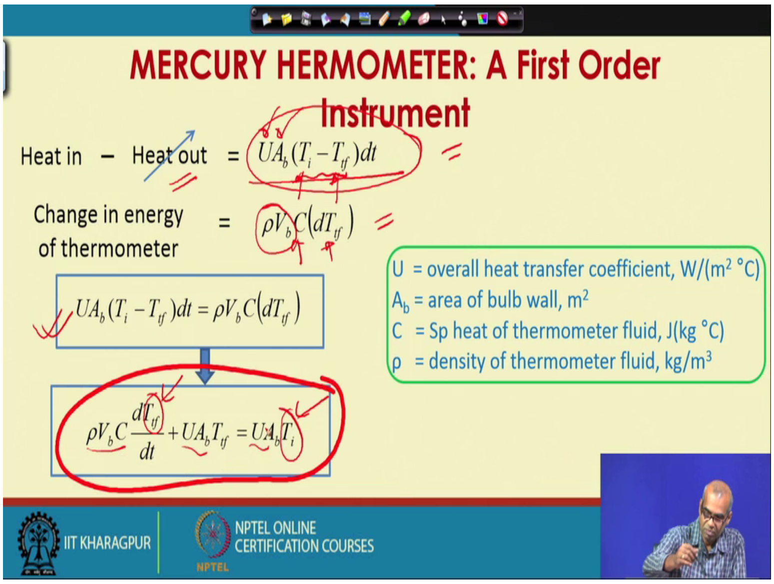

So, rho into V b is the mass of the mercury, rho is the density of the mercury V b is the volume of the bulb, so there is the volume of the mercury, so rho V b is the mass of the mercury into specific ate into temperature difference. So, let us equate this with this; that is what I am doing here and if I rearrange, I get this look at here this is a first order differential equation, which relates T t f with t i this is output and this is input. So, rho V b c, U A b these are all the system parameters. So, combinations of the systems parameters are those; a 1, a 0, b 0 those kind of terms.

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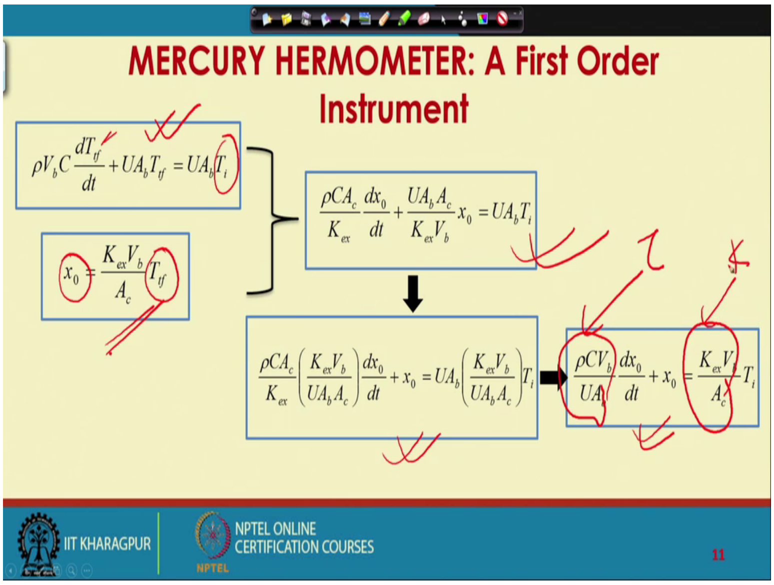

So, this is the equation we just derived; I can also write this equation in terms of; d x 0 d t, so this equation output is the temperature of the thermometer of the bulb temperature of the bulb of the temperature indicated by the thermometer that is related to this input t i. Now since this temperature of the bulb of the temperature indicate by thermometer is also related to the x 0, which is the mercury level in the capillary and they are related by this equation; I can also replace t f making use of this equation and can write this equation. So, again this is also a first order ordinary differential equation which can be rearranges as this and finally as this.

So, if you look at this equations these entire thing becomes the time constant tau and these entire thing becomes the static sensitivity k.

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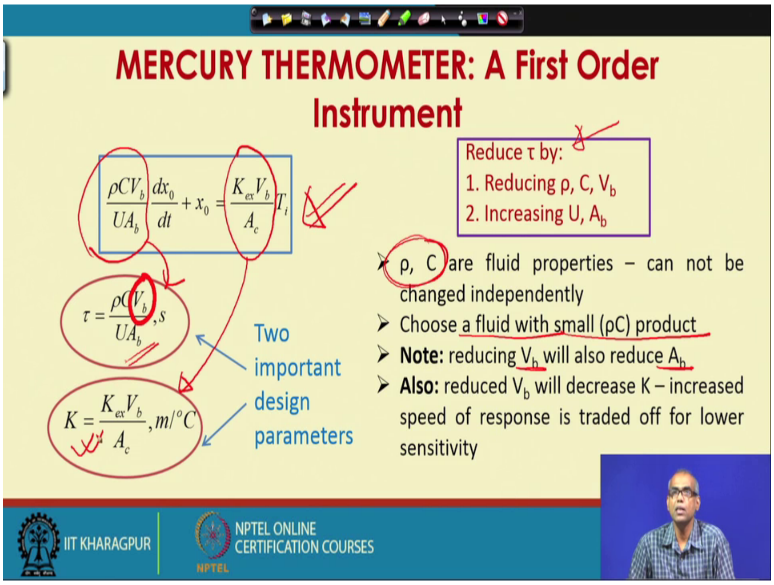

So, this is the equation we just obtain these I told you that this is tau time constant and this is the statistics sensitivity K. Now these are two important design parameters, you want the mercury thermometer to have low time constant because if the instruments time constant is low the speed of response will be high, imagine that at one time constant you will get 63.2 percent of the final response, so smaller the time constant value faster is the response larger is the time constant value slower or slaggy is the response. So, you want my thermometer to have a low time constant, now I know which combinations I know the combination of the parameters that gives me time constant tau, so I can appropriately tune those parameters to design a thermometer you have low time constant look at how time constant is rho C V b by U A b.

