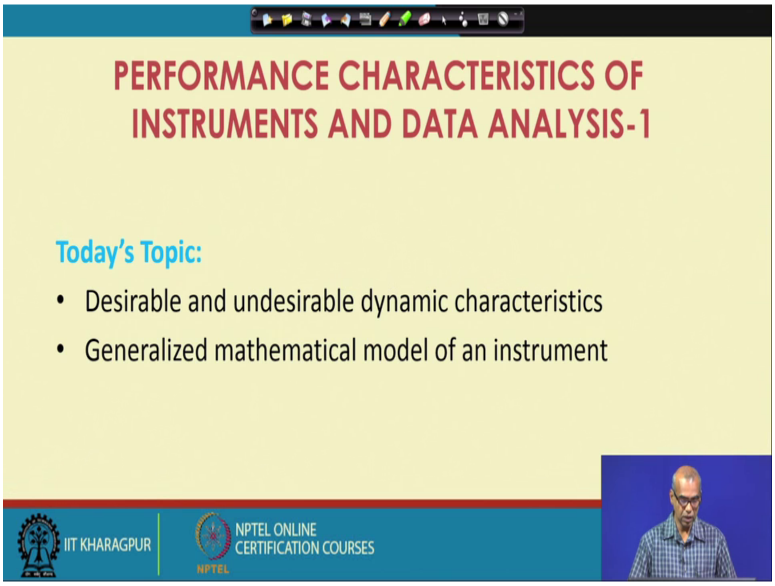
Chemical Process Instrumentation
Prof. Debasis Sarkar
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 08
Performance Characteristics of Instruments and Data Analysis – I (Contd.)

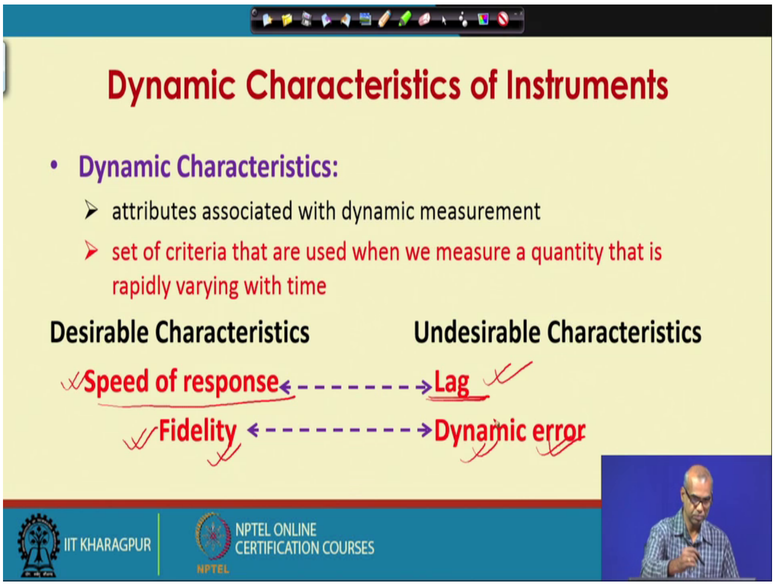
Welcome to week – 2, lecture – 8. In our previous lecture, we have talked about static characteristics. In today's lecture, we will talk about dynamic characteristics and also we will talk about general mathematical models for instruments.

(Refer Slide Time: 00:36)



So, this is today's topic, desirable and undesirable dynamic characteristics and generalized mathematical model of an instrument.

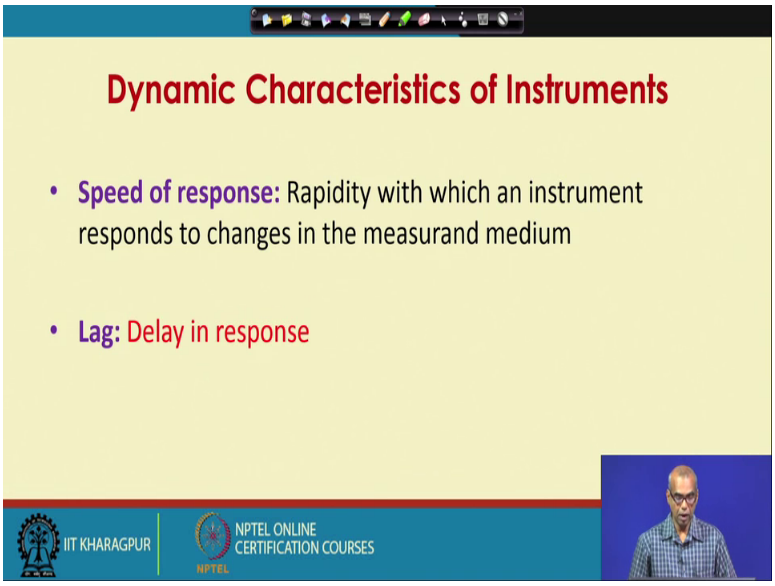
(Refer Slide Time: 00:47)



So, we start with dynamic characteristics of instruments. We have defined dynamic characteristics as a set of criteria that are used when you measure a quantity that is rapidly varying with time. So, these are the attributes associated with dynamic measurements. So, we will talk about 4 different dynamic characteristics, speed of response, lag, fidelity and dynamic error.

