Chemical Process Instrumentation

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Lecture – 13
Performance Characteristics of Instruments and Data Analysis-II (Contd.)

Welcome to lecture 13. In the previous lecture, we have talked about various types of errors that are associated with your measurement. In this lecture, we will talk about how to combine various component errors to calculate the overall accuracy in the measurement. See, when you measure a quantity the instrument that are using may have several components and each component may have some uncertainty or error associated with it. So, the question we ask; now is how do I combine this individual errors to compute the overall accuracy in the measurement.

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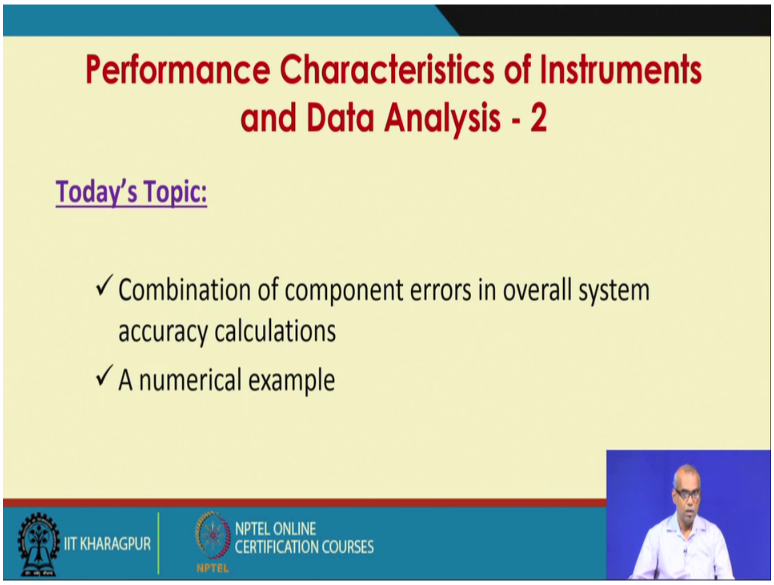

So, the today's topic is combination of component errors in overall system accuracy calculations and then after introducing this concept you will also take a simple numerical example for demonstration.

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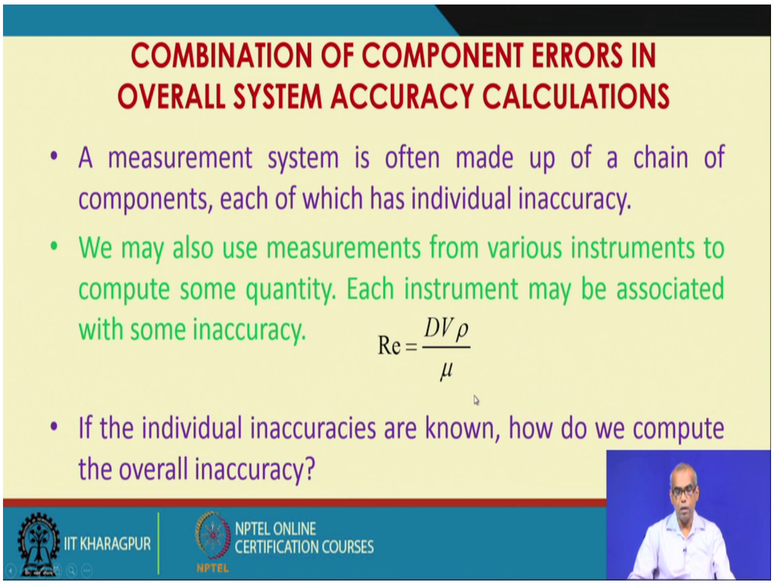

So, measurement system is often made up of a chain of components each of which has individual accuracy we may also use measurements from various instruments to compute some quantity and each instrument may be associated with some inaccuracy for example, let us consider that Reynolds number is DV rho times mu which is diameter of the tube through which a liquid is flowing into velocity of the liquid if the density of the liquid divided by viscosity.

Now you can use different instruments to measure diameter velocity density and viscosity and each measurement may have some uncertainty associated with it and then when you compute Reynolds number from the measured diameter velocity density and viscosity, how do I combine; this uncertainties in individual measurements to get an estimate of overall accuracy in computation of Reynolds number. So, this is the question we ask.

So, that if you know the diameter is measured with this uncertainty if density is measured with this uncertainty. So, on and. So, forth how do I combine this uncertainties together to get an estimate of overall accuracy in the computation of Reynolds number? So, the first question we ask is if the individual inaccuracies are known; how do I compute the overall inaccuracy.