So, you can reduce tau; by reducing, the terms that are appearing in the numerator that is rho C and V b rho is the density of the mercury, C is the specific heat of the mercury and V b is the volume of the bulb, we can also increase U and A b to reduce time constant. Now rho and C mercury density and heat capacity at the properties of the mercury, so they cannot be change independently once you have choose the mercury once you have choose chosen mercury or any other thermometer fluid the density under specific heat is fixed. So, I cannot independently lower down this I can independently choose this, so I choose a thermometer fluid with small product rho C, so the density times the heat capacity should be lower.

I can also reduce V b; that is, the volume of the bulb.

So, volume of the bulb when I reduce it is also reduce the area of the bulb, but you want to reduce the volume of the bulb and you want to increase the area of the bulb to decrease tau, so you have to take a suitable V b by A b, also if you look at the sensitivity expression; if I reduce the volume of the bulb, the sensitivity will be decreased. So, if I want to in if I want to lower down the time constant which will give me first response by decreasing the volume of the bulb, this also decreases the sensitivity. So, if I want to reduce the time constant to obtain the faster response by decreasing the bulb; that means, I design a thermometer with small thermometer bulb that will give me the first response, but it will also decrease the sensitivity of the instrument. So, the increased speed of response is traded off for lower sensitivity.

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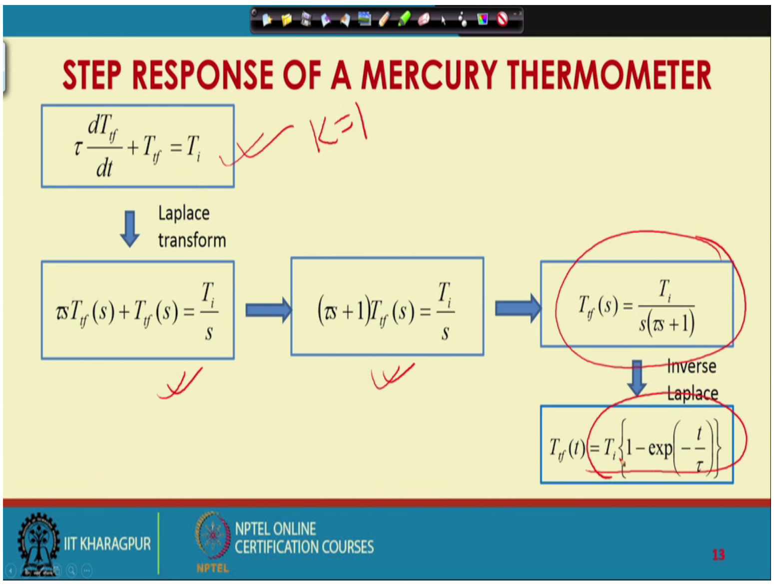

Consider this equation for the thermometer that taken the sensitive K equal to 1 here, take the Laplace transformation the rearrange this; you will get the Laplace transform output versus input relationship, if you take inverse you will get this as output in time domain. Note that: the form of the equation that you have got, this equation you got previously as well.

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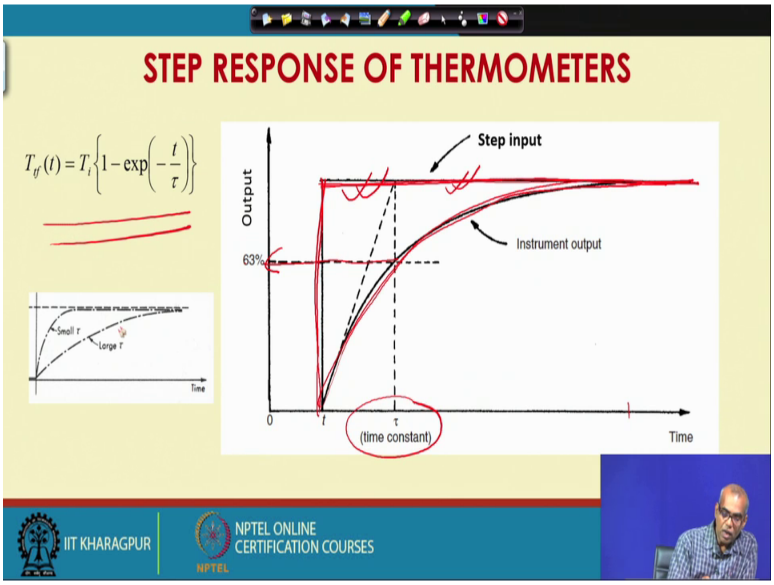

So, this is the equation for thermometer response; if I give a step input of magnitude 1 or inverse step input my response is this, at T equal to infinity it matches with this. If I gives us step input of this; I am expecting the output finally to be this, my response goes like this which asymptotically matches with this line at T equal to infinity. At one time constant value I get 63.2 percent of this final response. So, for large time constant you see the response will be slow for small time constant the response will be fast. So, we will stop here.