Chemical Process Instrumentation
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Lecture – 11
Performance Characteristics of Instruments and Data Analysis-II

Welcome to week 3, lecture 11. In this week, we will start our discussion on performance characteristics of instruments and data analysis part 2. So, part 1; we have seen in the previous week. So, we will start performance characteristics of instruments and data analysis part 2.

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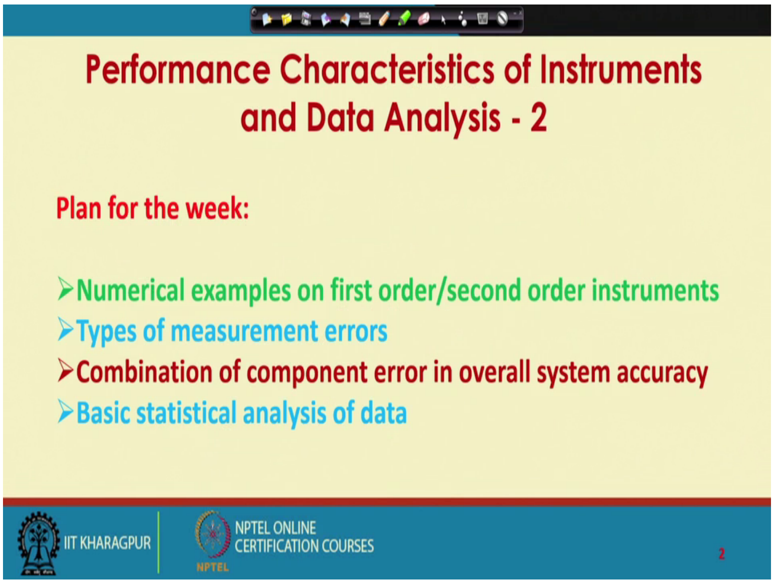

So, this is our plan for the week in the first lecture we will talk about some examples numerical examples on first order and second order instruments in subsequent lectures we will talk about different types of measurement errors; no measurement is error free. So, we will talk about what are the different types of errors that are associated with our measurements then we will talk about combination of component error in calculation of overall system accuracy and finally, we will talk about some basics statistical analysis of data.

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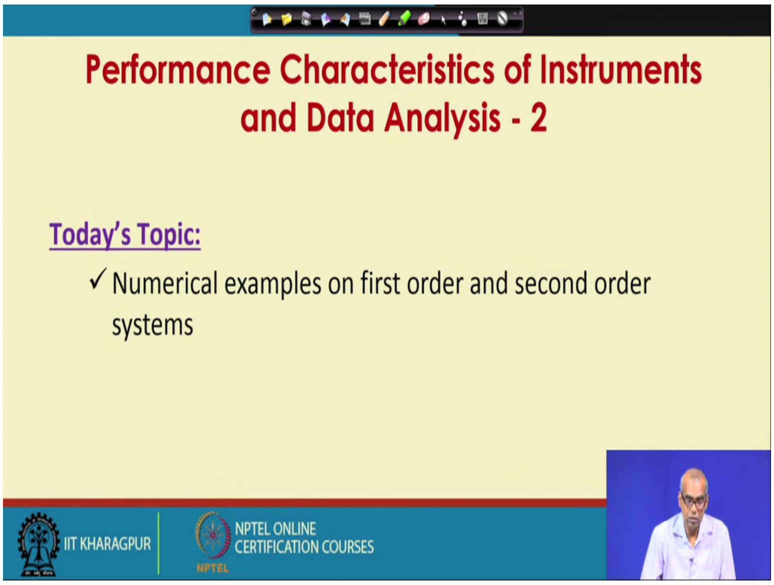

So, in today's class, we will take few numerical examples on first order and second order systems or instruments.

