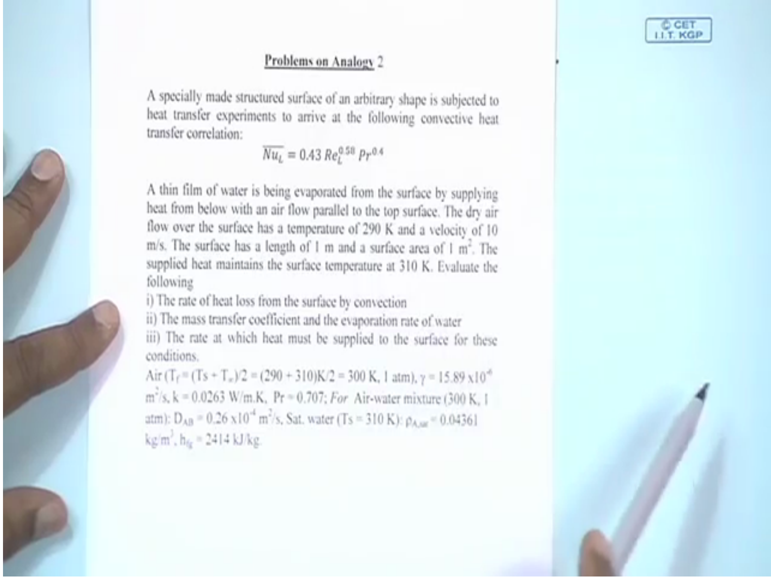
# Transport Phenomena.
## Professor Sunando Dasgupta.
## Department of Chemical Engineering.
## Indian Institute of Technology, Kharagpur.
## Lecture-55.
## Analogy-Tutorial II.

So we are going to start today with the solution of a few more problems, so this is again going to be tutorial session and we would solve a number of problems looking at different aspects of analogy and how one relation can be used in a completely different process as long as some of the basic conditions are met. So the 1st problem that we are going to take a look is the one in which we have a surface and on the surface heat transfer experiments were performed to arrive at a relation, correlation for Nusselt number. And we understand from our study that the Nusselt number correlation that one would expect, one that would like to express the experimental data must contain Nusselt number would be function of Reynolds number and Prandlt number.

The exact relation between Nusselt number, Reynolds number and Prandlt number that was obtained in this specific case by analysing the large number of experimental data. So with this knowledge, then what were going to do, we are going to do a mass transfer operation, we are going to predict the performance of the mass transfer experiment based on our knowledge of the heat transfer experiments. So this is a perfect example of use of knowledge gathered in one type of transport process and projecting it to obtain the useful relations or numbers with respect to a completely different type of transport process.

(Refer Slide Time: 1:54)



Problems on Analogy 2

A specially made structured surface of an arbitrary shape is subjected to heat transfer experiments to arrive at the following convective heat transfer correlation:

$$\overline{Nu_L} = 0.43\, Re_L^{0.58}\, Pr^{0.4}$$

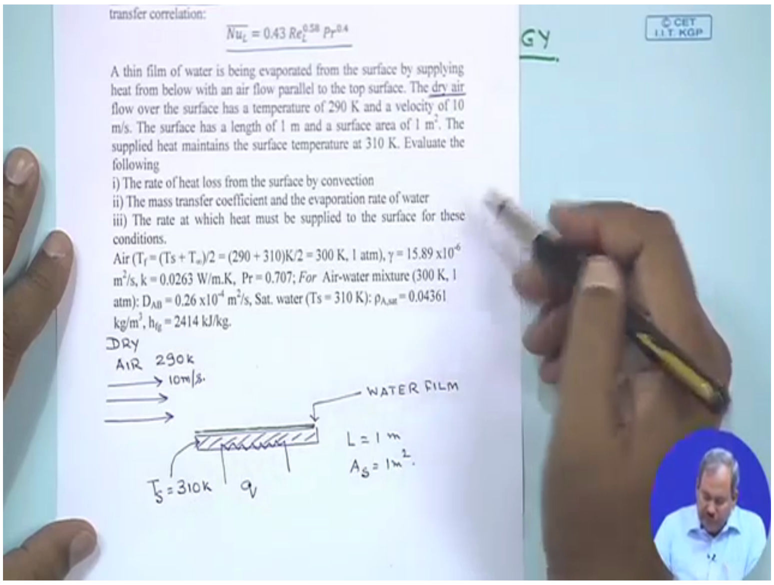
A thin film of water is being evaporated from the surface by supplying heat from below with an air flow parallel to the top surface. The dry air flow over the surface has a temperature of 290 K and a velocity of 10 m/s. The surface has a length of 1 m and a surface area of 1 $m^2$. The supplied heat maintains the surface temperature at 310 K. Evaluate the following