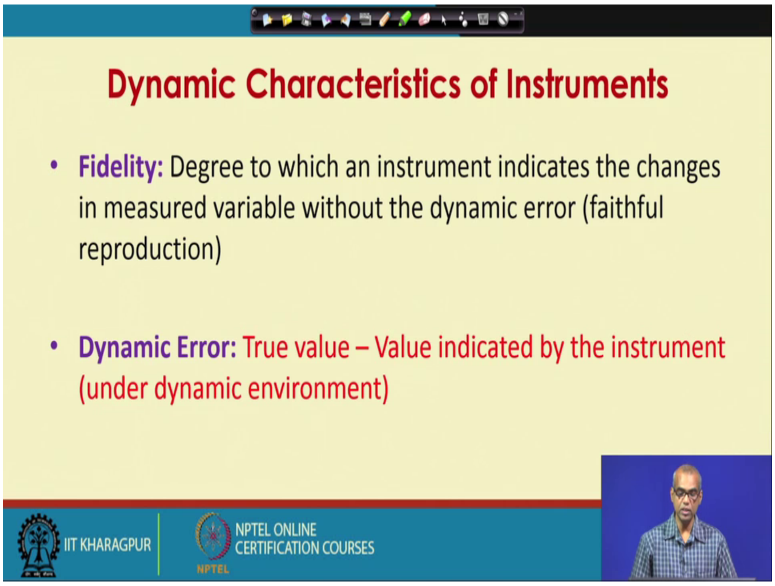
Speed of response is a desirable dynamic characteristic. So, we want our instrument to have this characteristics whereas, lag is an undesirable dynamic characteristics. Similarly, fidelity is a desirable dynamic characteristics and corresponding undesirable dynamic characteristics is dynamic error. So, if speed of response is good, lag will be less similarly for this dynamic characteristic.

(Refer Slide Time: 02:18)



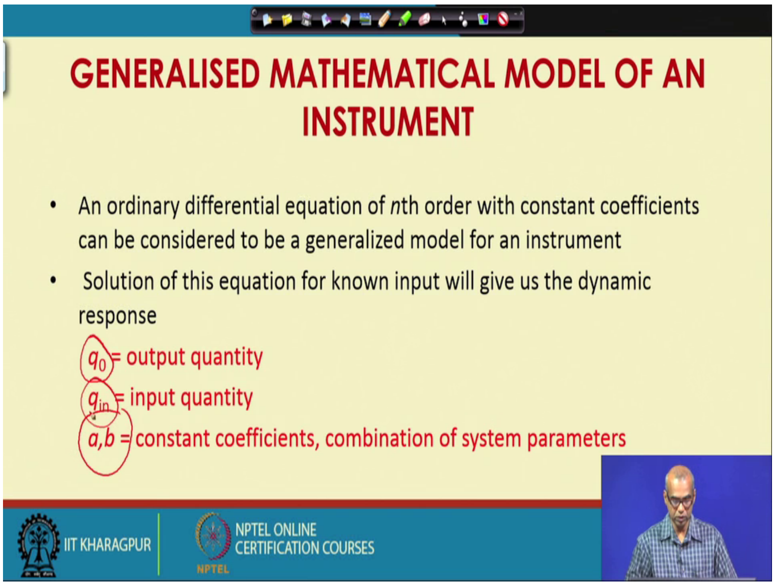
So, speed of response is defined as the rapidity with which an instrument responds to changes in the measured medium. So, lag is opposite to that. So, this is delay in response.

(Refer Slide Time: 02:36)



Fidelity is degree to which an instrument indicates the changes in measured variable without the dynamic error; that means, it is a representation of faithful reproduction and dynamic error is defined as the difference between the true value and the value indicated by the instrument under dynamic environment.

(Refer Slide Time: 03:02)

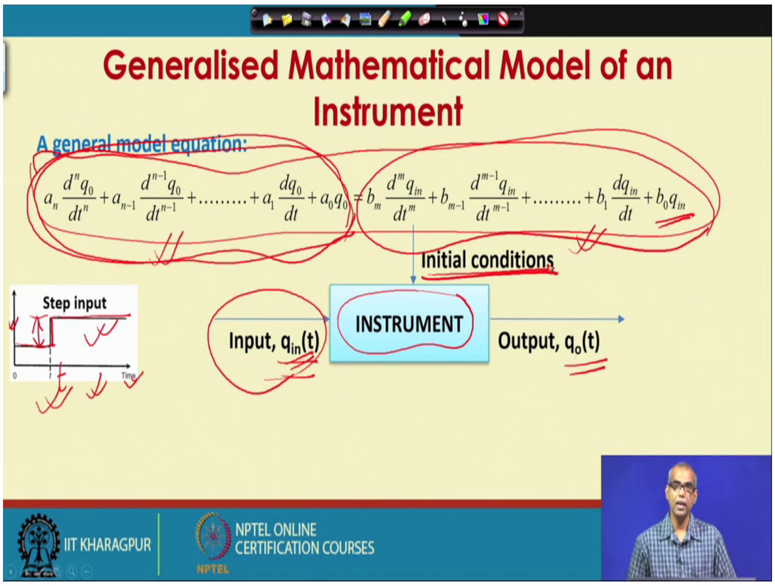


Now, we will talk about generalized mathematical model of an instrument. So, the question you ask here, is it possible to express the working of an instrument in other words is it possible to relate the input and output of an instrument by a set of mathematical equations. Usually, a differential equation is used so that you can get output for a given input with respect to time.

So, an ordinary differential equation of nth order with constant coefficients can be considered to be a generalized model of an instrument. So, this is our apotheosis that an ordinary differential equation of nth order with constant coefficients can be considered to be a generalized model for an instrument. So, if we consider any general instrument we assume that an ordinary differential equation of nth order with constant coefficients will be able to relate the relationship that exists between the input to the instrument and output from the instruments. So, let us first write down an nth order general ordinary differential equation with constant coefficients.

