Transport Phenomena
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Lecture 8
Example of Shell Momentum Balance (Contd.)

So we will continue with examples of shell momentum balance. This class I will show you two problems. I will not solve them completely. I'll leave that to you and I will provide you with answer. But you would see that it's no longer easy to think of the right kind of shell. And what happens if it's an unsteady state problem in which velocity has not stabilized with time? How do we tackle such problems? What if, let's say I have a plate and a liquid on top of it. Everything is stationary.

So time T less than zero, nothing moves. The plate is stationary, the fluid is stationary. At time T equals zero suddenly the bottom plate is set in motion. So as the bottom plate is set in motion, the layer just about it, the liquid layer just above it, due to no slip condition will start to move. But the top layer slightly above it still doesn't know that a motion has been initiated somewhere down below. So it would take some time. Before by viscous transport thetop layer would realize that there is motion towards the right.

Now therefore you would see that the velocity is not only a function of let's say Y, distance from top plate, but it is also a function of time. Because at this location if you fix the location Y location, the velocity keep on changing with Y as the effect of the motion of the bottom plate will be fulfilled more and more at this Y location till we reach the steadier state. But whenever this we have a flow suddenly set in motion. The flow will no longer remain one-dimensional.

How to handle that problem with a simple shell balance? It is not possible. In some cases, like previously we have dealt with the flow in a tube inpresence of a pressure gradient and in presence of gravity. What if it is not a straight tube? What if the shape of the tube changes, the diameter does not remain the same, the diameter is constant for somelength and then it changes its shape? What if the flow is no longer inside the tube but it comes from inside the tube and then starts to spill over and flow along the outside of the tube? All these are possibilities.

All these are situations which one must know how to analyze in order to provide solutions for everyday problems. Solutions for problems that many of us would encounter. So we cannot restrict ourselves to the simplest possibility case of one dimensional steady flow. So before we move on to a systematic study of the set of equations that can provide a complete solution for such cases. Two more examples to see how still at a rudimentary level we can use the shell momentum balance.

So the first problem that we are going to deal with is about flow in a narrow slate. Let's say a narrowslate is formed by two walls. The distance between them is quite small compared to their width or compared to their length. So it's as if these two papers are kept very close to each other, such that the gap between the two is small as compared to the width of it or the length of it.

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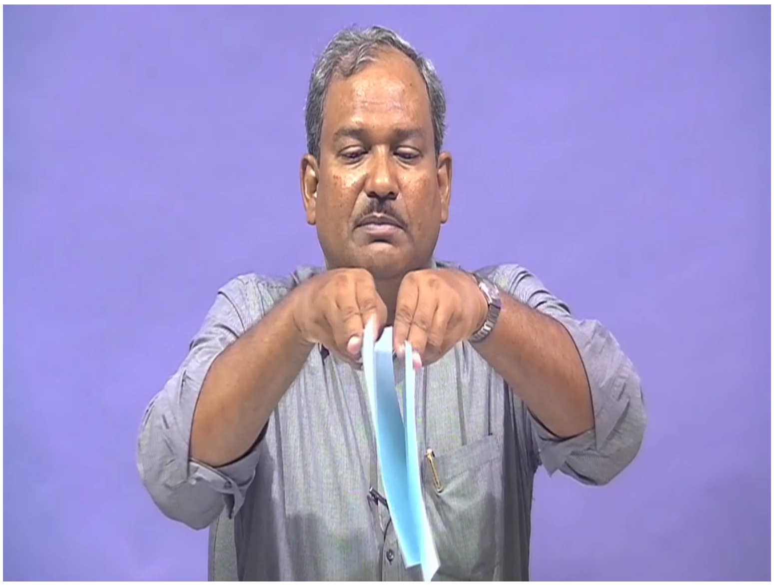

So I have another paper which is very close forming a slate in between. And let us assume that towards the upper portion of the slate there is some applied pressure which is more than the pressure at the bottom. And it's obvious that if I can keepit vertical, then there would be effect of gravity forces is as well. As a result of which the fluid starts to flow in between these two slates. Inbetween the slates formed by the two plates. So the downward velocity obviously is going to be function of how far it is from either of these two plates.

So if I assume one dimensional flow once again as simplification. If I assume that it's acted upon by gravity and pressure difference. However in steady 1D incompressible flow, then the velocity is a function only of how far they are from the side walls of the slate. And it's not going to be a function of where it is in terms of the Z location, let's say. So we are (de) going to start with our analysis of flow in a slate.

Pressure gradient, gravity both are present and the slate is very narrow such that the variation with in this direction is important, variation in this direction can be neglected and it’s 1D (ste) steady flow, so there is no variation in the direction of flow.

So I am trying to draw the slate. So this is a slate where two plates are situated at a distance of 2B apart. The origin of the co-ordinate system is here, where this is the Z direction, this is Y and this is the X direction.So I have flow through this and flow comes out through this. The width of it is W, the length of the two plates is equal to L. And we can see that vZ is the only non-zero component of velocity and vZ is a function of X. VZ is not a function of Z. And since it's a narrow slate, dependence of vZ on Y can be neglected.

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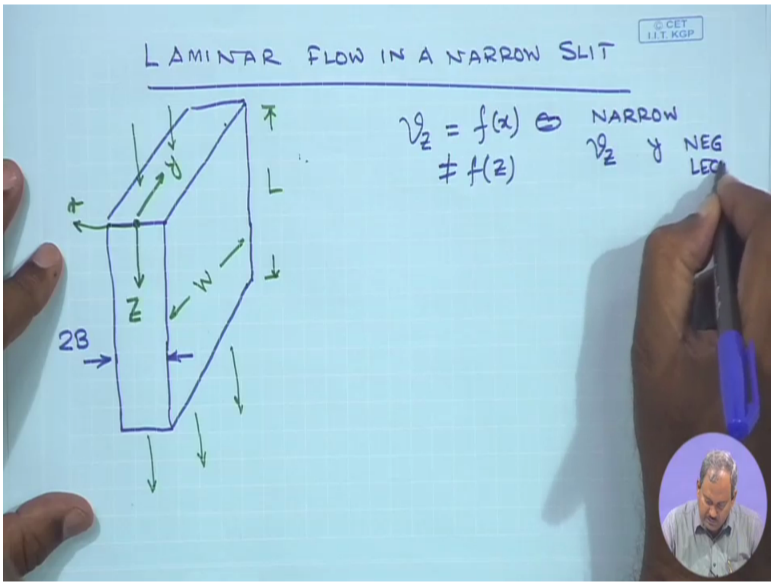

What I mean to say is that this is too long compared to this gap. So therefore what happens near Y equals 0 or at Y equals W that portion can be neglected. So the flow is principally one dimensional and it's a steady state case. So these are our assumptions.