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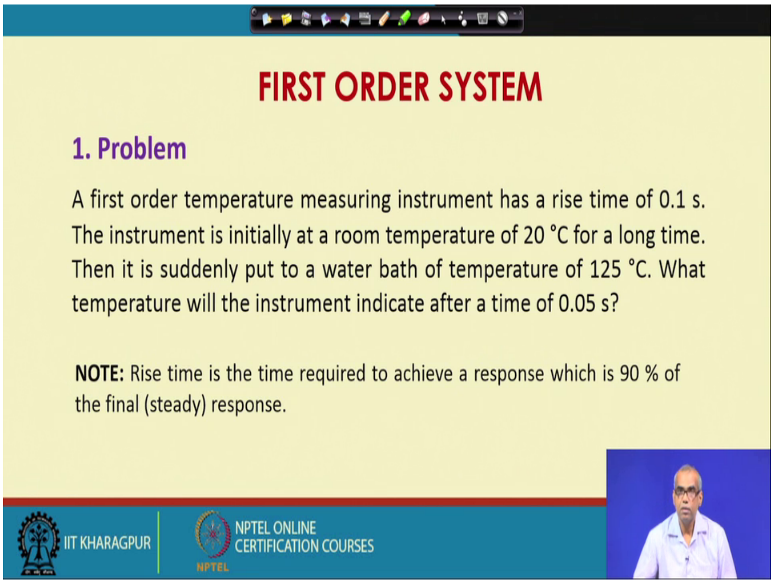

So, let us take our first example or first numerical problem, it is about the response of a temperature measuring instrument which is a first order instrument. So, the statement of the problem is as follows a first order temperature measuring instrument has a rise time of point one second the instrument is initially at a room temperature of 20 degree Celsius for a long time then it is suddenly put to a water bath of temperature of 125 degree

Celsius what temperature will the instrument indicate after a time of 0.05 second we need to define rise temperature here rise temperature is the time required to achieve a response which is 90 percent of the final or steady response.

So, what is asked in this problem is that it is given that an instrument has a rise time of point one second. So, 90 percent of the final response can be achieved in 0.1 second, then it is said that instrument was there in the temperature of 20 degree Celsius for a very long time. So, we will assume that the temperature was showing steady temperature of 20 degree Celsius, then it is suddenly put into 125 degree Celsius into a medium with 125 degree Celsius; that means, the instrument which was showing 20 degree Celsius has now suddenly be put to a medium whose temperature is 125 degree Celsius what it means is that you have given a step input of magnitude 125 minus 20 equal to 105 degree Celsius.

So, it is a question of giving a step input to the first order instrument, then we ask the question; what will be the response after a certain length of time. So, a first order instruments is characterized by 2 parameters, we have seen time constant tau and the sensitivity K here the input and the output both are temperatures both are same signals. So, the K value we take as one. So, first thing will be to we must find out the value of time constant tau. So, the question you ask now is can I find out the value of time constant from the data given the magnitude of safe input is given can only find out the time constant value. So, let us see how you do it. So, what is given is initial temperature 20 degree Celsius, it is put suddenly to a temperature of 125 degree Celsius. So, there is a step input is given.

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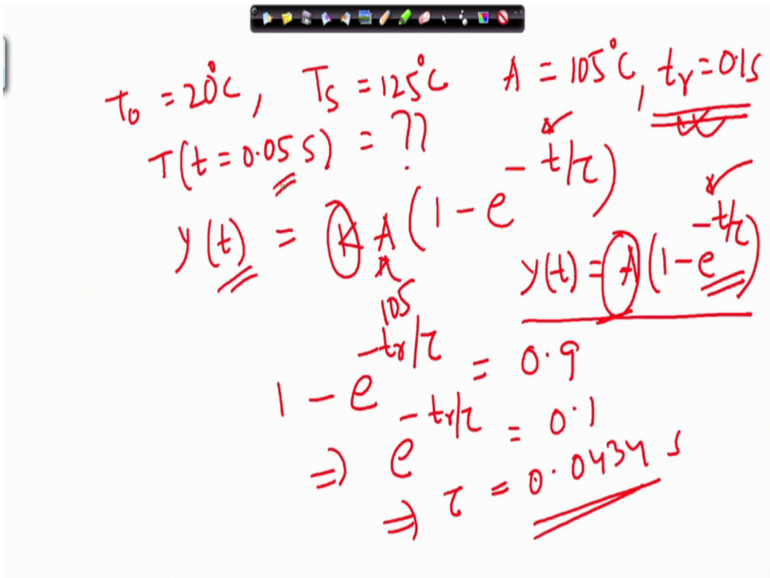

So, magnitude of step if I call it, A is 105 degree Celsius. So, 125 minus 20 rise time is given as point one second I have to find out the response at 0.05 second is how much; we know the step response of a first order instrument is given as KA 1 minus e to the power minus t by tau here K is one a is 105 t is 0.05 if I find out tau I can find out the response. So, how do I find out tau another information that is given is that a rise temperature, we know the rise temperature is rise time sorry the rise time is given as 0.1 second. So, rise time is the time at which 90 percent of the final response is achieved.