So, solution of this equation for known input will give us the dynamic response. So, let us define before we write the ordinary differential equation of nth order. Let us define; q 0 as output from the instrument, q in is the input to the instrument and a, b these are the constant coefficients. Essentially, this will be the combinations of system parameters. So, different instruments will have different a's and b's.

(Refer Slide Time: 05:12)



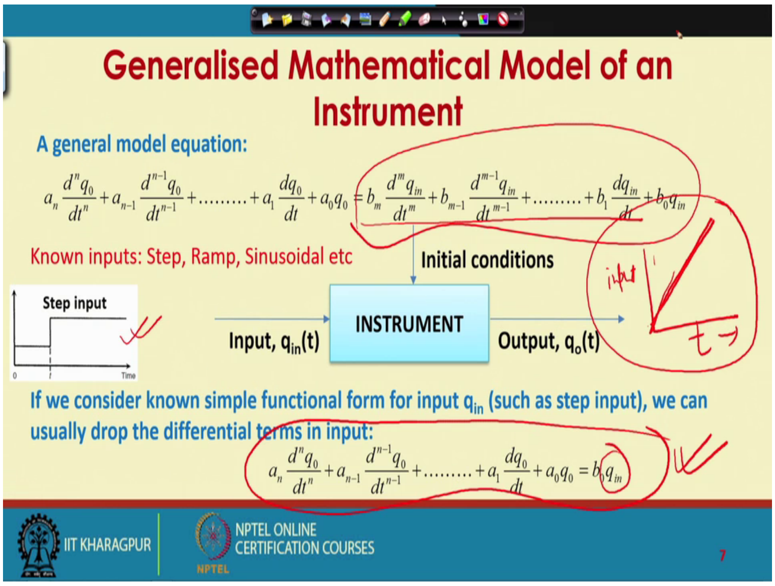
So, let us now look at this a general nth order ODE. q 0 is output. So, the differential equation on the left hand side represents the output and the differential equation on the right hand side represents the input. So, an nth order differential equation represents the output and similarly, an mth order differential equation represents the input. Note that, both input and output are functions of time that is why we are representing output by any nth order differential equation and input by any mth order differential equation.

So, if now, I consider that this black box represents instrument and input goes to this instrument let us say instrument may be a step input which says that input was steady up to time t. At time t, I have given a sudden change in the input values. So, this is the input values axes and this is time. So, at time t, I have given a step input of this magnitude.

So, q in represents this function. So, the mathematical function representing this step input is basically q in and output q 0 t will be the solutions of this differential equations, when we put this q in which is the mathematical representation of the step input and we know the values of a's and b's which are the constant coefficients of the ordinary differential equations. So, this is differential equation, it requires approximate appropriate initial conditions for its solutions. So, this is a general scheme for mathematical modeling of an instrument.

You basically relate output and input by a general ordinary differential equation with constant coefficients. So, different instruments will have different sets of these constant coefficients and we need to know the functional form of the input. For example, in the case shown it was a step input. So, the mathematical represented of step input has to be supplied then the solutions of the differential equation will give you the output which is represented as q 0 which is a function of time.

(Refer Slide Time: 08:49)



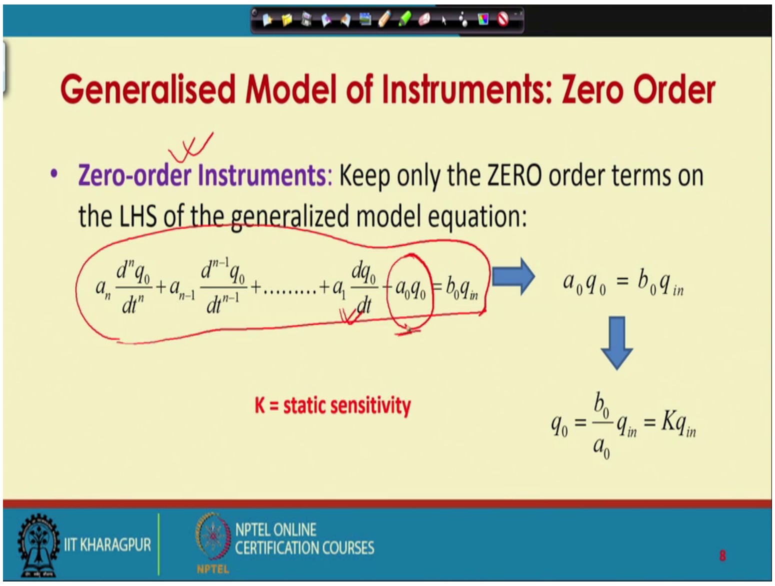
So, if you consider simple method, simple functional form such as this step input, for input, we can usually drop the differential terms in the input. What you mean here, that if we know the functional form of q in then I do not have to express q in as a general mth order ordinary differentially equation with constant coefficients because I know the functional form of the input.

So, commonly we use simple functional forms for inputs such as step input, there is ramp input which is a linear variation with time, there is sinusoidal input which changes as a sine wave so on and so forth. So, in that case we can drop the differential equation on the right hand side and can put the functional form of the input as it is. For example, if I use step input or if I use this is an example of ramp input, this is time and this is input. So, I can the functional form of this input will be that of a straight line, this let us say this goes through the origin.

In that case I do not need to represent q in or the output or the input to the instrument as an ordinary differential equation, because I can explicitly put the functional form of the input and then we drop these differential terms. In that case, a generalized mathematical model for an instrument will be represented by this and for common engineering applications we will make use of this. So, the let the output be represented by nth order differential equation and on the right hand side let us take the functional form of the input.