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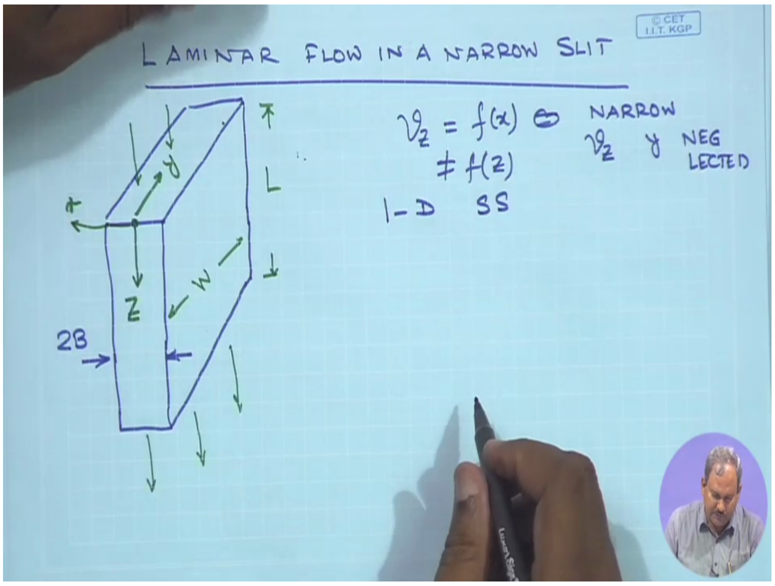

So again we need to first find out what is the expression for tau to be used in here. What is tau and we can see its Z momentum, but Z momentum is being carried by viscosity in the X direction. So therefore the subscript for tau should be tau XZ. And tau XZ would act and since your velocity varies with X so your shell is going to be this. Shell is going to be of thickness delta x. Right because that's how we draw our shell. Whereit's a dimension direction in which thedependent variable, velocity in this case, keeps on changing.

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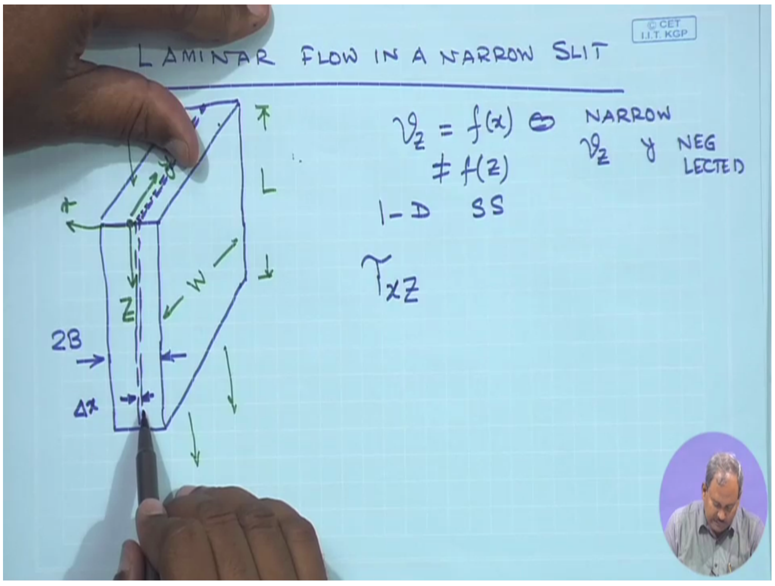

So that's the case. Then tau XZ works on an area WL which is evaluated at X and then the same thing goes out is same tau XZ times WL. But here we have X plus delta x.

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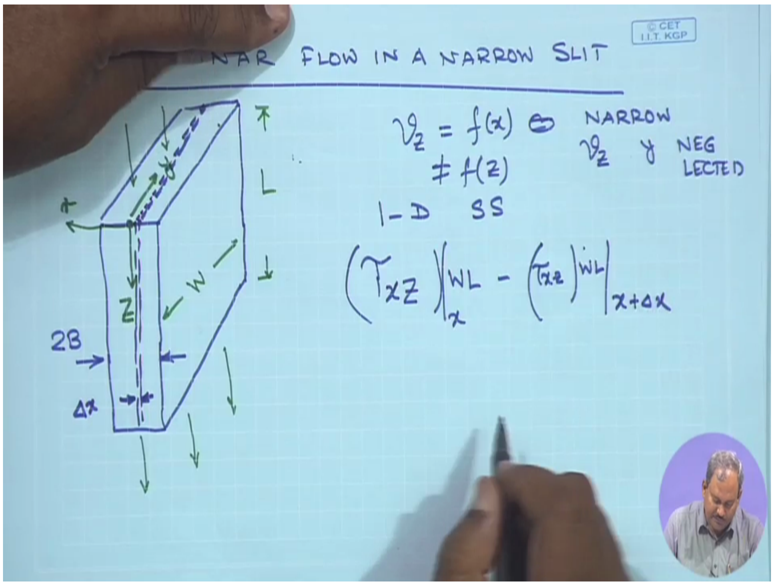

The amount of convective mass which comes in through the top and leaves at the bottom are equal. So I do not write time because they are going to get cancelled anyway. So the mass or the rate of momentum coming in because of flow at the top of the shell and the bottom of the shell are equal. So there is no net contribution ofconvective momentum into this case.

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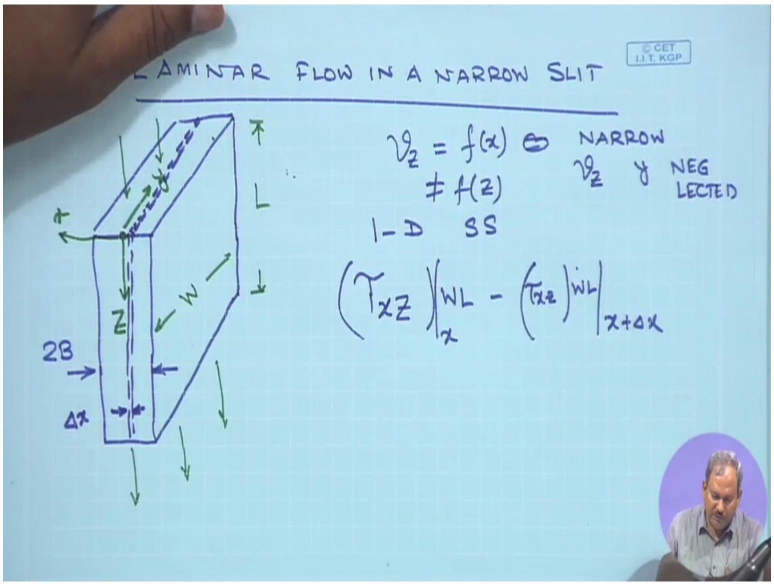

But there is an effect of pressure and there is an effect of gravity. So what is the effect of pressure? Let us assume the pressure over here is p0 and the pressure over here is pL. So the force due to pressure on the top surface that forces the liquid to move in the Z direction would simply be W delta x p0. So that is the area W times delta x and p0 is a pressure and the one that's working at the bottom would be simply W delta x times pL, which is at the bottom.

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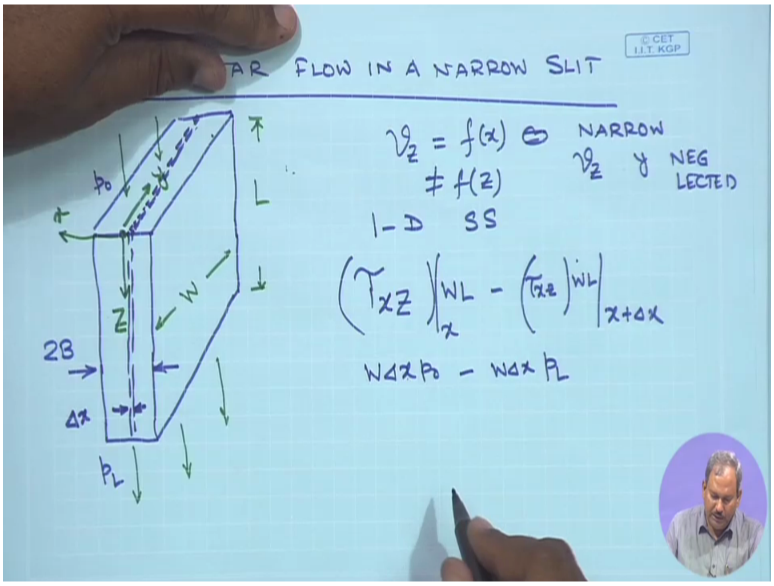

And we have the effect of gravity. The effect of gravity can we simply obtained by finding out what is the volume of the entire shell which is W delta x L. What is the mass of it? Multiplied with G. So you get the conductive transport of momentum, the pressure forces, the surface forces which are acting on it and the gravity which is acting on the shell. At steady state the sum of all these must be equal to zero. So you should be able to express it in terms of quantities that we you know what are the steps.