So, from these equation y t equal to let us I would like my right a now because K is 1; 1 minus e to the power minus t by tau what is the final response the final response is a because final response will be attained at t equal to infinity is the system will attain steady state this part becomes 0. So, I t equal to a. So, y t equal to A is the final response. So, 90 percent will be achieved in t r time. So, what we can write is one minus e to the power minus t r by tau is equal to 0.9. In other words, e to the power minus t r by tau will be 0.1 and from here I will get tau as 0.0434 second; once this is obtained I will make use of this equation again with a equal to 105 and then we can find out the response at 0.05 second. So, this problem has 2 parts first part I have to find out the time constant making use of the information on rise time.

And the second part I may use this obtain time constant value to find out the response at t equal to 0.05 second in a straight forward manner from the equation of the step response.

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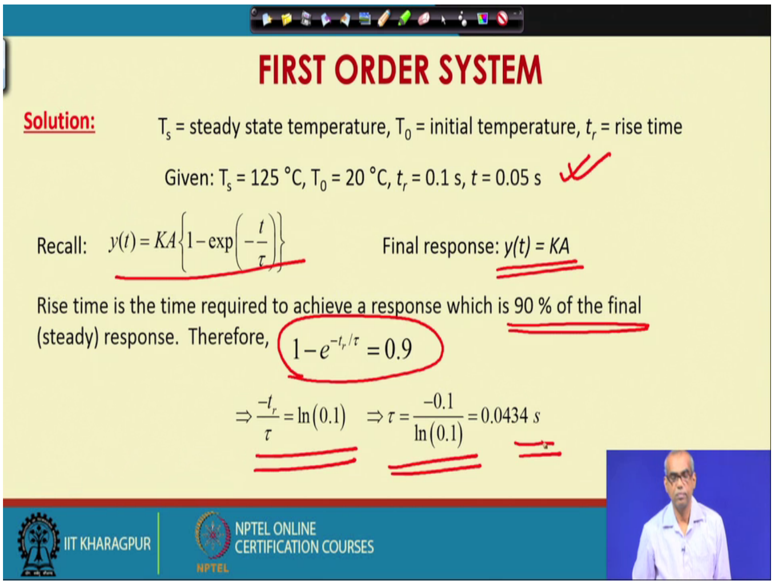

So, now, let us look at here. So, these are the values or informations given this is the equation I was talking about here the value of K is 1; that is fine. So, the final response is y t equal to KA or considering K equal to one it is K. Now rise time is that time required to achieve a response which is 90 percent of the final or steady response. So, I can write this from where I write this and then rearrange to get tau as this once you have tau you put the value of tau.

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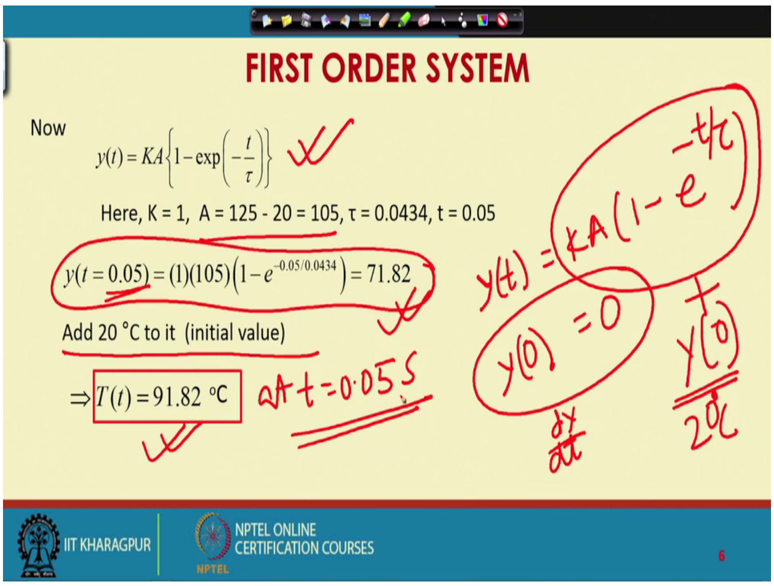

In this equation with a equal to 105 and you will get the response at time t equal to 0.05 second here you need to understand one more thing.