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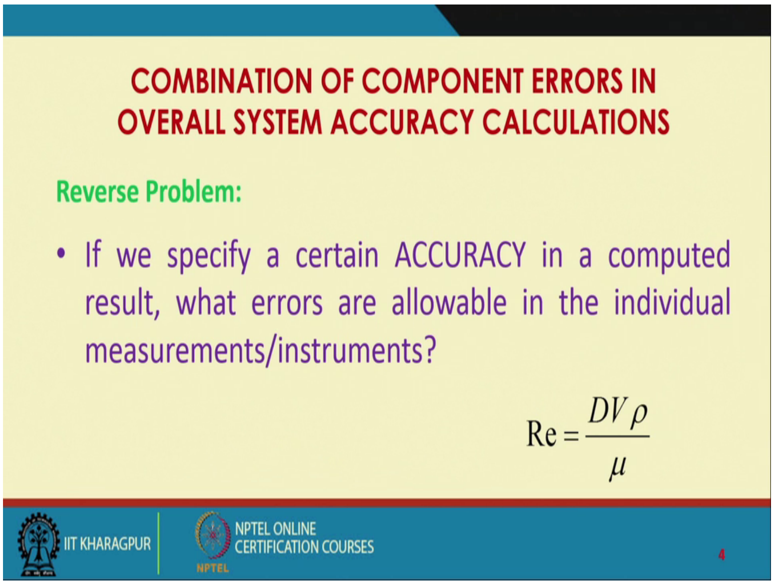

This is the forward problem. So, I know the individual inaccuracies how do I combine the reverse problem is let us say I want to specify that I want the computation of Reynolds number with maximum allowable inaccuracy is this. So, maximum allowable inaccuracies x percent what were the accuracies we each we must compute diameter velocity density and viscosity. So, this is known as reverse problem.

So, forward problem you know the individual component in accuracies combine them to compute the overall systems accuracy in the reverse problem you specify that I want the final measurement with this specified inaccuracy or specified accuracy. So, what are the maximum inaccuracy that can be tolerated in individual measurements?

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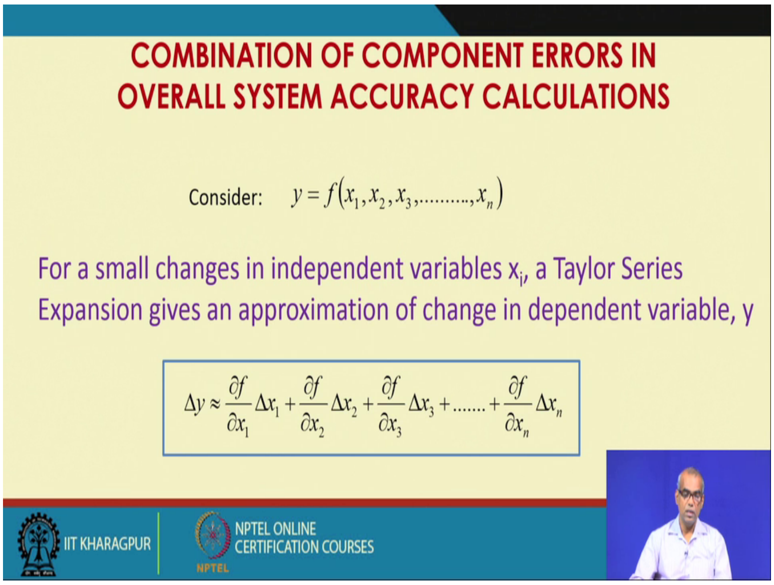

So, let us consider that a measurement can be expressed as a function of various individual measurements such as x 1, x 2, x 3 up to x n for a small change in independent variable x i a Taylor series expansion gives an approximation of change in the in dependent variable y. So, y is a function of x 1, x 2, x 3 up to x n. Now, if I make small changes in this x i a Taylor series expansion gives me an approximation of change in dependent variable y.

So, that can be written as shown delta y which is change in the dependent variable is del f del x 1 into del x 1 plus del f del x 2 del x 2 up to del f del x n.