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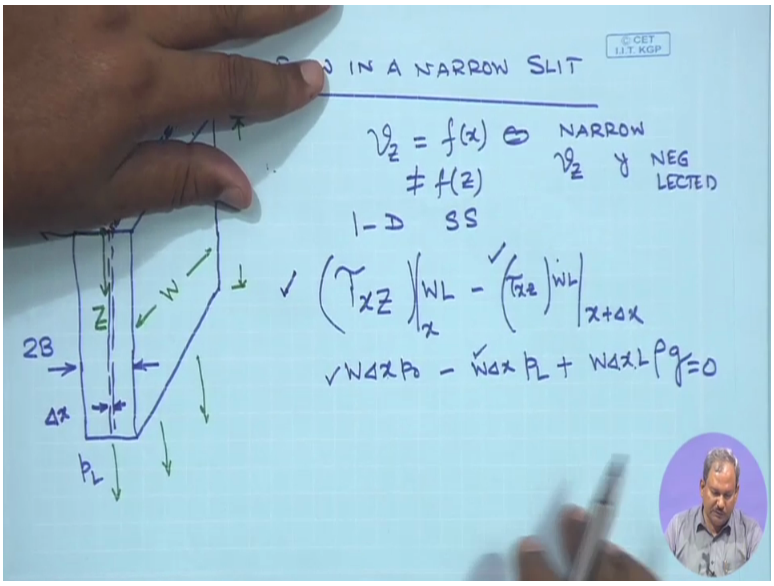

Next in the (boun) the boundary conditions that one can use in this case is that at X equals to B, the velocity is going to be zero. VZ is going to be zero. That's a no slip condition. And you can make another simplification is that you can clearly see that the velocity is going to be maximum at X equals zero. Because at the center plain, the velocity is (max) going to be maximum and therefore if the velocity is maximum at X equal to zero, then tau would be equal to zero at X equal to zero.

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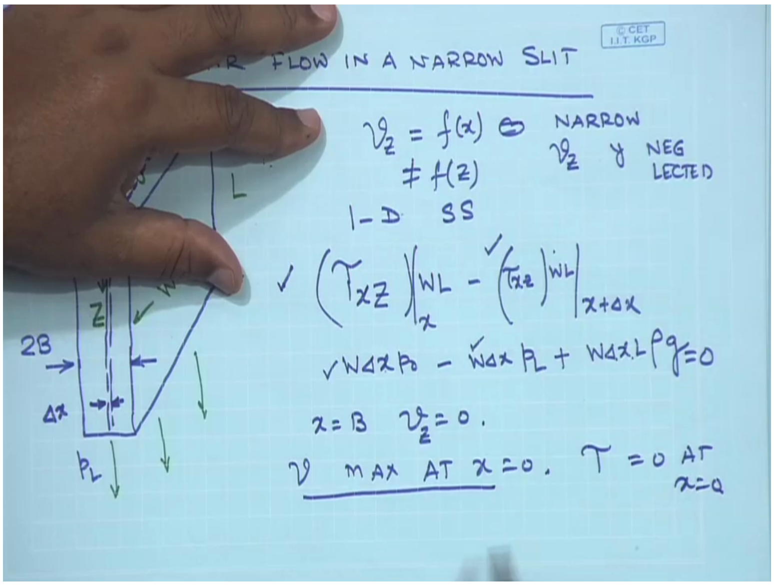

Because in order for velocity to be maximum, dVdX, D vZdX is zero, multiply it with mu and what you get is tau XZ.Tau XZ would be equal to zero. So you can use this could be your boundary condition one, this could be a boundary condition two and use of these boundary simplifications which I am not going to do any work which would give you an expression for the velocity and this is for you to check that your velocity expression would be equals to B square by 2 mu p0 minus pL by L times 1 minus X by B whole square.

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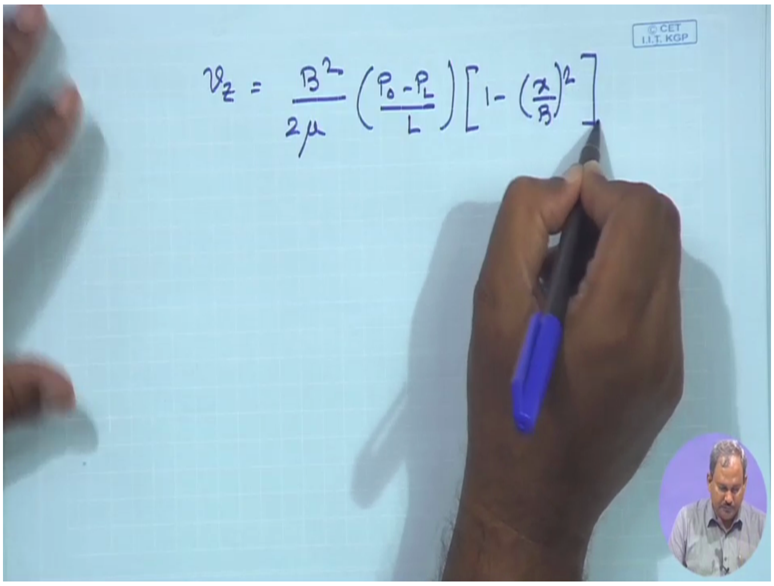

So these capital P zeros are the same this thing.The governing equations would simple be D dX of tau XZ equals p0 plus rho G0 minus pL plus rho GL divided by L.And this one I call it as p0 and this one I call it as pL by L. So this is this is this is how I have work in the expression for vZ.

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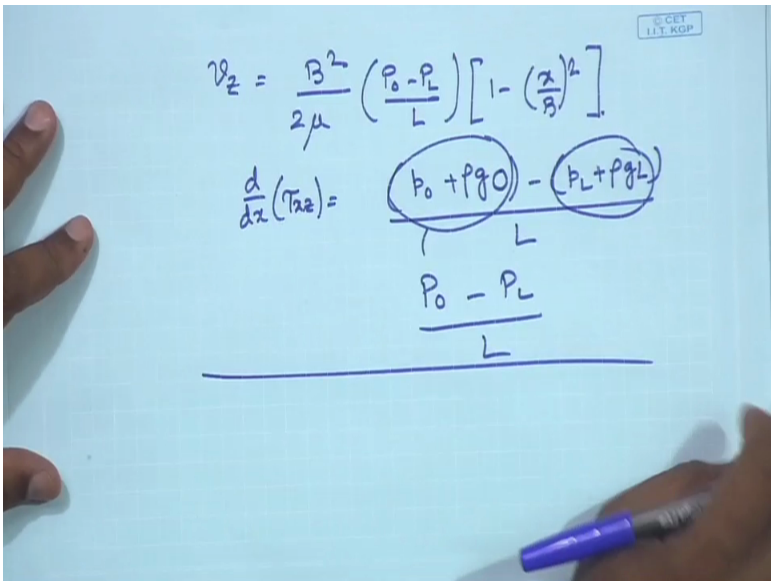

Once you have obtained the expression for vZ, you can obtain an expression for average velocity which should turn out to be, this is for you to check again, p0 minus pLby L. And I would also like you to find out what is the relation between vZ and Vmax and if you work out you would see that the relation is going to be this. So you should check this out as well.