i) The rate of heat loss from the surface by convection

ii) The mass transfer coefficient and the evaporation rate of water

iii) The rate at which heat must be supplied to the surface for these conditions.

Air ($T_f = (Ts + T_\infty)/2 = (290 + 310)K/2 = 300$ K, 1 atm), $\gamma = 15.89 \times 10^{-6}$ $m^2/s$, k = 0.0263 W/m.K, Pr = 0.707; *For* Air-water mixture (300 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ $m^2/s$, Sat. water (Ts = 310 K): $\rho_{A,sat} = 0.04361$ $kg/m^3$, $h_{fg} = 2414$ kJ/kg.



transfer correlation:

$$\overline{Nu_L} = 0.43\, Re_L^{0.58}\, Pr^{0.4}$$

A thin film of water is being evaporated from the surface by supplying heat from below with an air flow parallel to the top surface. The dry air flow over the surface has a temperature of 290 K and a velocity of 10 m/s. The surface has a length of 1 m and a surface area of 1 $m^2$. The supplied heat maintains the surface temperature at 310 K. Evaluate the following

i) The rate of heat loss from the surface by convection

ii) The mass transfer coefficient and the evaporation rate of water

iii) The rate at which heat must be supplied to the surface for these conditions.

Air ($T_f = (T_s + T_\infty)/2 = (290 + 310)K/2 = 300$ K, 1 atm), $\gamma = 15.89 \times 10^{-6}$ $m^2/s$, k = 0.0263 W/m.K, Pr = 0.707; *For* Air-water mixture (300 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ $m^2/s$, Sat. water (Ts = 310 K): $\rho_{A,sat} = 0.04361$ $kg/m^3$, $h_{fg} = 2414$ kJ/kg.

So the 1st problem today, which is the 2nd problem that we are going to analyse in analogy 2 in Reynolds, modified Reynolds Chilton Colburn analysis is that there is a specially made structured surface of an any arbitrary shape is subjected to heat transfer experiments to arrive at the following convective heat transfer correlations. So this is the relation that was obtained by heat transfer experiment. Okay. And then in the next part what it says is that a thin film of water is being evaporated from the surface by supplying heat from below with an airflow parallel to the top surface.

So with the knowledge of the heat transfer, then what we have then is the same surface for which the heat transfer relation is known to us it is and it contains a very thin film of, thin film of water. So this is the water film that is there, so this is the water film and this is going to be evaporated by, heat is supplied from below, so you provide some heat from below through an electrical heating may be. And we also have an airflow parallel to the top surface. So this is the airflow which flows over the top surface. It has been mentioned that the air is dry, so it is dry air, essentially signifying that it does not contain any water in it.

So whatever water film, it is evaporating in dry air. And the temperature of this air is at 290 Kelvin and it has a velocity of 10 metre per second and the length of the surface is 1 metre and the surface area is 1 metre square. The supplied heat, the heat that is supplied maintains the temperature of the surface which I denote by TS to be equal to, to be equal to 310 Kelvin. So you now have a situation in which water is going to be, going to get evaporated, what is going to evaporate to a dry air which is flowing at 10 metre per seconds at a temperature of 290 Kelvin and you are supplying, the supplied heat maintains the temperature of the solid plate on which the water film rests at a temperature of 300 Kelvin.