The commonly used functional forms are step input, ramp, sinusoidal so on and so forth. There is impulse, you will learn more about such inputs or such relationships for different systems when you study process control in some other class.

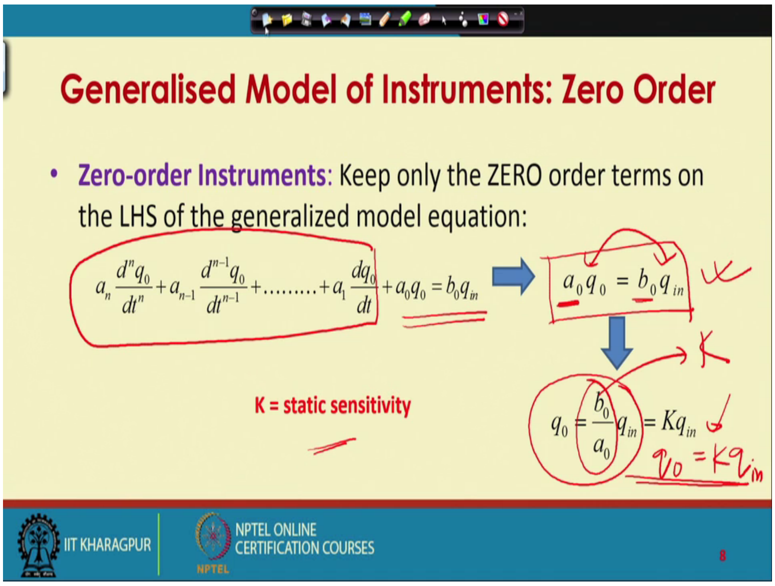
(Refer Slide Time: 11:38)



So, now, if an nth order differential equation represents any general instrument, I can say that zero-order instrument will be represented by a zero-order differential equation with constant coefficients. A first order instrument will be represented by first order ordinary differential equation with constant coefficients. A second order instrument will be represented by second order ordinary differential equation with constant coefficients so on and so forth. So, I will start with the general ordinary differential equation of nth order representing an nth order system and then from that equations by putting n equal to 0 or n equal to 1 or n equal to 2, we will get the models or the modeling equation for zero-order, first order, second order instruments and so on and so forth.

So, let us look at here, so, this is we are saying now, that a generalized mathematical model for an instrument which is commonly used. So, for a zero-order instrument we will keep only the zero order terms on the left hand side of the equation. So, what is zero-order? Only this is the zero-order term. This is first order term, before this will be second order term so on and so forth. So, this is the zero-order term. So, if I retain only zero-order term, what I will have is I will drop this entire part and will have only this.

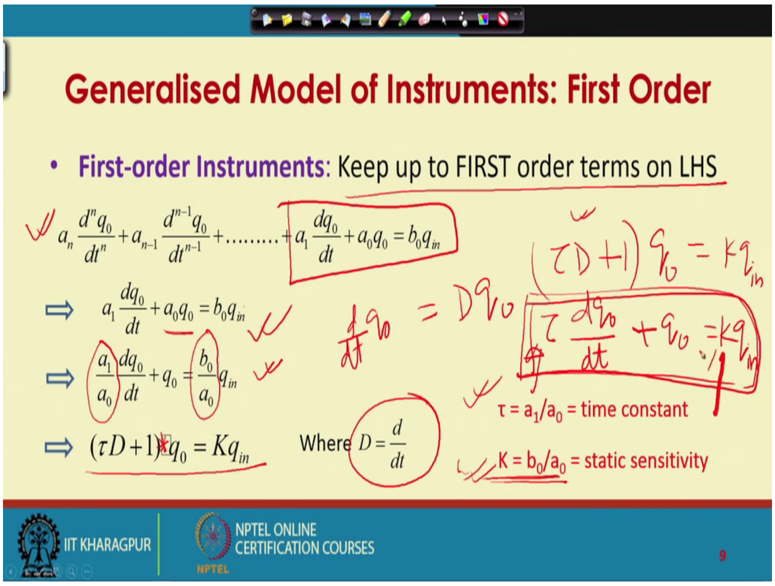
(Refer Slide Time: 13:55)



So, a 0 q 0 is b 0 q in. So, this is an algebraic equation. Please note, that this is not an ordinary differential equation, this is an algebraic equation. So, this algebraic equation relates input and output and a 0 and b 0 are the constant coefficients. So, if I write output q 0 as b 0 by a 0 into q in and represent b 0 by a 0 as K, what I get is q 0 is K q in. This K, please note, that this is output by input. So, this is by our previous definition represents static sensitivity.

So, this equation represents a zero-order instrument which is an algebraic equation.

(Refer Slide Time: 15:23)



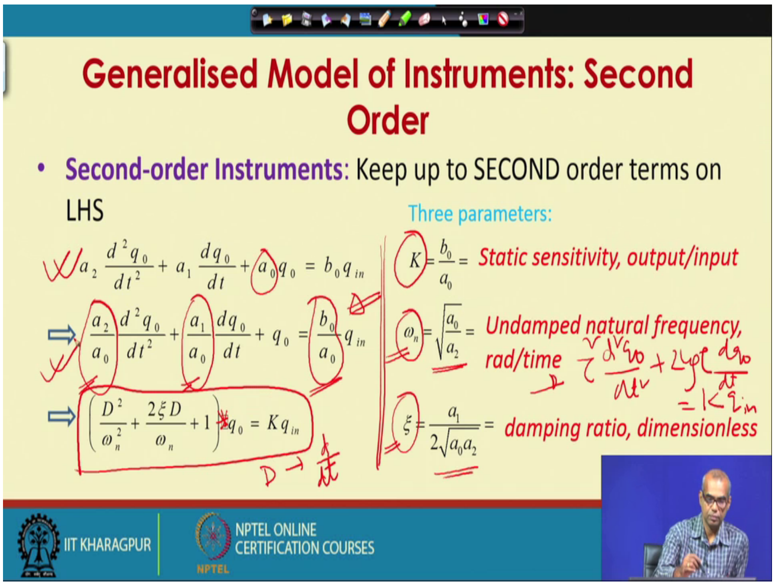
So, first order instrument, let us extend the same idea, a first order instrument will be represented by first order differential equation. So, we will keep up to first order terms. We will keep only first order terms on the left hand side. So, this is the general equation. If I keep only first order terms I will retain only this.