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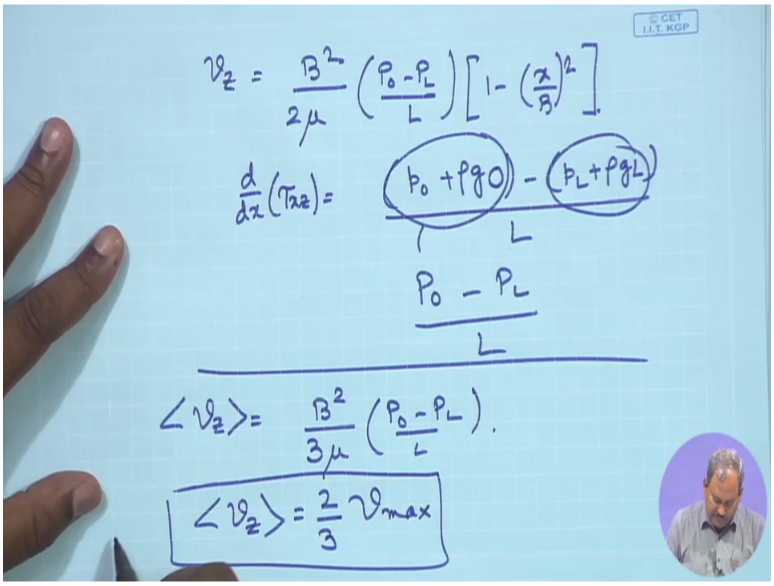

And the volumetric flow rate would be equal to 2 by 3 p0 minus pL by mu L timesB cube W. Again another practice problem for you. So you are going to find out what is a velocity? What is the average velocity?What is the relation between average velocity and the maximum velocity? And what is the relation between the volumetric flow rate and the imposed condition? The geometry and the property, the viscosity in here.

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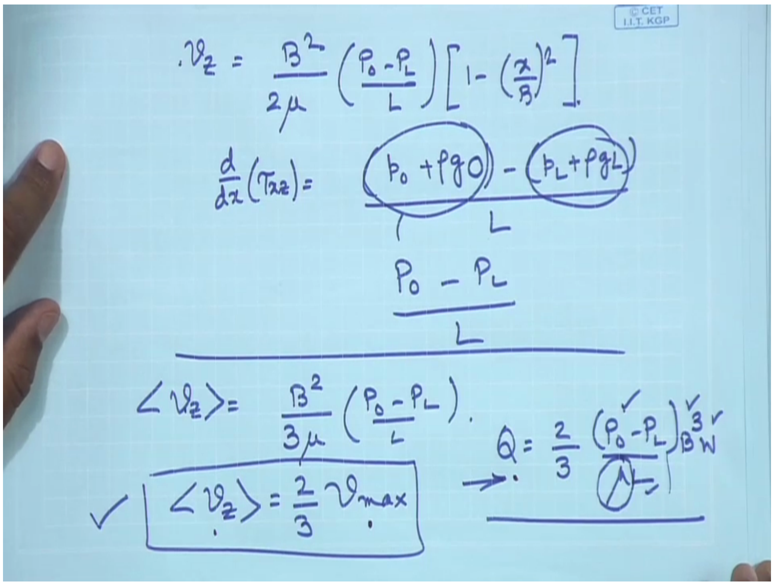

And these are the four things which you should do it on your own and check if you are getting the correct result or not. And if there is any question you can always ask the TA to about whatever I am teaching so far, if you have any questions, you should always contact the two TAs.

Contact me and we will try to clarify any doubts that you may have either in the concepts, whatever I teach here or in the problems that I am giving you or will give you (fu) in future and (prac) for you to practice on. So please ask questions, send us your queries and we will try to answer them. The last problem before we formally close this session, shell momentum balance and go into something deeper is a case in which there is a pipe and through this pipe. It's a poor example of a pipe.

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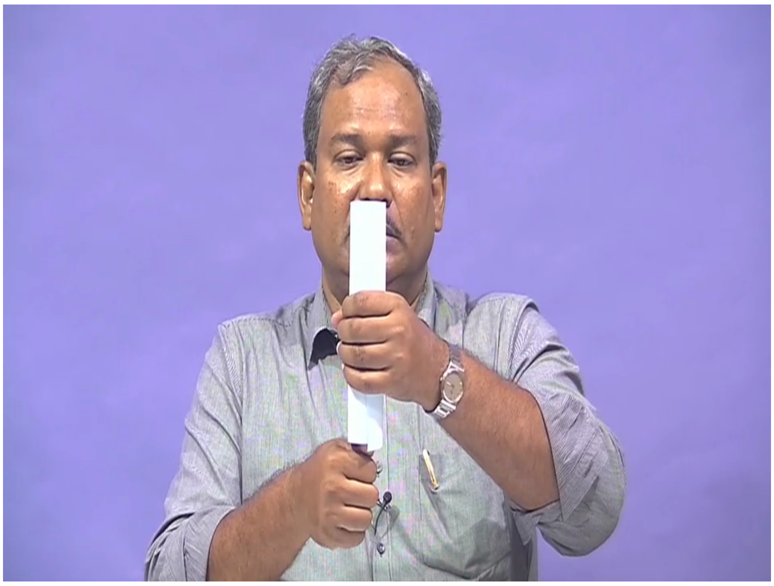

As liquid comes in, there is a pressure gradient which forces the liquid to come to the top. As it comes to the top, it spills over and as it spills over, it starts to fall along the outside of the pipe wall. Creating a film of some known thickness on it. So we have flow in at the top it reverses its direction and starts to flow along the walls of the pipe. We need to find out, we need to analyze this problem. So youhave to be careful in here.

You are not dealing with what is happening inside the pipe, you are dealing with what is happening on the outside of the pipe. And in the outside of the pipe, the region is at the radius and beyond. Not the point where (bet) from zero to R. So zero to capital R is not your domain of interest. It’sR to some film thickness, that is what you are going to analyze and trying to find out what is the velocity, trying to find out what is the flow rate and so on.

When you see the falling film outside of it, even though you have pressure which is forcing the liquid to move up, come to the top and then spill over. When it starts to fall, it's a freely falling film. There isno imposed pressure gradient on the system. The liquid is falling on its own. So there is no pressure gradient, only gravity which is present. And what are the boundary conditions? The boundary conditions are also clear. At over here there is no slip.

So the film in contact with at the outside of the wall, the no slip condition will simply tell you that the velocity of the following film in contact with the outside of the pipe wall is zero. And if it is falling as a film then I must also have a liquid-vapor interface. The outside of the following film and the air beyond that. So that is liquid-vapor or liquid-air interface. And what is your boundary condition to be used for liquid-air interface? That the shear stress is zero.