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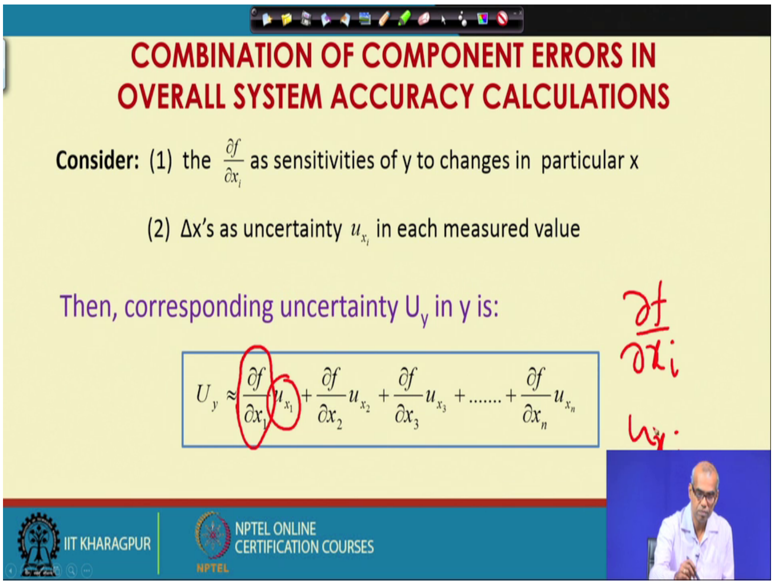

And del x n; now let us consider this del f del x del f del x 1 u x 1 this quantity. So, in general del f del x i and u x i quantities the del f.

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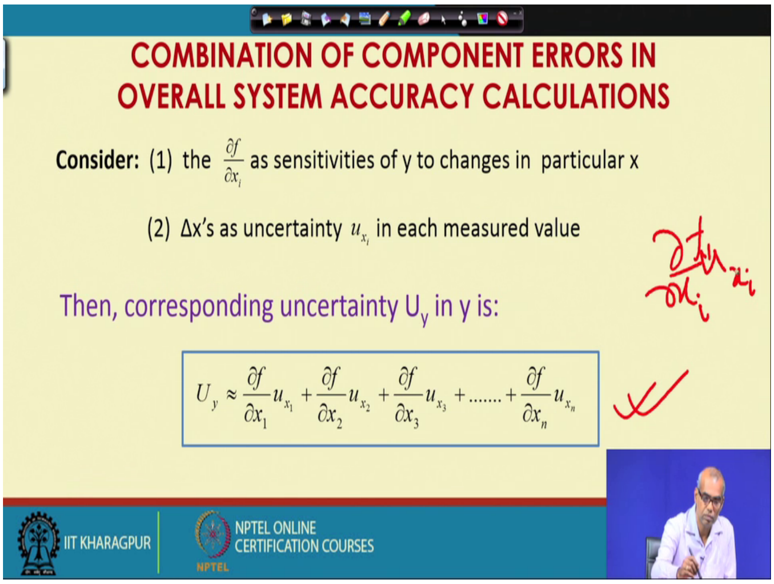

Del x i quantities can be considered as sensitivities of y to change in particular x and then this u x i; let us consider uncertainties in each measured value. So, the corresponding uncertainty will be sum of all this quantities. So, the uncertainty in y is sum of all this del f del f del x i u x i quantity.

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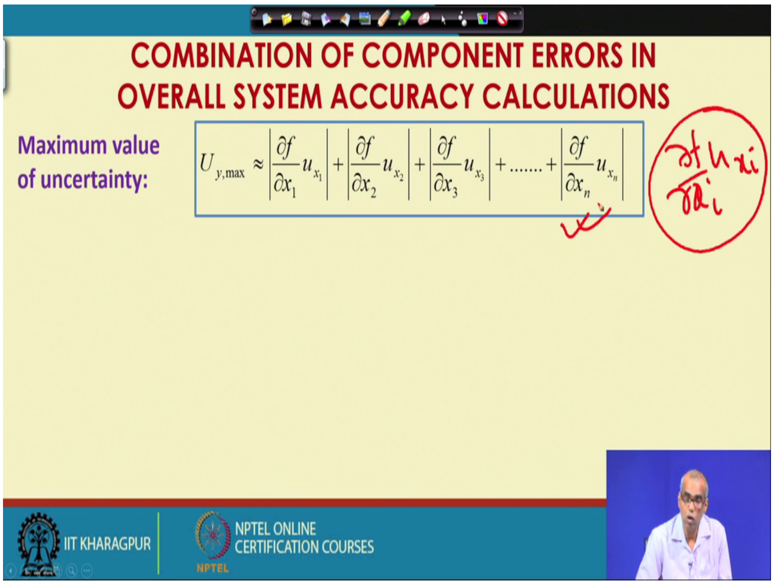

So, maximum uncertainty can be obtained, if I take the sum of absolute values of this quantities.

If I do not take absolute values we sum of such terms will have positive value sum of such terms will have may have negative values and we will get a much lower estimate or much higher estimate than the actual uncertainty. So, the maximum possible uncertainty is when I take the absolute values.