So, this represents an ordinary differential equation with constant coefficient which is a model for first order instruments. This equation can be rearranged as follows; divide this equation throughout by coefficient of zero-order term which is q 0. So, this equation we get if I divide this equation throughout by a 0, now, we define a 1 by a 0 as time constant of the instrument and b 0 by a 0 we have seen previously it represents static sensitivity.

So, in a zero-order instrument we had only one term which was K, the static sensitivity, in first order instruments we have 2 parameters one is a 1 by a 0 which is time constant represent commonly as tau, it has unit of time and another parameter is b 0 by a 0 which is known as static sensitivity and has an unit of output by unit of input. So, introduce the operator capital D which is d dt. So, we can write as d dt of q 0, so, d dt of q 0 can be written as Dq 0. So, this was q 0 here, a 1 by a 0 is tau. So, this can be written as tau D plus 1 into q 0 is Kq in.

This is a multiplication sign. So, a first order instrument is represented as tau dq 0 dt plus q 0 equal to K q in; setting small d dt as capital D, you will get this equation. So, this equation represents a first order instrument which has 2 parameters; one is time constant, another is static sensitivity.

(Refer Slide Time: 19:15)

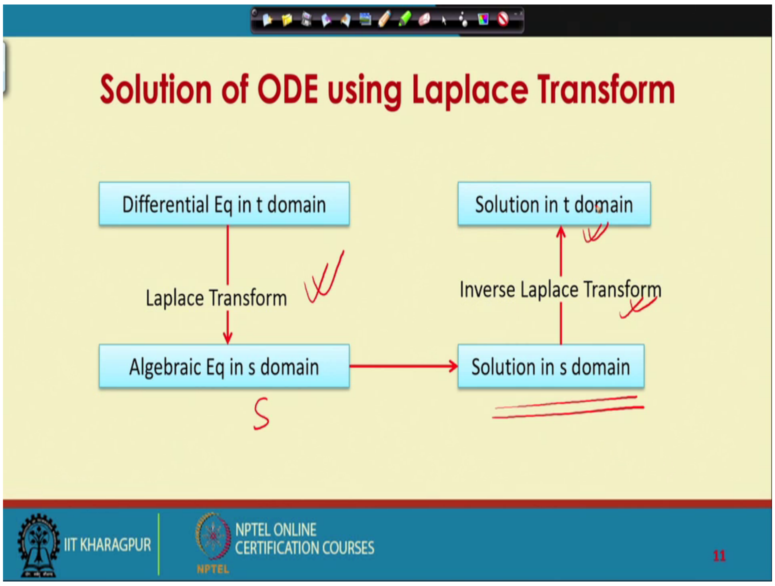


Let us extend it further through second order instruments. So, for second order instrument will keep up to second order terms on the left hand side of the generalized equation, generalized nth order equation, if we do that we will get this equation.

Again, divide this equation throughout by the coefficient of zero-order term q 0. So, I get a 2 by a 0 here, a 1 by a 0 here and b 0 by q 0 here. Note that b 0 by a 0 is same as static sensitivity K. This 2 terms is written as follows; we introduce a term called natural frequency which is square root of a 0 by a 2 and we introduce another term called zeta which is a 1 by 2 square root of a 0 a 2.

So, if we do that, I will write this differential equation as this. Here, again I have introduced capital D as d dt. So, in this equation we have 3 parameters K, which is sensitivity, natural frequency which is this and zeta which is the damping ratio. So, this is also written as tau square d square q 0 dt square plus 2 zeta tau d q 0 dt equal to K q in. If you introduce these terms you will be able to write this from here.

(Refer Slide Time: 22:32)

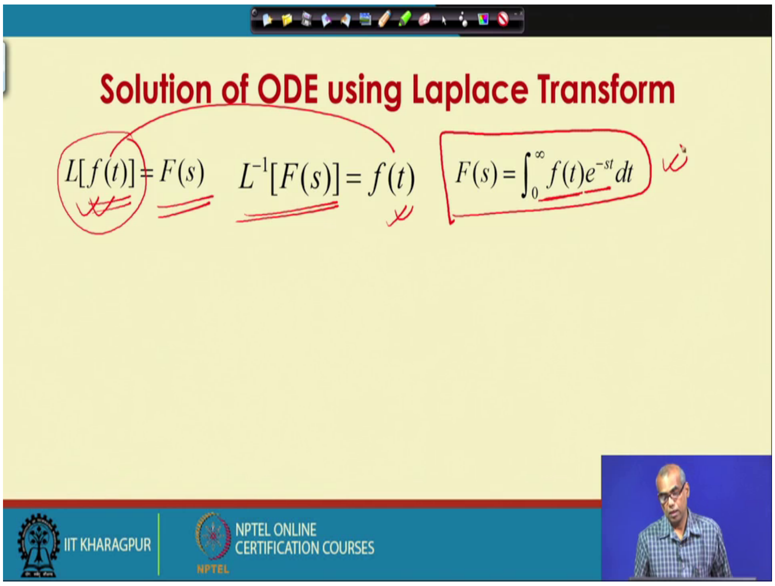


Now, we will see that the instruments are represented by ordinary differential equations, leaving aside the zero-order instrument. So, to know the output of an instrument for a given input we should be able to solve an ordinary differential equation. There are different techniques for solutions of ordinary differential equations. Linear differential equations can be solved analytically; non-linear differential equations are ordinarily solved, numerically. In this lecture, as of now we are talking about linear instruments. So, the instruments are modeled by linear ordinary differential equations.