So this falling film will have two boundary conditions. The first is no slip at the pipe wall and no shear at the edge of the film in contact with air. These two are the boundary conditions. But there is one conceptual thing that I would like to mention once after I draw the picture and will provide you with the answer for you to try on your own.

So this one looks like, I am only drawing half of it. Let's say this is the outer wall of the pipe. I have the same thing on this side which I am not drawing.

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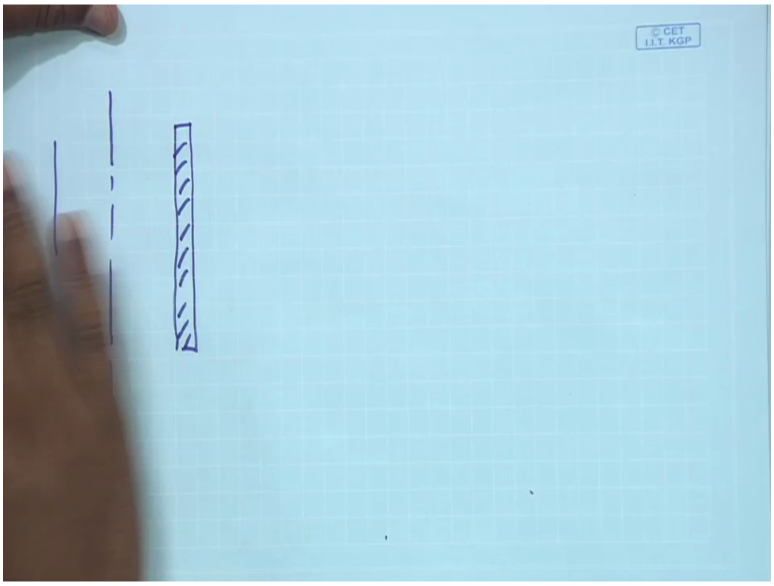

I have a flow of liquid. Then it comes over here and starts falling as a film. Let's say that this is R and this is aR. So that defines essentially the thickness of the film. Nowyou first have to find out what is tau? What are the subscripts of tau? If this is your Z direction and this is your R direction, then obviously this is Z momentum getting transported in the R direction. So it's tau RZ.

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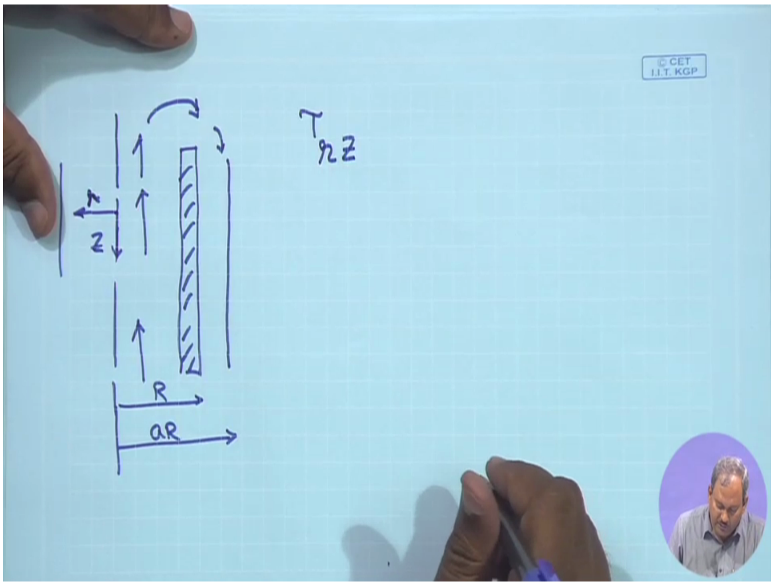

So any shell that you are going to choose must be of thickness delta r. So here you have at R. Here you have at R plus delta r. So at the same time the convective flowin and the convective flow out are equal and they will cancel each other. So I need to only find out what is the shear coming in, the force due to shear, the force due to shear going out. There is no pressure, only gravity which is acting on it.

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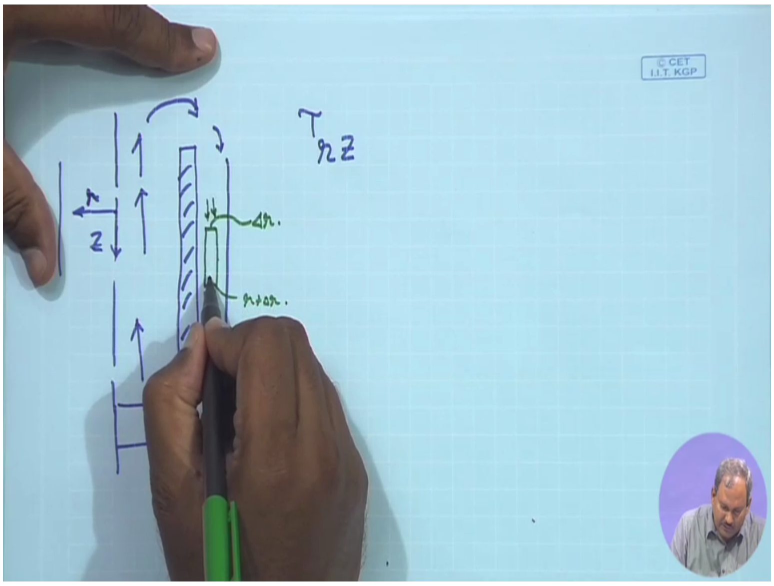

So the area on which this tau RZ is acting on is simply twice pie R times L. Where L is the length of this imaginary shell wrapped around the tube.

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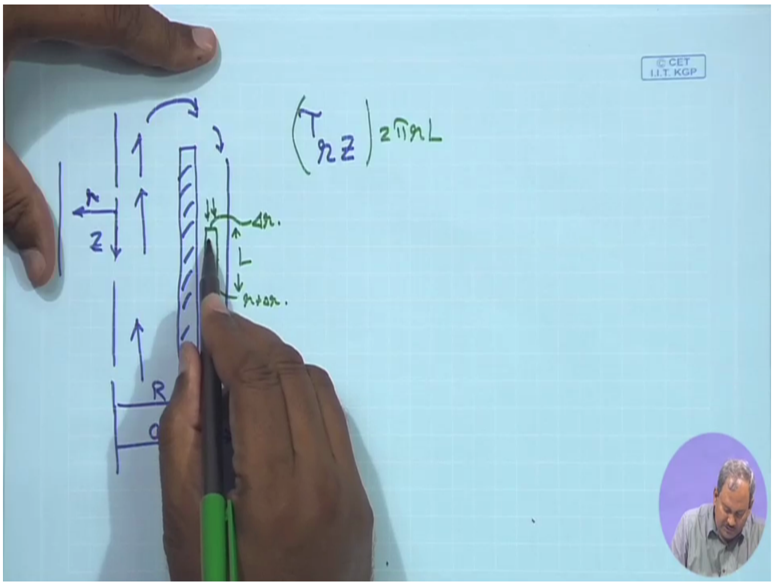

So if this is the tube and I have something wrapped around it, the thickness of it is delta R and the length of it is L. So I am making a balance on this ring kind of structure around that tube. I am trying to find out what are the shear contribution? What is the contribution of the convective momentum, the gravity and so on? So what you have in here is twice pie RL,that's the area evaluated at R minus same thing at R plus delta r times twice pie R plus delta r times L. No contribution from gravity, sorry no contribution from convection, no pressure difference.