When this equation y t equal to KA one minus e to the power minus t by tau was derived by taking Laplace transformation and then inverse Laplace transformation it was assume that y 0 is 0 you remember what happens when I take Laplace transformation of d y d t this is s y s minus y 0. So, in this case the particular numerical problem we are taking about y of 0 is not 0 initially the thermometer was showing 20 degree Celsius. So, that is consider as t equal to 0 initial time. So, y 0 or y at t equal to 0 is 20 degree Celsius. So, whatever we obtain from this we have to add y 0 which is 20 degree Celsius. So, I get seventy one point eight 2 degree Celsius from here I must add 20 degree Celsius to it which was the initial value. So, the thermometer will show 91.82 degree Celsius at t equal to 0.05 second. So, this is the final answer.

So, now let us take another example.

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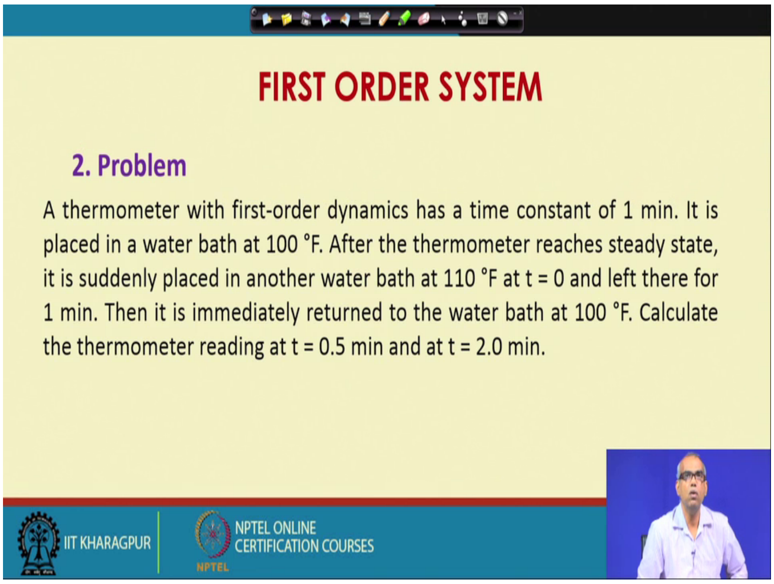

Again it is an example of first order instrument a thermometer a thermometer with first order dynamics has a time constant of one minute it is placed in a water bath at hundred degree Fahrenheit after the thermometer reaches steady state it is suddenly placed in another water bath at 110 degree Fahrenheit at t equal to 0 and left there for one minute then it is immediately returned to the water bath at 100 degree Fahrenheit calculate the thermometer reading at t equal to 0.5 minute and at t equal to 2 minute. So, try to understand the problem you have a thermometer which is an first order instrument. So, the thermometer was showing at temperature of 100 degree Fahrenheit, it has a time constant of 1 minute. So, it was steady at hundred degree Fahrenheit now suddenly and I call that time as time t equal to 0, suddenly the thermometer is put to a water bath with 110 degree Fahrenheit temperature.

What does it mean I give a step input of magnitude 10; 110 minus 100 and keep it there for certain time keep it there actually for 1 minute; see 1 minute is equal to time constant is also here and then I return back the thermometer to 100 degree again. So, this is also another step input, but initially it was arising step input positive step input now it is a falling step input or negative step input. Now it is from 110 to 100 and initially, it was 100 to 110 both are magnitude 10, but in one case, it is plus 10; another case, it is minus 10. So, now, I have to find out the response is that t equal to 0.5 minute and t equal to 2 minute. So, I have to calculate a response before on time constant value and at 2 times constant values. So, let us see how we do that.

So, basically there are 2 step inputs given and you can make use of the expression for step response directly all the required informations are given in the problem let us look at the solution.