In process control and instrumentation a popular technique called Laplace transform function exists. A popular technique called Laplace transformation which can be used for solutions of ordinary differential equation. So, what the Laplace transformation does is, it converts the ordinary differential equation to an algebraic equation. Now, there is something called inverse Laplace transformation, so, if I do the inverse of the Laplace transformation I will get the final output. So, what happens is the differential equation is in t domain means time domain.

So, if you take Laplace transformation, you will get algebraic equation in s domain. Now, you solve the algebraic equation in s domain, you take the inverse Laplace transformation you get back the solution in t domain or time domain. So, if we know how to take Laplace transformation of differential equations even if you know how to take inverse Laplace transformation of an algebraic equation, I will be able to solve linear ordinary differential equation with constant coefficients. Remember, the Laplace transformation will work on linear differential equations because Laplace transformation is a linear operator.

(Refer Slide Time: 25:32)



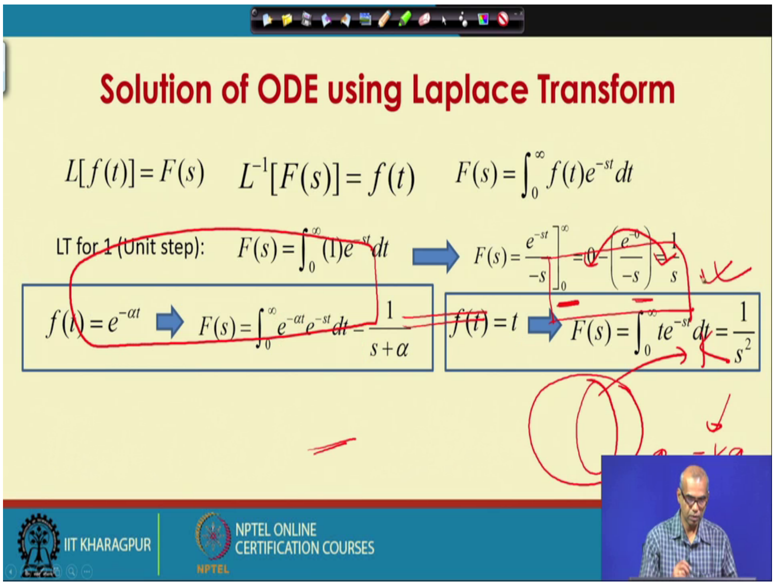
So, let us quickly see few points about Laplace transformation. So, we represent Laplace transformation a function f which is a function of time as this and after Laplace transformation of function f t what we get is an algebraic equation in s domain we represent that as F of s.

So, if I take inverse Laplace transformation of F s, I should get back this functioning time domain. Laplace transformation is defined by this integral. So, you wish to take the Laplace transformations of function f t multiply that by e to the power minus s t and then integrate between 0 to infinity. So, the Laplace transformation of function f t is integral 0 to infinity f t e to the power minus st dt. So, this integration has to be carried out.

Now, you have standard text books of process control they are at tables showing the Laplace transformation of various functions as well as inverse Laplace transformation of various equations or functions. So, looking at those tables you should be able to solve ordinary differential equations using Laplace transformation.

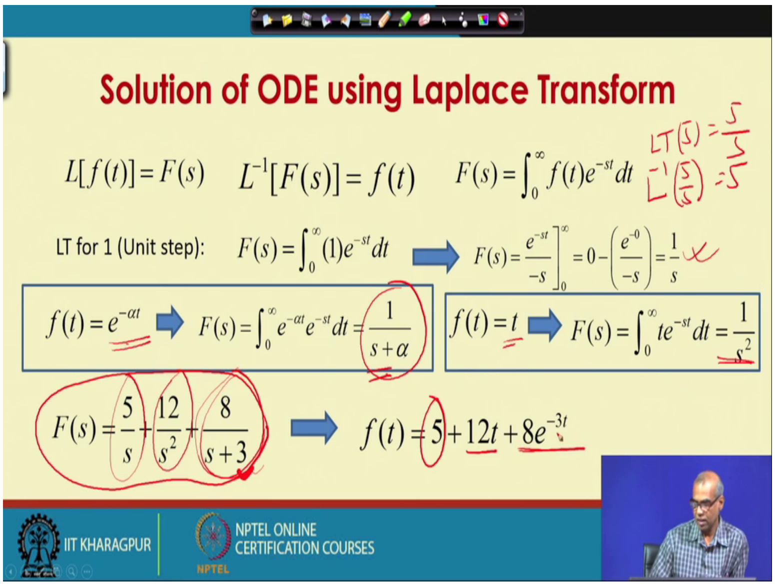
So, let us say Laplace transformation of unit step is this, let us put 1 in place of f t and if we do this integration I will get 1 by s.

(Refer Slide Time: 27:57)



Similarly, if you want to take the Laplace transformation of function e to the power minus alpha t, you put e to the power minus alpha t in place of f t and then you get 1 by s plus alpha. So, Laplace transformation of e to the power minus alpha t is 1 by s plus alpha. Similarly, Laplace transformation of function t, f t equal to t is 1 by s square.