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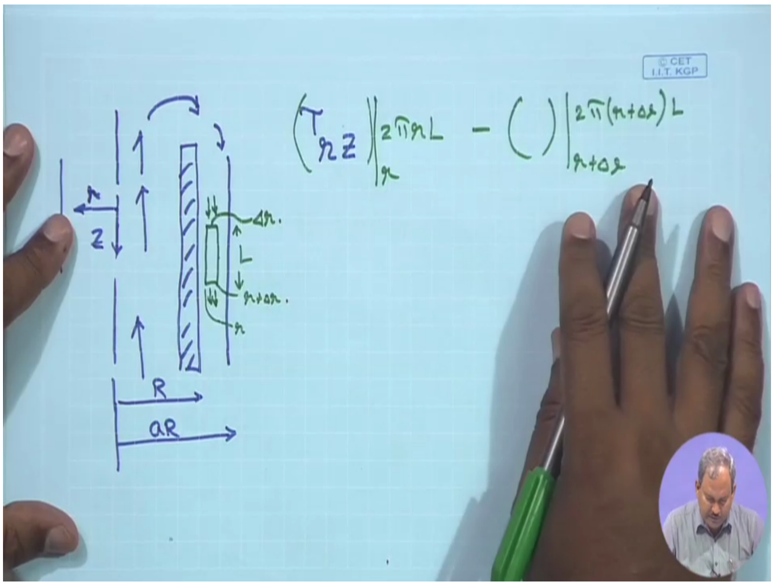

The only other thing is gravity for which we need to find out what is the (ar)volume? Multiply that volume with rho and with G, in order to obtain the body force. And at steady state this is equal to zero. So after you divide both sides by del r, the usual practice by now you are experts of that. So what you get is, DdR of R tau RZ is equal to rho G times R or tauRZ is equal to rho GR by 2 plus C1 by R.

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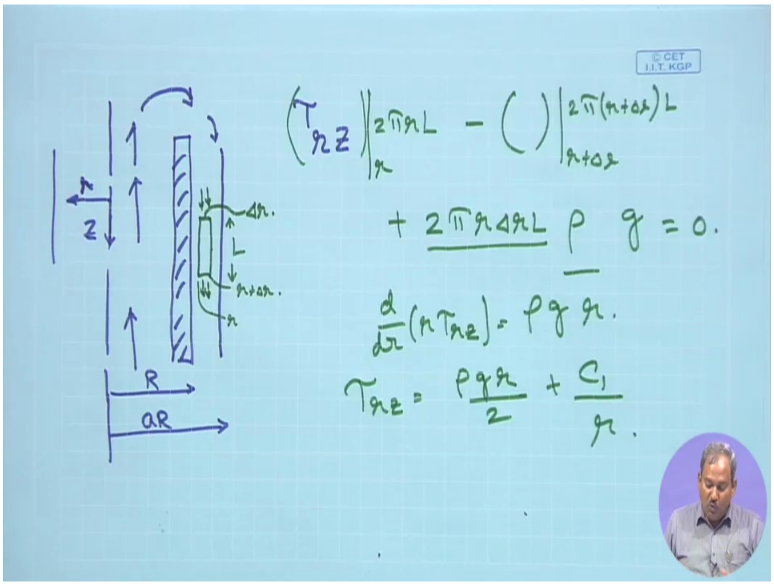

What is the boundary condition to be used here? Can you say something about boundary condition? Like if you look at this expression, you will be tempted to use the condition that at R equals 0, tau RZ has to be finite and therefore C1 is equal to zero. This is the first thing that may come to your mind.

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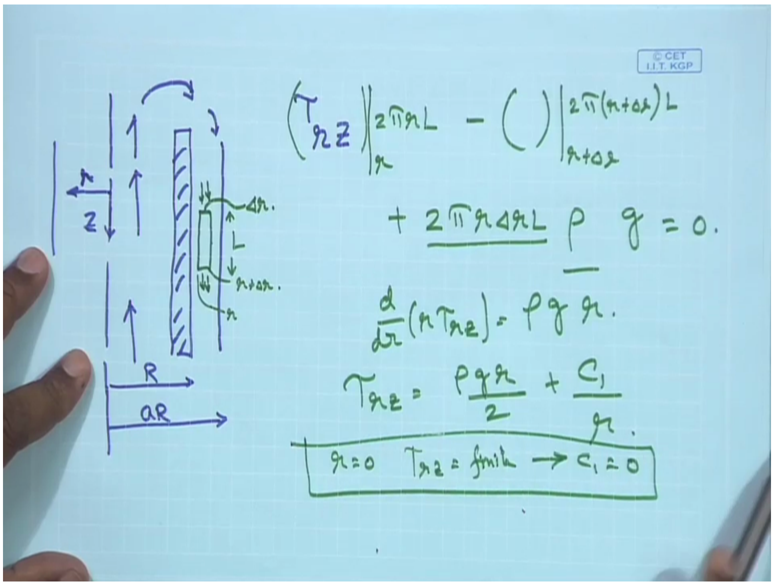

But this is wrong. Why this is wrong? Because the equation, the governing equation that you have written is valid for a space outside of the tube, not inside of the tube.

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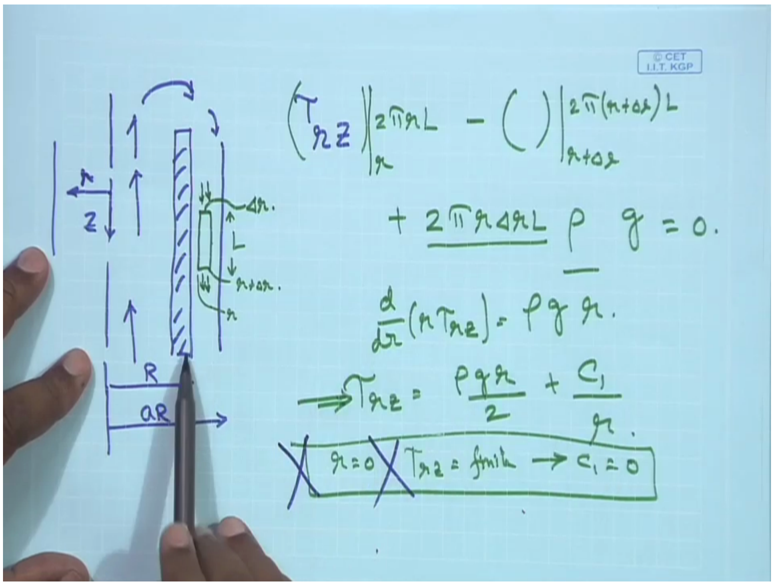

So the equation that you have written essentially tells you what happens in the falling film of the liquid outside of the tube. So your domain of applicability of the governing equations starts at r equals capital R and extends all the way to A times R, which is specifically the outside of the film. It is not valid for any value of r which is less than capital R. Therefore you cannot use a condition R equals 0 to a (bound) governing equation which is not valid for R equals 0.

This is an important lesson which we must keep in mind is that whenever we write a governing equation, whenever which choose the boundary condition, we should be careful about what is the region of the applicability of this governing equation. Whatever boundary condition that I am choosing, is that valid for the case that we are handling? Is that within the domain of applicability of the governing equation? That is something one has to keep in mind in order to solve for this.

So this cannot be a boundary condition in that case. Then the only option here is, we have to substitute the tauRZ and using Newton’s law and when you do Newton's law and do the integration you would get vZ equals minus rho GR square by 4 mu minus C1 L and R plus C2. And C1 and C2 in this expression can be obtained by the boundary conditions which are no slip.