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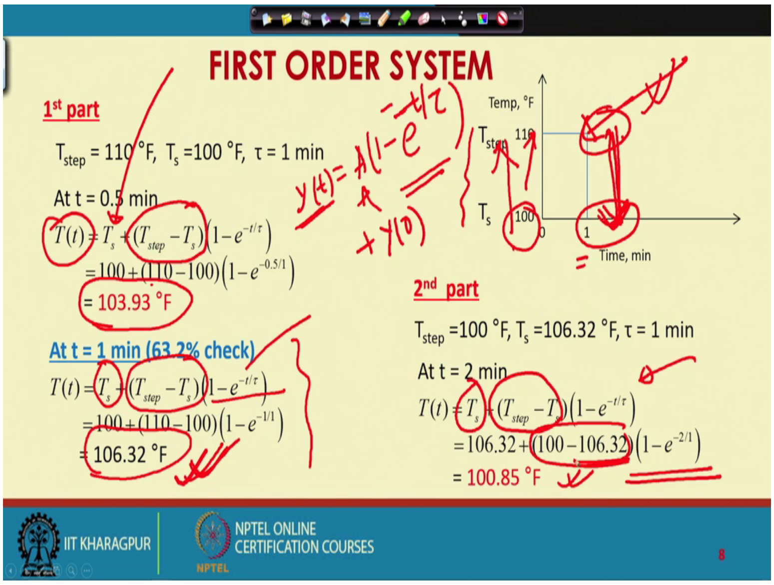

So, initially the thermometer was 100 degree Celsius medium, I suddenly give it a step input for magnitude 10 by bringing it to a medium or water bath of 110 degree Celsius.

So, keep it there up to one minute and then again give a falling step input by bringing the thermometer back to water bath of 100 degree Fahrenheit. So, in the first part I will make use of the equation which is y t is A into 1 minus minus t by tau please this note that this is minus minus t by tau y t is basically the temperature here. So, t; t A is the magnitude of step.

So, this is the difference between these 2 and 1 minus e to the power minus t by tau is this and what is this is that y 0, we talked about in the previous problem also. So, initial temperature that the thermometer was showing. So, in I have all the information now the initial temperature, it was showing as 100 the step input of magnitude 10; 110 minus 100 and 1 minus e to the power minus 0.5 by 1.5 is the time at which I want to 0.5 minute is the time at which I want to 0.5 minute is the time at which I want response. So, this is computed as 10.93 degree Fahrenheit. So, this is the response at 0.5 minute the thermometer was there in the water bath of 110 degree Fahrenheit for 1 minute. So, I must know; what was the temperature shown by the thermometer at time t equal to 1 minute.

Because that temperature will be the initial value for the step to compute the step response. So, in the cringe back the thermometer from 100 degree Fahrenheit to 100 degree Fahrenheit. So, let us now try to compute the temperature indicated by the thermometer after 1 minute, again is a straight forward application of these equation initial value we still 100 degree Fahrenheit magnitude of step 110 minus 100 and 1 minus e to the power minus t by tau this t is now 1. So, this you get as 106.32 you remember we talked about 63.2 percent that when the thermometer or a first order instrument is given when a step input at time t equal to one time constant 63.2 percent of the final response is achieved in this particular case my temperature my thermometer was showing 100 degree Fahrenheit.

Then suddenly, I put it to a medium with 110 degree Fahrenheit and I am trying to find out what is the response after one minute and one minute happens to be the time constant of the thermometer. So, in 1 minute 63.2 percent of the final response should be achieved. So, we have given a 10 degree a step input of magnitude ten. So, 63.2 percent of that must be achieved in one time constant. So, 63.2 percent is 6.32; 63.2 percent of 10 is 6.32. So, hundred plus 6.32 is 106.32 should be the temperature shown by the thermometer at one time constant. So, this is what we get. So, my answer is correct the second part I give a falling step input to the thermometer. So, again we make use of the same equation, but with appropriate values initial value is now this.

Because after 1 minute this was the temperature the thermometer was showing magnitude of step is now 100 minus 106.32 because this is the temperature this is the temperature that the instrument was showing after one minute. So, let me correct my previous statement I said that initially I give a magnitude a step into a magnitude 10 that is correct, but then if I say that I give another step input of magnitude minus 10 this is not correct because I waited here only for 110 constant value if I would have waited here for long time for steady state to attain then they should been have to correct. So, I give arise in step or positive step of 10 degree that is correct, but the magnitude of falling step will be this temperature minus the temperature that the thermometer was showing here I gave a step input of a magnitude 10.

So, the thermometer will finally, show 110 degree Fahrenheit only when the thermometer attends steady state, but here after one time constant value the thermometer was showing 106.32 degree Fahrenheit. So, the magnitude of the following step is 100 minus 106.32. So, this is that value and 1 minus e to the power minus t by tau. So, time t equal to 2 minute and you straight away compute as 100.85 degree Fahrenheit.