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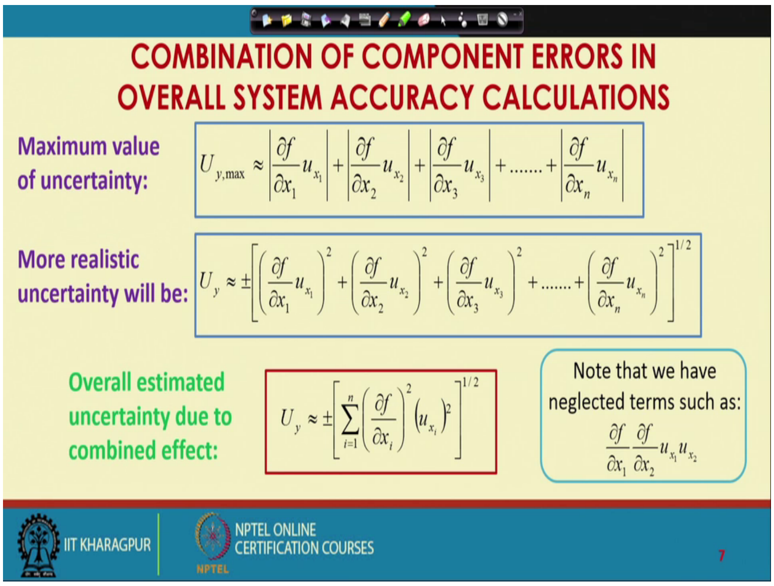

So, sum of all the absolute values will give the maximum value of uncertainty, but the most provable uncertainty will be or most realistic estimate of uncertainty will be if I take this value what will do is I square take R m s type of values right. So, this can be written compactly as this. So, how you obtain this from this you just square both the sides and written only this square terms sum of this square terms drop the cost terms like u x 1 u x 2 and so on and so forth then u square is becomes this term square.

So, u becomes square root of this. So, we have neglected terms like del f del x 1 del f del x 2 u x 1 and u x 2 such cross terms. So, if the individual inaccuracy is unknown; that means, del f del x i u x i all this u x i u x u x 1 u x 2 u x 3 all this things are known and if I can calculate the sensitivity terms such as del f del x 1 del f del x 2 and so on and so forth; I can compute the overall inaccuracy or uncertainty using this expression. Now let us consider the reverse problem which is more interesting.

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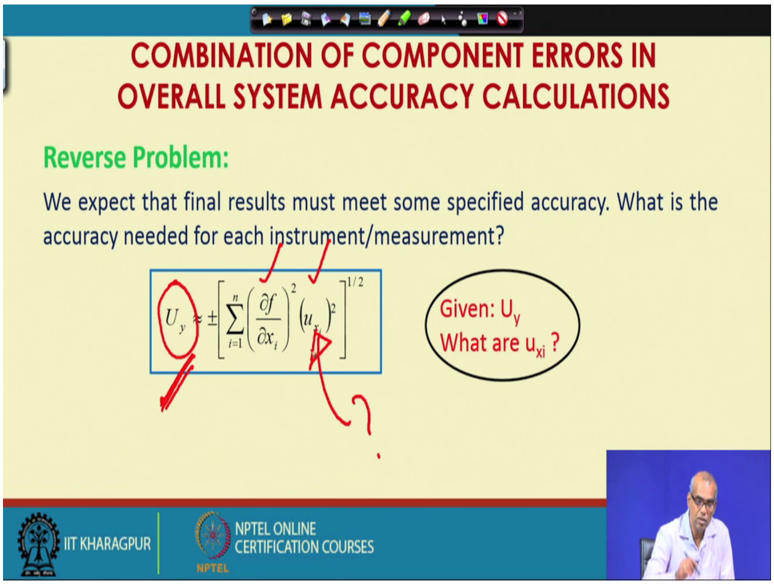

We expect that final results must meet some specified inaccuracy or accuracy we expect that final results must meet some specified accuracy what is the accuracy needed for each instrument or measurement. So, this is the expression we just obtain which tells us how to combine the individual errors.

So, individual uncertainties to get overall uncertainty in the measurement, but in case of reverse problem we say we are given this how do I calculate this in the forward problem we are given this and if we know this I can calculate this in a forward or step forward manner, but the reverse problem is if overall uncertainty is specified. So, U y is given what are the values of u x i, how do I compute that this is a reverse problem.