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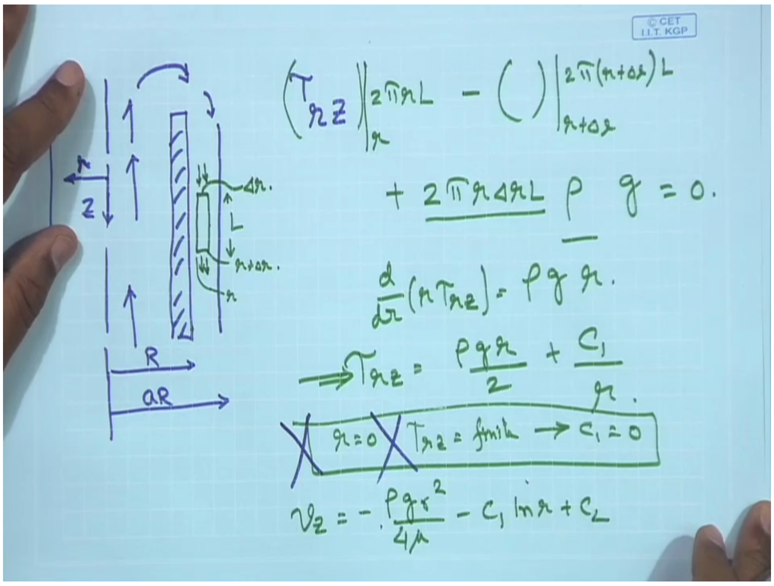

At r equals capital R, the velocity is equal to zero and at r equals aR, D vZdR is 0. So this is no slip and this one is no shear. So this is liquid-solid interface, this is liquid-vapor interface. No slip and not shear.

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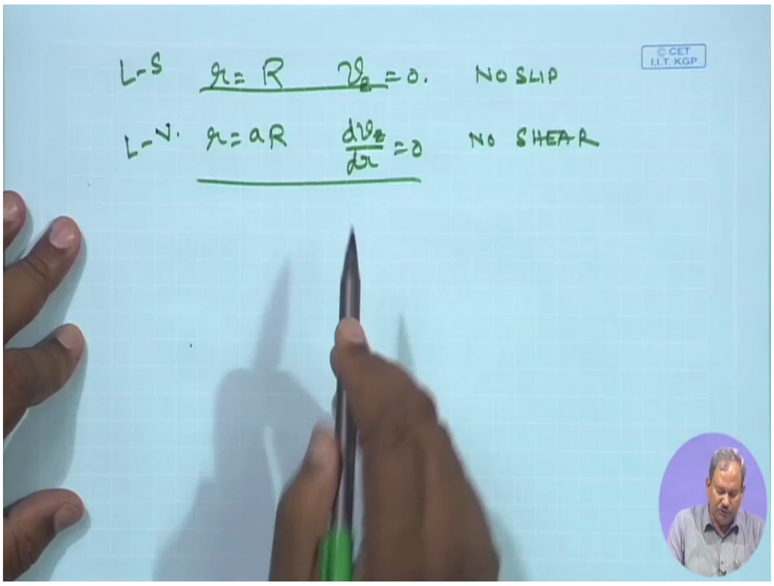

When you are do that and you put in the expressions, the final expressions you would get is equals R square by 4 mu, 1 minus small r by capital R whole square plus 2A square LN r by R. This is going to be the final expression for velocity for flow outside of a tube.

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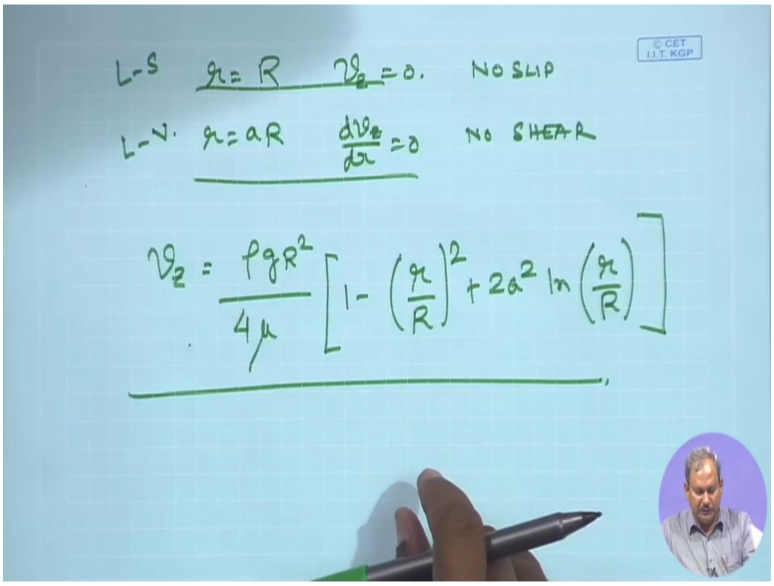

And I think you can see slowly that especially in the last problem mentally visualizing what will be the shell, is becoming an issue. There are cases as I have told you at the beginning of the classes. Unsteady state problems for a flow sudden is set in motion. Or in the previous problem we have made a very grave assumption that, I think I should point it out. Let's look at this figure more carefully.

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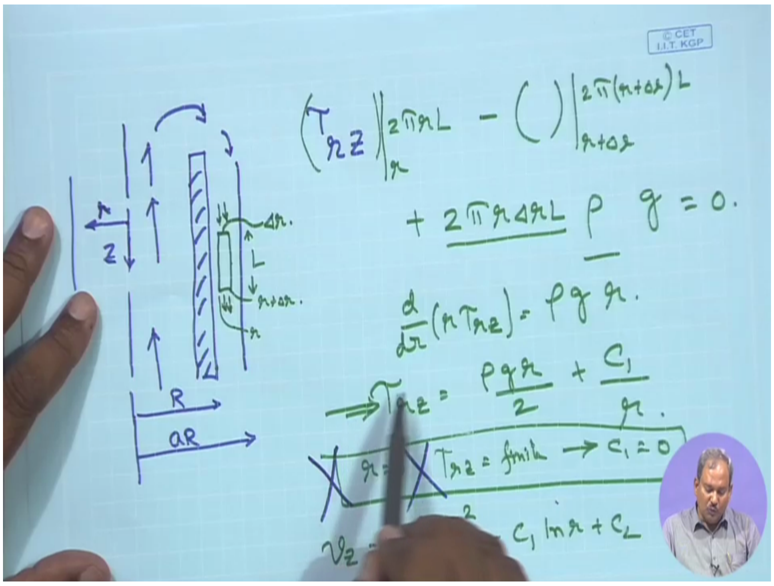

Whatever expressions that we haveused, it's essentially true for one dimensional flow. That means the flow is only in theZ direction. There is no flow in the R direction and there is no flow in thick direction. That may be true when we are somewhere over here.