So there are 3 parts to this problem, we will solve each one of them but this is what the system would look like. So we start with a known correlation from heat transfer experiments and then you are essentially asked to calculate several parameters which are related to mass transfer. The 1st part is the heat loss from the surface by convection, by only by convection, if you do not have any mass transfer and from this point onwards, 2nd and 3 would essentially require solution of the mass transfer part of the transport process as well. So you would 1st start with the rates of heat loss from the surface by convection.

I think the problem is clear to clear to all, the problem clear to all of you, you have a surface and the heat transfer correlations for that surface is known to us, now we cover the surface with a thin-film of liquid and you use a heater to increase the temperature of the substrate to some extent. So the temperature of the substrate is elevated as compared to the air which is flowing over it parallel to the surface. So I have air flowing over it at 290 Kelvin, whereas the substrate is maintained at 310 Kelvin. Even in the absence of the water film, since the temperature of the substrate and the temperature of the air, they are different, you are going to have convective heat transfer coefficient, convective heat transfer.

Since the 2 temperatures are different and you have an imposed flow on a hot surface and the temperature difference between the air which is flowing and the solid substrate would ensure that you have convective heat transfer, convective heat transfer in this case. So our job is to obtain the convective heat transfer, 2nd, this surface is now coated with a thin film of water, so now what is going to evaporate, so this is a case of simultaneous heat and mass transfer. The principal heat transfer is the one heat transfer is due to convective heat transfer, then I have a mass transfer, so I need to find out what is the mass flux of water vapour leaving the water film.

So the mass flux of the water in the vapour form which leaves the film, that needs to be calculated. So how do we calculate that, in order to that, it is again a convective mass transfer process, so in order to calculate that, I need to find out what is the convective mass transfer coefficient. But I do not know what, any relation, what is going to be a relation for convective mass transfer coefficient. So in order to obtain the value or the expression of the convective mass transfer coefficient, I need to use the analogy. So I would use modified Reynolds Chilton Colburn analogy and the one that is known to me is the heat transfer relation, the Nusselt number relation.

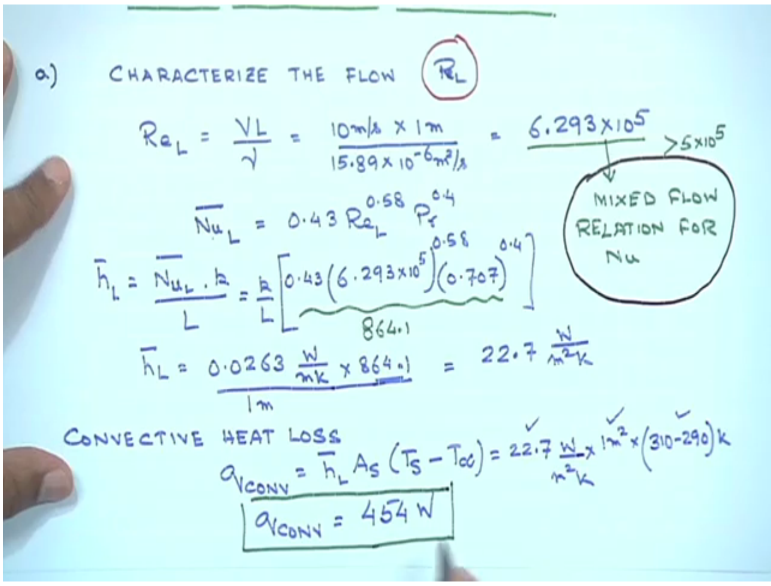
So we would use Chilton Colburn analogy to obtain a relation for the Sherwood number which is equivalent to Nusselt number in heat transfer. So the 2nd part would involve obtaining or evaluating the expression for the Sherwood number for the mass transfer case. Once I have the Sherwood number I would be able to obtain what is the value of the convective mass transfer coefficient or H suffix M. This H M, this is convective mass transfer coefficient should be multiplied with area and the concentration difference in to obtain the mass flux of water in vapour form leaving the interface.