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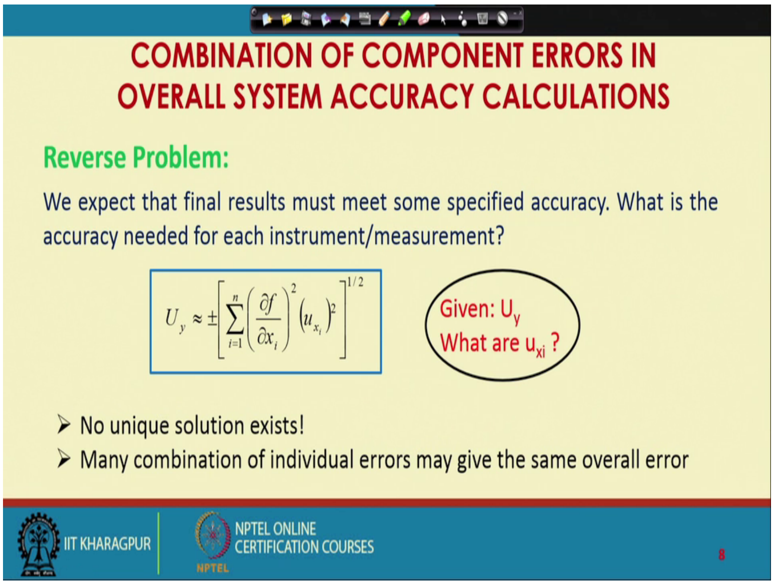

Please note that there is no unique solution for this problem because many combination of individual errors may give the same overall error.

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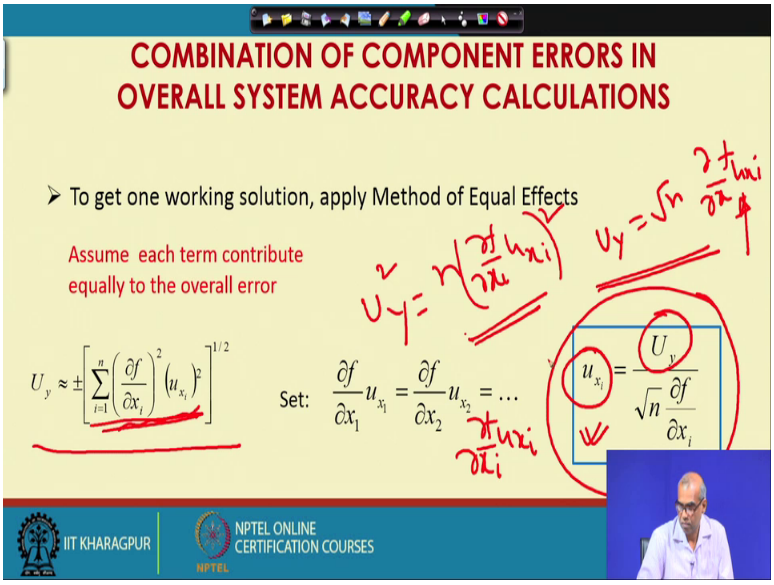

So, to get one working solution we apply something called method of equal effects what it means is we assume that each term contribute equally to the overall error. So, the overall uncertainty you have seen in the forward computation or the forward problem is this. So, it is the sum of combinations of terms like del f del x i u x i.

So, del f del x i u x i whole square. So, these are the terms now if I assume that each term contribute equally this is an approximation, but if I make this approximation I will set del f del x i u x i equal to del f del x 2 u x 2 and so on and so forth then I can then what I will get from here is if I assume all this things are equal note that this will be n times del f del x i u x i whole square. So, inside right. So, u y will be this u y u y square will be this in other words u y will be square root of n del f del x i u x i or u x i will be then u y divided by square root of n del f del x i which is this. So, you simply assume that each such term contribute equally. So, that you can write del f del x 1 equal to u x 1 equal to del f del x 2 equal to u x 2 and equal to let us say del f del x i u x i.

So, this sum then becomes this sum then becomes n into del f del x i u x i whole square you have square root here. So, if you now square both the sides u square will be equal to this from here u y will be this from which you can compute u x i equal to this u x i is the individual inaccuracies or individual uncertainty. So, given a specified uncertainty I can now find out the individual uncertainty.

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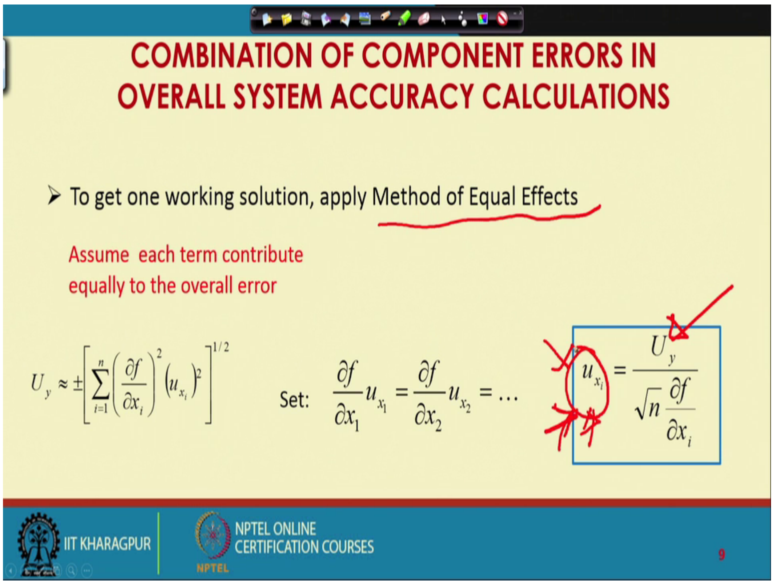