(Refer Slide Time: 28:43)



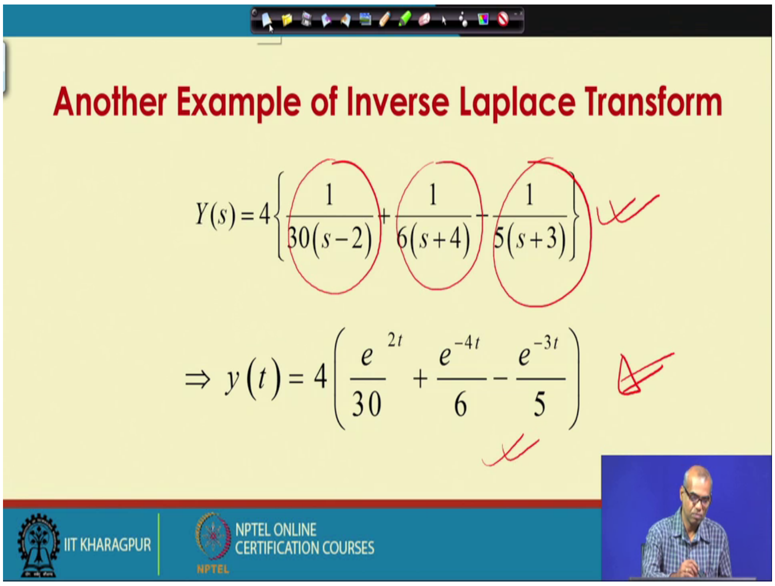
So, if I have F s which represents a Laplace transformed quantity, so, the right hand side part represents Laplace transformation of some function f t. So, I want to know what is the f t, from this Laplace transformed quantity.

So, I have to what I have to do is, I have to take this inverse of this. So, basically see I have to take inverse of this, I have to take inverse of this, I have to take inverse of this and add up. Now, look at here Laplace transformation of unit step is 1 by s, so, Laplace transformation of 5 by s is Laplace transformation of 1 is 1 by s. So, Laplace transformation of 5 is 5 by s, see Laplace transformation of 5 is 5 by s.

So, inverse Laplace transformation of 5 by s will be 5. Thus, how I get the inverse of lap 5 by s is as 5. Similarly, 1 by s square is a Laplace transformation of t. So, 12 by s square will be the inverse of 12 t and similarly, 1 by s plus alpha becomes inverse of sorry 1 by s plus alpha becomes Laplace transformation of e to the power minus alpha t. So, 8 by s plus 3 will be Laplace transformation of 8 into e to the power minus 3 by t.

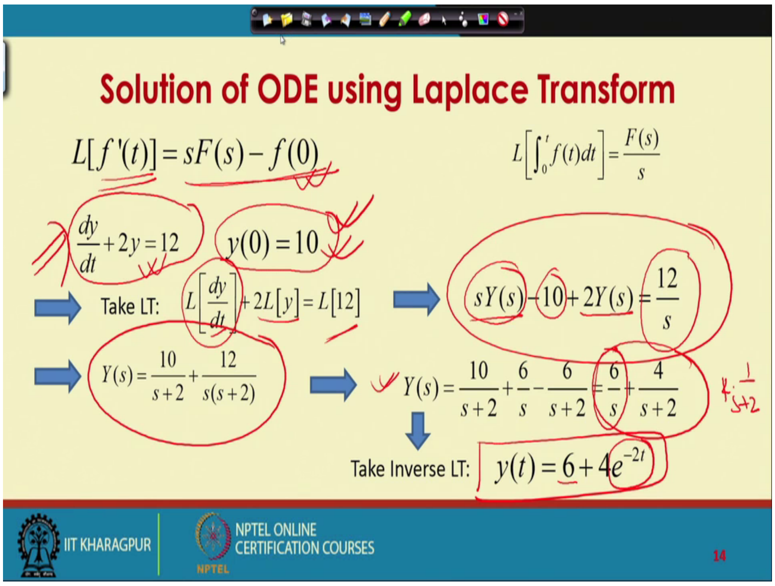
Consider 1 by s plus 3 first here. So, put alpha equal to 3, so, it becomes e to the power minus 3t. Now, multiply this 8, this can be written as 8 into 1 by s plus 3. So, from here and here you get that inverse of 8 by s plus 3 is 8 into e to the power minus 3t.

(Refer Slide Time: 31:47)



So, another example, if the Laplace transform quantity is this, inverse will be this. You consider each term individually, make use of the table or the equations shown in the previous slide you will be able to get this.

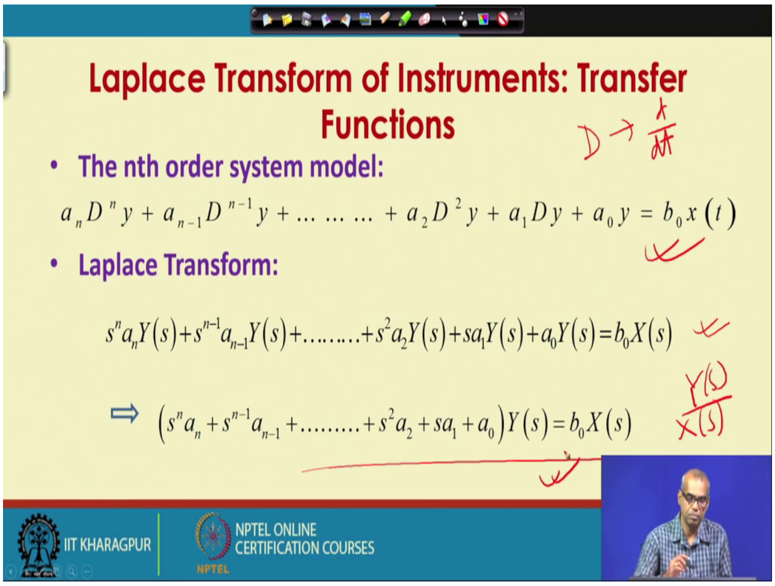
(Refer Slide Time: 32:21)