So H bar M times A, times Delta rho where rho is the concentration of, concentration change in between the water-air interface and at a point far from the interface, so if this difference is known to me, then H times H M times A times Delta rho would give me the mass of air, mass of water which gets evaporated which changes phase due to the concentration difference. So whenever we, when we evaluate the total amount of mass which leaves the surface, I need to, that amount of water while transforming, while getting transformed to the vapour state will require latent heat.

So the latent heat of vaporisation, it is going to take from the solid substrate. So therefore a part of this heat supplied by the heater is going to go for convective heat transfer and the major part, you would see later on, the major part of the heat supplied by the heater to the substrate would be used up for the evaporative process. So there is an evaporative process requiring heat transfer where there is a change of phase and a convective process in which we would also require we would also need the heater to supply sufficient heat to maintain the temperature of the solid substrate at a constant point. So whatever heat that is supplied by the heater would be the sum of the convective heat losses and heat required for evaporation of the amount of water which we have calculated through the analogy by evaluating the value of H bar M from the known expression of H or Nusselt number.

So that in essence is what we are going to do the theory behind this problem. But 1st we are going to start with what is the heat transfer coefficient and once we have the heat transfer coefficient, find out what is the convective heat loss from the surface at steady-state. So that is what we are going to do now.

(Refer Slide Time: 11:27)



a) CHARACTERIZE THE FLOW $\boxed{Re_L}$

$$Re_L = \frac{VL}{\nu} = \frac{10\,m/s \times 1\,m}{15.89 \times 10^{-6}\,m^2/s} = \underline{6.293 \times 10^5} > 5 \times 10^5$$

MIXED FLOW RELATION FOR Nu

$$\overline{Nu}_L = 0.43\, Re_L^{0.58}\, Pr^{0.4}$$

$$\bar{h}_L = \frac{\overline{Nu}_L \cdot k}{L} = \frac{k}{L}\left[\underbrace{0.43\left(6.293 \times 10^5\right)^{0.58}(0.707)^{0.4}}_{864.1}\right]$$

$$\bar{h}_L = \frac{0.0263\,\frac{W}{mK} \times 864.1}{1\,m} = 22.7\,\frac{W}{m^2K}$$

CONVECTIVE HEAT LOSS

$$q_{conv} = \bar{h}_L A_s (T_s - T_\infty) = \overset{\checkmark}{22.7}\,\frac{W}{m^2K} \times \overset{\checkmark}{1\,m^2} \times \overset{\checkmark}{(310-290)}\,K$$

$$\boxed{q_{conv} = 454\,W}$$

So what we do have is for the part A, that is there in the problem. The 1st thing is we need to characterise the flow. And when I say characterising a flow I am talking about what is the value of Reynolds umber. So Reynolds number is the one which characterises the flow and this Reynolds number is essentially based on the entire length of the pipe. So the Reynolds number therefore is going to be V times L by the kinematic viscosity, the V is provided as 10 metre per seconds into length is 1 metre and the kinematic viscosity is also provided and you would see its value is going to be 6.293 into 10 to the power 5. It has been provided the relationship for Nusselt number, the average value of Nusselt number is, based on the entire length of the substrate is given as 0.43 Rel to the power 0.58 times Prandlt number to the power 0.4.