So, here a point you want to make is given the individual uncertainty I can find out what will be the tolerable uncertainty in the individual components, but they are all equal we are assuming that is now because of this method of equal effects approximation we could find this now does it mean that when designing an instrument if one component inaccuracy exceeds this value is that is an going to fail may not be.

So, because see if one components inaccuracy is worse than this by as a little margin you may have another component whose inaccuracy is much lower than this. So, when these 2 are combined the overall accuracy can still be achieved. So, this is a thing that we have to keep in mind.

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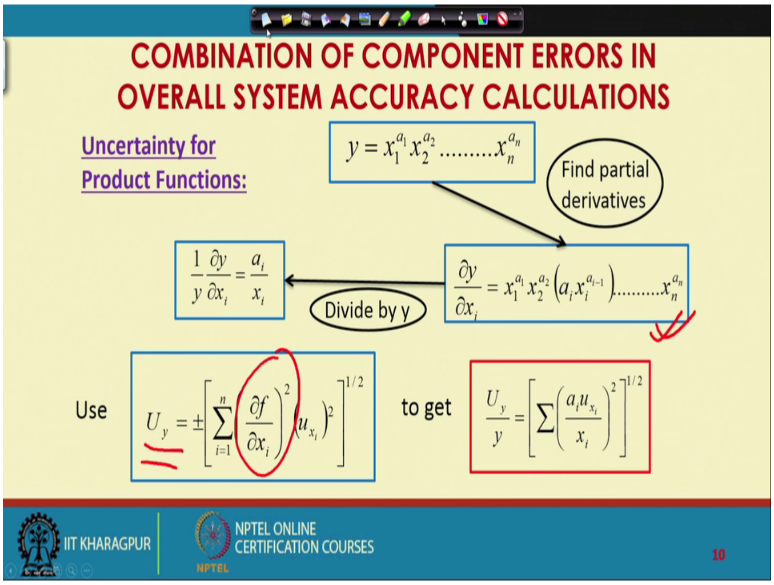

Uncertainty can be easily obtain from functions like this which is product function. So, all we need to do is the; we have to find out this partial derivatives. So, you find out partial derivatives once you have obtain the partial derivatives you can compute the overall uncertainty.

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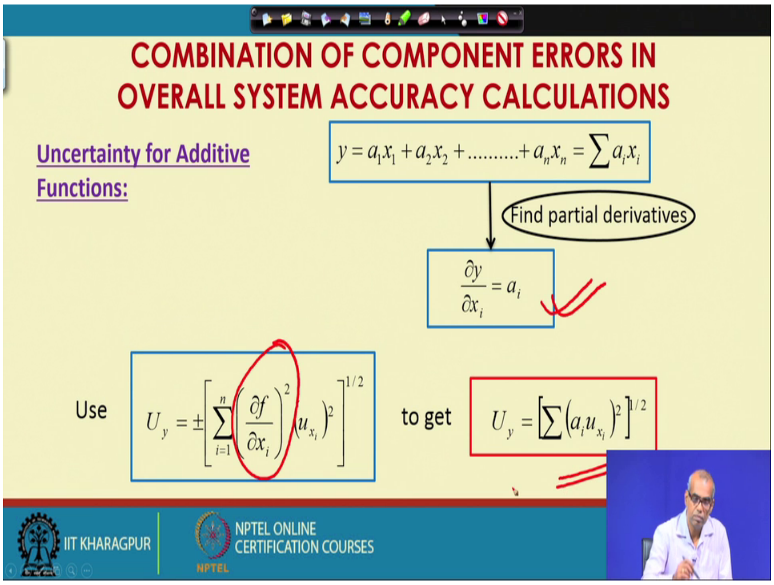

If your function is additive you find out again the partial derivatives if you have the partial derivatives you make use of this expression to find out the overall uncertainty.