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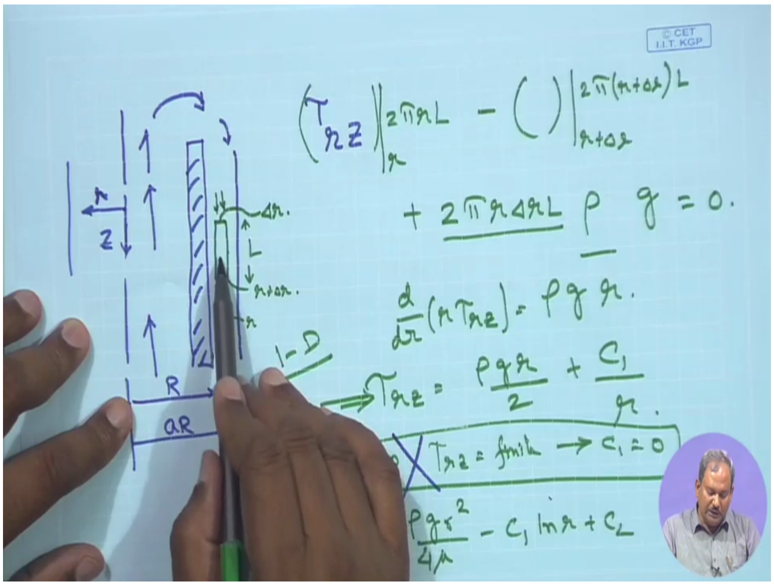

But what happens and this region, when the flow changes from its motion in the minus Z direction, it suddenly changes and start flowing in the reverse direction. So thereis bound to be 2D effects near this region which cannot be modelled by a simple shell balance like this.

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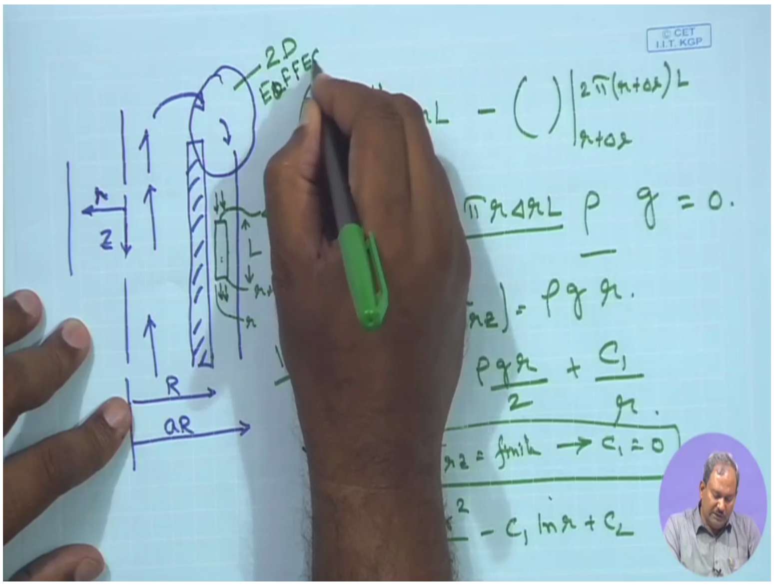

So the applicability of this shell balance for this specific problem is way below the top of the plate, where all these 2D effects have subsided.

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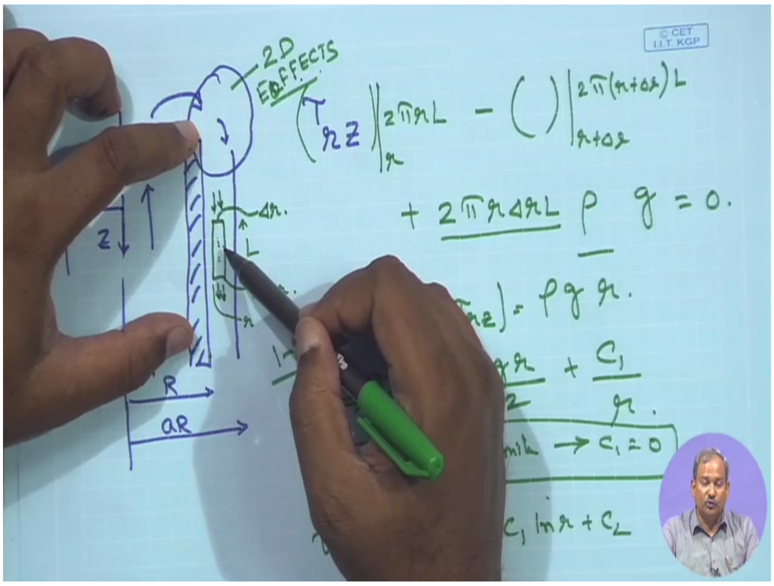

So you cannot have a 2D effect, you cannot account for 2D effects by simply expressing it in terms of a shell momentum balance. So this is valid for Newtonian steady state 1D flow. But leaving aside section near the top plate where your mode of analysis is no longer valid.

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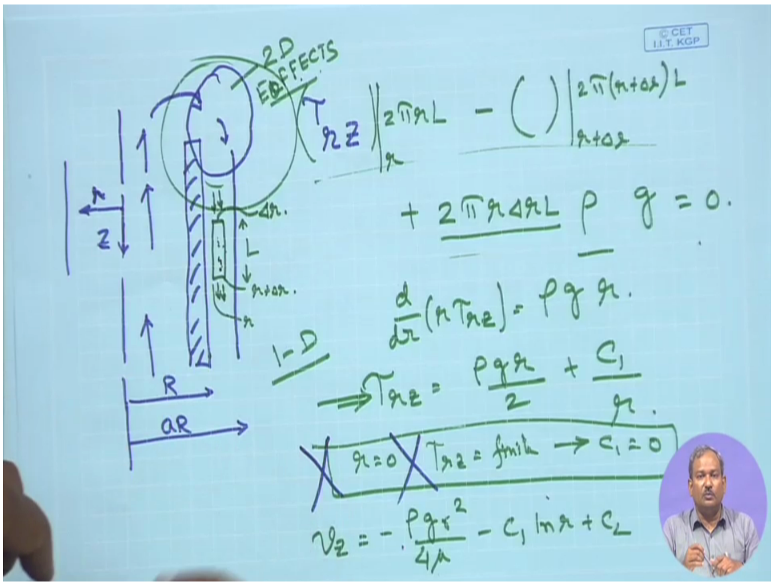

So again showing a potential problem limitation of the shell balance approach. So for unsteady state cases, for cases where there is change in the flow direction resulting in situations where you can get multidimensional effects, the shell balance method fails. We cannot have shell balance method. And similarly we are resuming that it's laminar flow. That means all the transport of momentum is due to viscosity only. Its viscous transport of momentum.

If it’s a turbulent flow, then most of the momentum could be carried not by this viscosity or the molecular momentum but it would be due to the formation of eddies. So eddies is a packet of fluids which generate in turbulent flow and which carry with the momentum from one point to the other. So the transport of momentum by eddies will supersede that by the simple molecular transport in laminar flow which we have modelled up to this point.

So we can clearly see a need for a more generalized treatment for situations where we have multidimensional effects,the effect of unsteady behavior. Andnot laminar, but beyond laminar there may be a turbulent flow as well. How do we take in the account the additional momentum transport due to the formation of eddies. So there has to be a general treatment. There has to be an equation which would be complicated to begin with but if you cancel the terms which are not relevant then they would reduce to a very compact neat expression.

And we do not have to worry about the shell. We do not have to worry about the multidimensional effect and so on. So in our next part I would try to explain to you what is the momentum balance for an open system giving rise for the case of mass? If youthink of the conversation of mass, what is the equation of continuity? And finally what is equation of motion or more commonly known as the Navier Stokes equation?

So Navier Stokes equation would do exactly the same thing that we have done so far but in a more structured manner. So the next classes we will deal with the concept of Navier Stokes equation and more importantly how to use them for solving the problem that we have already solved and beyond. Thank you.