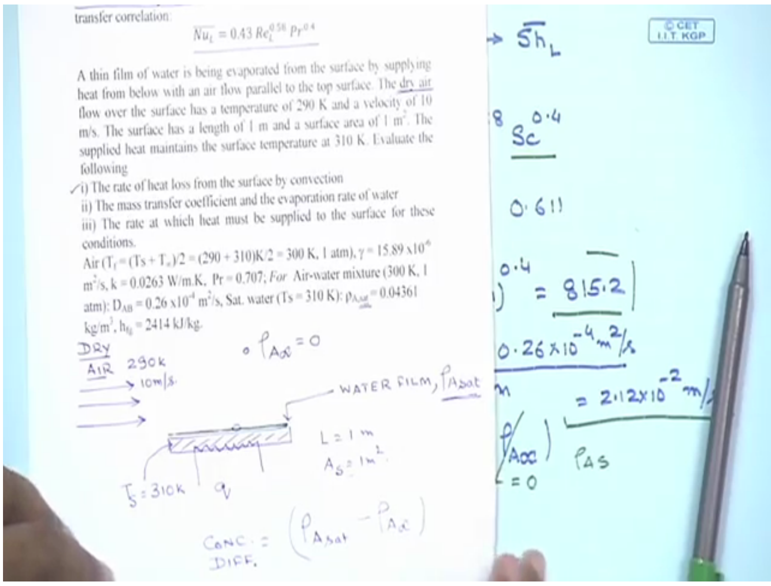
So this is a relation which has been obtained through the analysis of experimental data, if it is not provided, then we realise that this value being greater than 5 into 10 to the power 5, this value of Reynolds number corresponds to turbulent flow. Now nothing, anything, it is unlikely that the flow is going to be turbulent from the very beginning, so in absence of the Nusselt number relation, the other choice is to use the mixed flow relation mixed flow relation for Nusselt number. But since, since it is better than 5 into 10 to the power 5, in order to obtain Nusselt number, I could have in absence of this relation, I could have used mixed flow relation for Nusselt number, which were not doing right now, since relation for Nusselt number is already provided to us.

So from here the average heat transfer coefficient can be calculated as thermal conductivity by length and when you put in the, put in the numbers over here, so it is 0.43, Reynolds number is 6.293 into 10 to the power 5, Prandlt number is given as, this is air, so Prandlt number is roughly is going to be 0.707 but the value of Prandlt number has also been provided, you can calculate that and this is 0.4 and this is your Nusselt number multiplied by K by L and we know what is the value of K, so H bar L is K, this entire thing, that is, the Nusselt number this would turn out to be 864.1.

So your heat transfer coefficient would simply be equals 0.0263 Watts per metre Kelvin which is for the K and the length is simply 1 metre times 864.1 which is the dimensionless number and this would simply turn out to be 22.7 watts per metre square Kelvin. So using the relations provided we can obtain what is the average value of convective heat of coefficient. So the convective heat loss from the surface, from the solid surface is simply Q, I will write, I will put the substrate convection to show that this is the convective loss only which would be average value of heat transfer coefficient times surface area times temperature of the surface - temperature of the air far from the surface.

And we know from our description of the problem, the temperature of the surface is 310 Kelvin and that of air is 290 Kelvin, the area is provided as 1 metre square, so the, when you put the values in here, this is going to be 22.7 Watts per metre square, area is equal to 1 metre square and the temperature difference is going to be 310 - 290 Kelvin Watts per metre square, Kelvin is missing here, so this is H, K, Dell T and the value of Q convection would turn out to be 454 watts. So that is the answer to the 1st part of the problem, that is the heat lost from the surface by convection, we are not considering mass transfer your, so if I find out by convection, this is going to be the amount of heat loss.

(Refer Slide Time: 17:27)



transfer correlation:

$$\overline{Nu_L} = 0.43\, Re_L^{0.58}\, Pr^{0.4}$$

A thin film of water is being evaporated from the surface by supplying heat from below with an air flow parallel to the top surface. The dry air flow over the surface has a temperature of 290 K and a velocity of 10 m/s. The surface has a length of 1 m and a surface area of 1 m². The supplied heat maintains the surface temperature at 310 K. Evaluate the following