So, now let us quickly see, how I solve an ordinary differential equation using Laplace transformation. So, Laplace transformation of f prime t which is d dt of f t is sF s minus f of 0. So, this is the initial condition. So, let us say I have an ordinary differential equation given as dy dt plus 2y equal to 12 and initial condition is given as y 0 equal to 10, y at t equal to 0 is 10. So, take Laplace transformation of this. So, you have to take Laplace transformation of dy dt, you have take Laplace transformation of 2y, I have to take Laplace transformation of 12. So, it becomes sY s for this 2Y s for this and 12 by s for this.

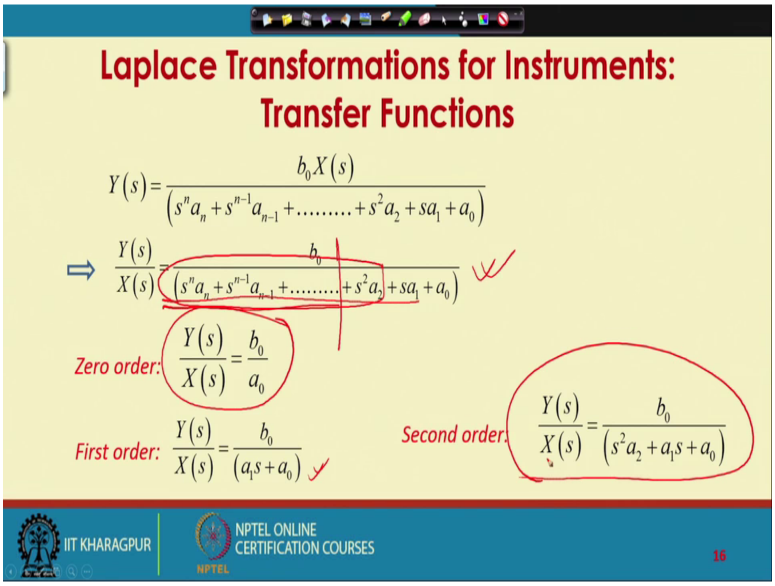
So, Laplace transformation of dy dt is sF s minus f 0. So, this is that sF s and this is y 0 which is 10. So, I rearrange this to get this, which can be further written as this. So, I have to take inverse of now 6 by s to get y t. I have now an algebraic expression Y s because it is Laplace transformation. Now to get back y t, I have to take inverse of this. So, I have to take inverse of 6 by s. So, 6 into 1 by s; 1 by s is Laplace transformation of 1. So, 6 by s, inverse of 6 by s will be 6. Similarly, 4 by s plus 2 considered it at 4 into 1 by s plus 2. So, 1 by s plus 2, from 1 by s plus 2 you get e to the power minus 2t as inverse multiply by 4. So, the solution to this differential equation for this initial condition is y t equal to 6 plus 4 into e to the power minus 2t.

(Refer Slide Time: 34:58)



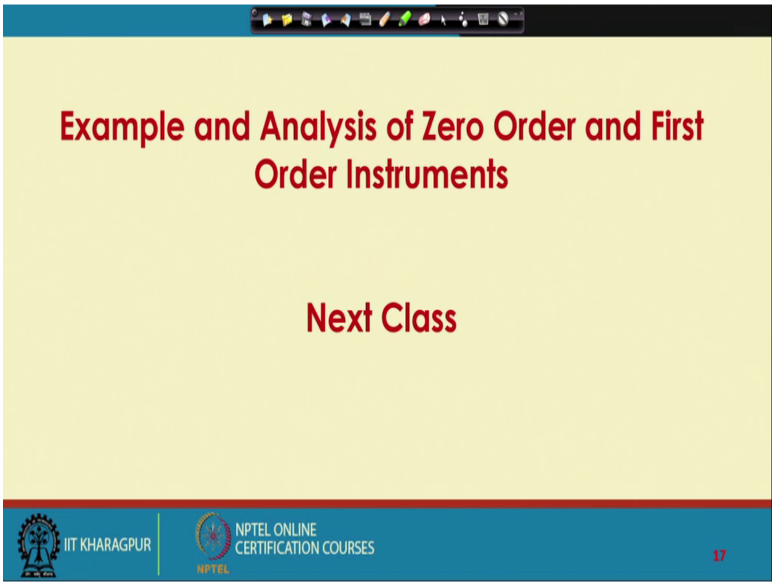
So, nth order differential equation, as we have talked about can be represented by this here. The only change I have made is in place of d dt, I have used the differential operator capital D. You can take Laplace transformation and you will get this. Now, this Laplace transformation can be written like this which allows you to write something called Y s by X s which represents the transfer function for the instrument.

(Refer Slide Time: 35:52)



So, this is I was talking about. So, for zero-order instrument to get the transfer function you have to drop all this terms, so, we will get b 0 by a 0. For first order instruments, we have to drop all these terms, so, you get b 0 by a 1 s into a 0 and similarly, for second order instruments you have to drop up to this. So, you get the transfer function as this. So, transfer function is Laplace transformation of output divided by Laplace transformation of input. So, we will stop here.

(Refer Slide Time: 36:34)



And in the next class we will talk about example analysis of zero-order and first order instruments.