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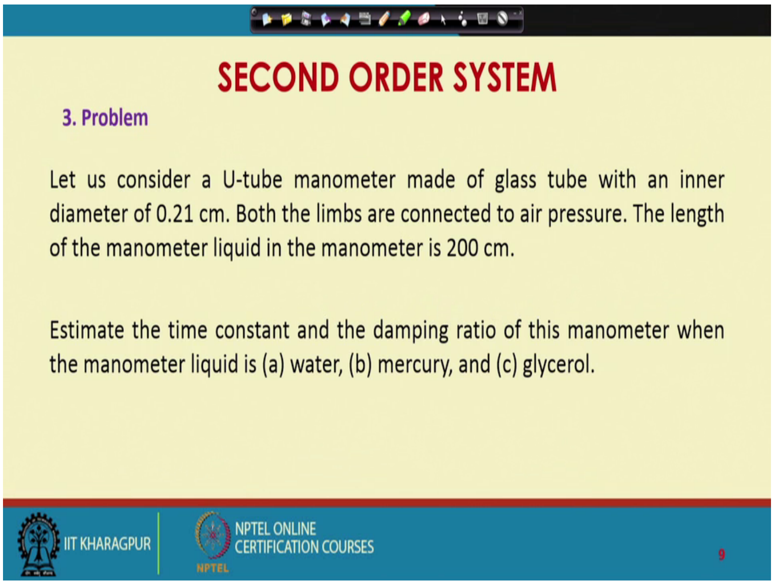

Third problem; let us now consider u tube manometer. So, as a second order instrument, we will talk about now a problem of response of second order instrument. So, let us consider u tube manometer made of glass tube with an inner diameter of 0.21 centimeter both the limbs are connected to air pressure the length of the manometer liquid in the manometer is 200 centimeter.

We use to estimate the time constant and the damping ratio of this manometer when the manometer in liquid is water mercury and glycerol. So, information that is given is diameter of the manometer tube the length of the manometer liquid and we have to find out the time constant and the damping ratio of the manometer for 3 different types of manometer liquids. So, we need the density values for this manometer liquids you also need the viscosity values for this manometer liquid.

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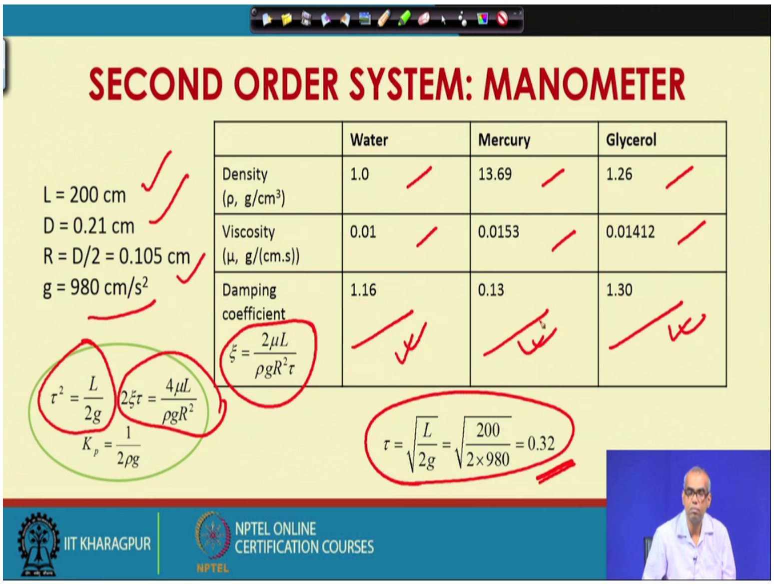

So, let us imagine we have found out this values from literature density of water we know mercury glycerol viscosity of water mercury and glycerol values are given as L is 2 hundred centimeter diameter is 0.21 centimeters radius is half of that and g acceleration due to gravity is given as this; now the time constant tau we know tau square is L by 2 g. So, it can be computed in a straight forward manner from this equation tau is square root of L by 2 g L is given as 200 centimeter.