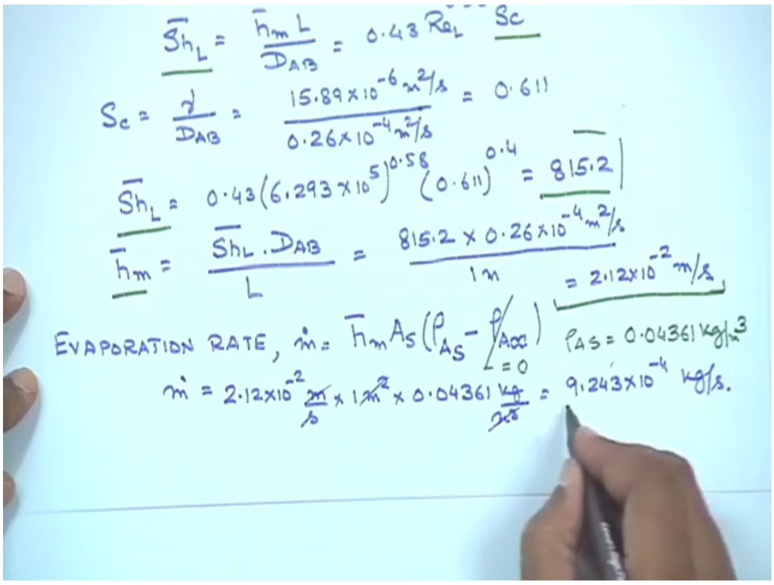
✓ i) The rate of heat loss from the surface by convection

ii) The mass transfer coefficient and the evaporation rate of water

iii) The rate at which heat must be supplied to the surface for these conditions.

Air ($T_f = (Ts + T_\infty)/2 = (290 + 310)K/2 = 300$ K, 1 atm), $\gamma = 15.89 \times 10^{-6}$ m²/s, k = 0.0263 W/m.K, Pr = 0.707; *For* Air-water mixture (300 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ m²/s, Sat. water (Ts = 310 K): $\rho_{A,sat} = 0.04361$ kg/m³, $h_{fg} = 2414$ kJ/kg.

CONC. DIFF. = $(\rho_{Asat} - \rho_{A\infty})$



$$\overline{Sh_L} = \frac{\overline{h_m} L}{D_{AB}} = 0.43\, Re_L^{0.58}\, Sc^{0.4}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{15.89 \times 10^{-6}\, m^2/s}{0.26 \times 10^{-4}\, m^2/s} = 0.611$$

$$\overline{Sh_L} = 0.43(6.293 \times 10^5)^{0.58}(0.611)^{0.4} = \overline{815.2}$$

$$\overline{h_m} = \frac{\overline{Sh_L} . D_{AB}}{L} = \frac{815.2 \times 0.26 \times 10^{-4}\, m^2/s}{1m} = 2.12 \times 10^{-2}\, m/s$$

EVAPORATION RATE, $\dot{m} = \overline{h_m} A_S (\rho_{AS} - \underbrace{\cancel{\rho_{A\infty}}}_{=0})$  $\rho_{AS} = 0.04361\, kg/m^3$

$$\dot{m} = 2.12 \times 10^{-2} \frac{\cancel{m}}{s} \times 1 \cancel{m^2} \times 0.04361 \frac{kg}{\cancel{m^3}} = 9.243 \times 10^{-4}\, kg/s.$$

Now I think we can go to the 2nd part of the problem, the 2nd part of the problem tells us that we need to find out what is the evaporation rate, so we need to find out evaporation rate. In order to find out evaporation rate, I need to find out what is H bar M, in order to find H bar M, I need to find out what is the average value of the Sherwood number based on the entire length of the substrate. So I use heat and mass transfer analogy and I realise that due to heat and mass transfer analogy, the relations, the expressions of Nusselt number and the expressions of Sherwood number are interchangeable.