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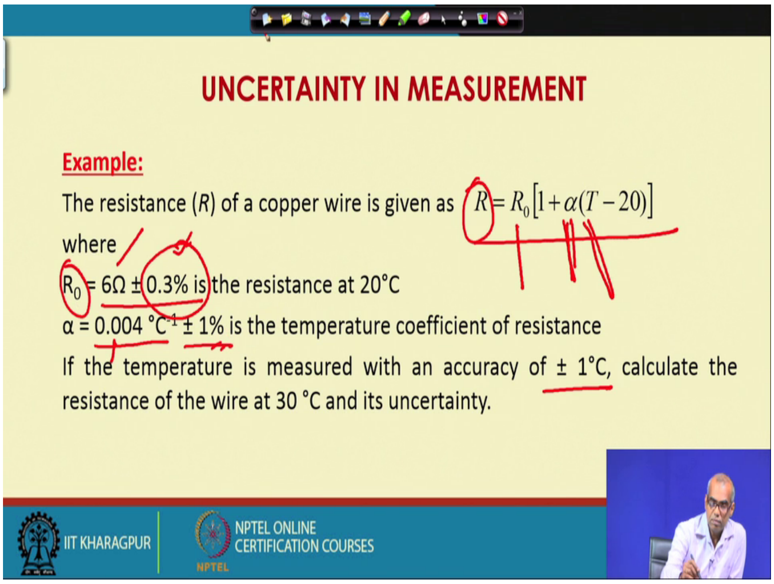

Now, let us take a simple example to illustrate the idea the resistance of a copper wire is given as R equal to R 0 into 1 plus alpha into T minus 20, here R 0 is the resistance of the wire at 20 degree Celsius and is expressed as 6 ohm plus minus 0.3 percent. So, point 3 percent is the uncertainty associated with resistance at 20 degree Celsius which is 6 ohm. So, we express the resistance at 20 degree Celsius is R 0 equal to 6 ohm plus minus

0.3 percent alpha is the temperature coefficient of resistance and this is expressed as 0.004 degree Celsius inverse plus minus 1 percent. So, plus minus 1 percent is the uncertainty associated with alpha, if the temperature is measured with accuracy plus minus 1 percent. So, this temperature is measured with plus minus 1 percent accuracy, calculate the resistance of the wire at 30 degree Celsius and its uncertainty.

So, I know the uncertainty associated with R 0 I know the uncertainty associated with alpha and I know the uncertainty associated with the temperature. So, all these uncertainties must be combined to get the uncertainty in this. So, note the nominal values are 6 ohm 0.004 and these are the uncertainty associated.

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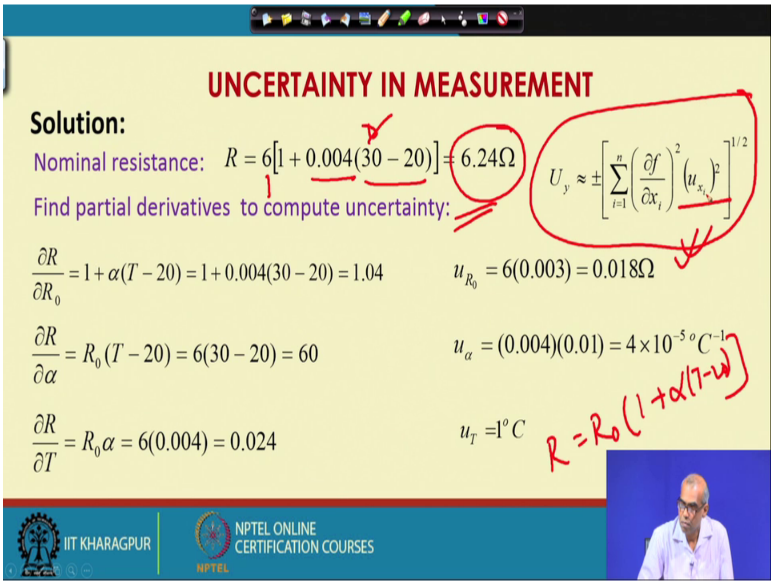

So, this is a step forward computation by making use of the expression that we have seen. So, this is the equation which gives us the overall accuracy when you know the individual inaccuracies in the components. So, first let us find out the nominal resistance at 30 degree Celsius the equation is R equal to R 0 into 1 plus alpha into T minus 20 degree Celsius. So, R 0 1 plus alpha into T minus 20 T is 30 degree Celsius.

So, the nominal resistance is 6.24 ohm, here we do not consider the uncertainty is associated with it now to find out uncertainty associated with these value I have to make use of this equation note here these values are given the uncertainty associated with R 0 was given as 0.03 percent, let us let me go back R 0 is given as 0.3 percent R 0 is given as 0.3 percent in accuracy uncertainty alpha is given as plus minus 1 percent and temperature is also measured with accuracy plus minus 1 percent.