So, tau is 0.32; now the damping coefficient is obtained from 2 zeta tau equal to four mu L by rho g are squared for me you represent viscosity values are given tau is computed now. So, you can directly make use of this equation which has been the; arrange to obtain zeta. So, you get zeta equal to 1.16 zeta equal to 0.13 and zeta equal to 1.30. So, this is overdamped this is overdamped this is underdamped.

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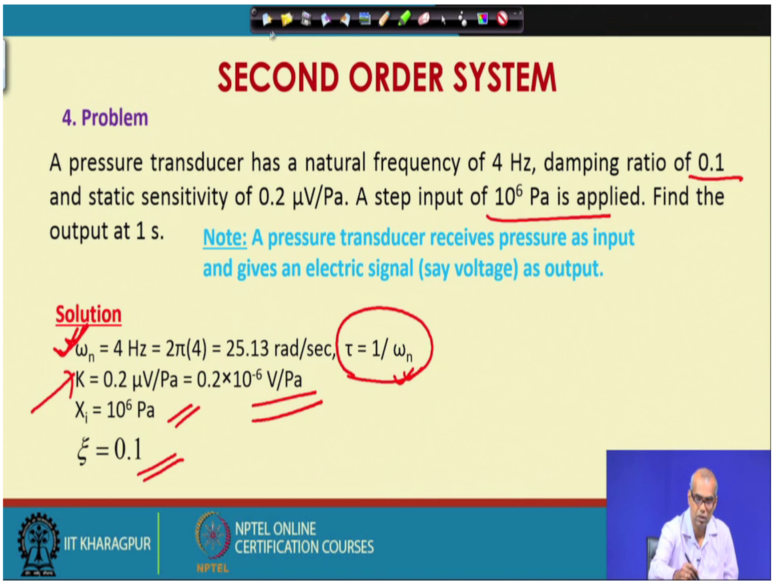

Finally let us take another problem on second order instrument a pressure transducer has a natural frequency of four hertz damping ratio of 0.1 and static sensitivity of 0.2 micro volt by Pascal a step input of 10 to the power 6 pascal is applied find the output at 1 second a pressure transducer we will talk about later for that time being; let us consider that the pressure transducer receives pressure as input and gives an electrical signal as output that electrical signal let us say voltage.

So, all the informations are given omega n is given tau can be obtained as one by omega n. So, frequency of oscillation is given from which the tau can be time constant tau can be obtained statistic sensitivity is given K as 0.2 micro volt per pascal you have to where expressing it in terms of volt per Pascal input is given as 10 to the power 6 pascal zeta is given as point one damping ratio. So, this is the case of underdamped instrument. So, we have to make use of the expression.

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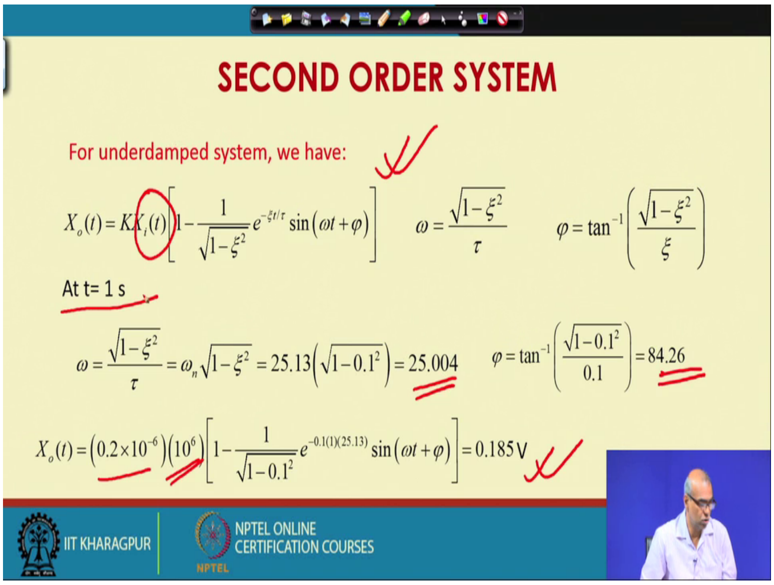

For underdamped response. So, we put all this values we put all this values in the expression for underdamped response. So, this is the expression for underdamped response I put all the values with input as 10 to the power 6 Pascals K is given all other terms are now known. So, the output at one second can be computed directly from the expression as 0.185 volt.

So, we will stop here for lecture 11, in this lecture we have taken 4 different numerical problems on responses of first order instruments and second order instruments.