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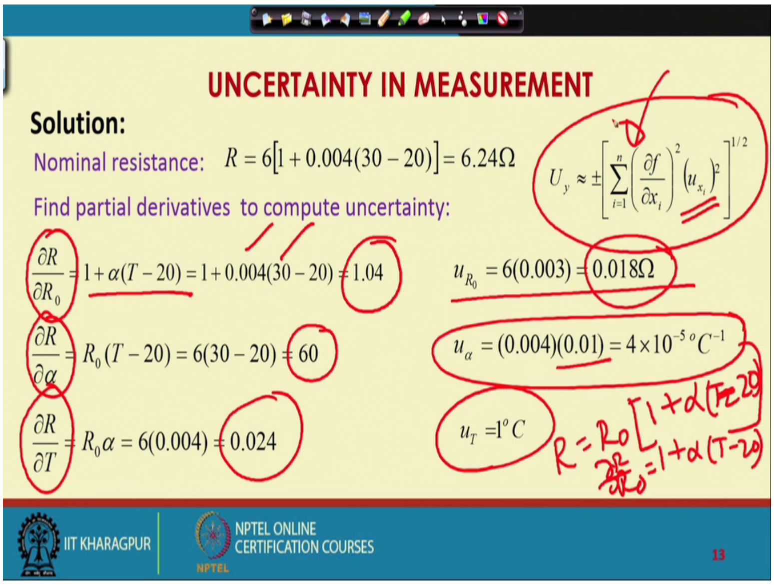

So, the uncertainty associated with R is 6 into 0.003; 0.3 percent. So, this value uncertainty associated with alpha is 1 percent. So, this value and uncertainty associated with temperature measurement is plus minus 1 degree Celsius. So, this is given as this.

So, these are now known the only thing that needs to be known is the sensitivity values del f del x i values now del f del x i can be computed from this equation. So, what is my equation is R equal to R 0 into 1 plus alpha into T minus alpha into T minus 20. So, make use of this equation to find out del R del R 0 del R del alpha and del R del T for example, if you take del R del R 0 it becomes one plus alpha into T minus 20. So, one plus alpha into T minus 20. So, you put the values of alpha put the values of T you get del R del R 0 as 1.04. Similarly you compute del R del alpha as 60 del R del T as 0.024; once you have all this values you are now in a position to make use of this equation to compute overall uncertainty in your measurement.

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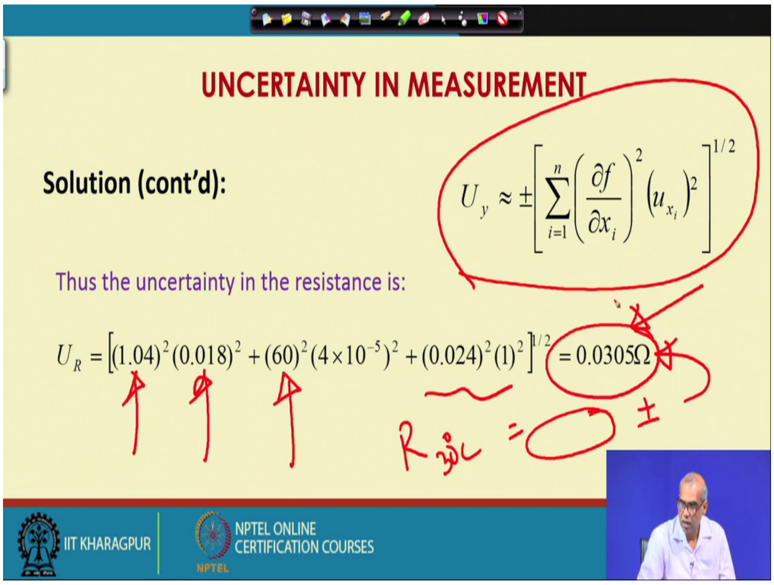

So, this is what I have done here I am making use of this equation I have now the value of del f del R 0 and the uncertainty with R 0, I have the value for alpha del and I have the value for T temperature. So, I get the uncertainty in the resistance is 0.0305 ohm. So, I will express the resistance at 30 degree Celsius as the nominal resistance that I get and plus minus this value.

So, this is my estimate of uncertainty in the computed resistance. So, this is how you will be able to compute the overall accuracy in your measurement if you know the individual uncertainties and making use of method of equal effects; you will also be able to find out what is the maximum tolerable inaccuracy in individual components to attain a measurement with specified overall accuracy. So, we stop our lecture 13 here.