What I mean by interchangeable is that the heat transfer relation which has been obtained can simply be used as the mass transfer relation provided we change Sherwood number, the Nusselt number by Sherwood number, Reynolds number will appear in both the relations and the Prandlt number is going to be replaced by the corresponding number in mass transfer which is Schmidt number. So what we have then here is, Sherwood number average based on the entire length is simply going to be H bar M, convective mass tougher coefficient by the diffusion coefficient would simply be equals 0.43 Reynolds number L to the power of 0.58 and SC to the power 0.4, so you have changes are Sherwood number in place of Nusselt number and Schmidt number in place of Prandlt number.

The Schmidt number is nu by DAB which is the kinematic viscosity divided by the mass diffusivity and this value is provided as 15.89 into 10 to the power - 6 metre square per second, the value of DAB is provided as 0.26 into 10 to the power - 4 metre square per second. So the kinematic diffusivity, kinematic viscosity or the momentum diffusivity and this is the mass diffusivity which the ratio of these 2 is nothing but the Schmidt number. And the value would turn out to be equal to 0.611. So you are sure would number based on length would simply be equals, equal to 0.43, Reynolds is 6.293 into 10 to the power 5 to the power 0.58 and Schmidt number 0.61 by to the power 0.4.

So this would be equal to 815.2, so this is going to be the value of Sherwood number for the case of mass transfer. So without performing any mass transfer experiments or without performing any complicated analysis for the case of coupled heat and mass transfer, we obtained what is the Sherwood number by simply using the heat and mass transfer analogy. So once we have the Sherwood number, the number, the Sherwood number available to us, then H bar M, that is the convective mass transfer coefficient would simply be Sherwood number based on length times DAB by L. So this is 815.2 which is dimensionless, the DAB is the diffusion coefficient which is provided, number of that is provided in metre square per second and then you have L is in 1 metre.

So this H bar M would be equal to 2.12 into 10 to the power - 2 metre per second. So the value of convective mass tougher coefficient is also known to me and once that is done, then the evaporation rate, M dot, if I call it as M dot would simply be equal to the convective mass transfer coefficient, the surface area, the species, the concentration, the mass concentration of the component A which is water at the surface - rho of air, rho of water at an infinite distance from the surface. So this is the standard formula equivalent to that of Newton's law of cooling, so it is exactly like Newton's law of cooling where the mass flux is expressed in terms of a convective mass transfer coefficient, the surface area and the difference in the species concentration at the surface and at infinity.

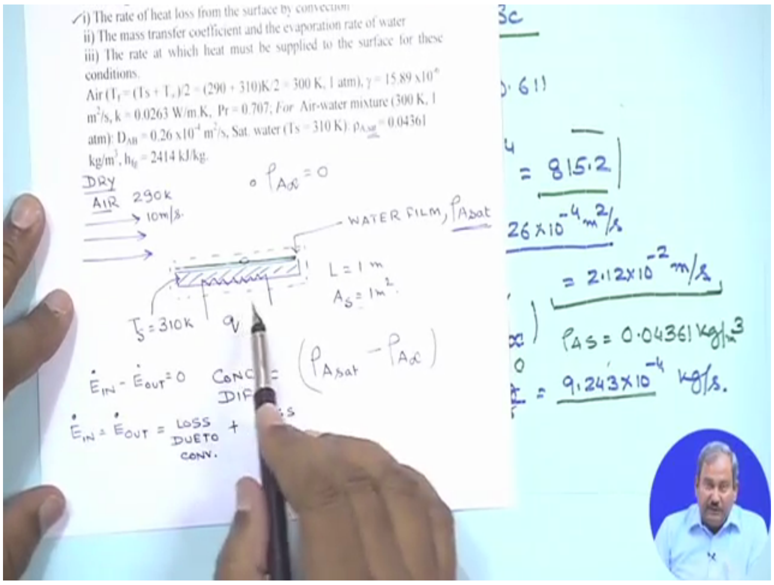
We realise that the air which flows over the surface is dry so therefore this would be equal to 0 and the value of rho AS is provided in the problem, the value of rho A sat is 0.04361 KG per metre cube, what we are assuming here is that at this point, the water molecules on the vapour side, on the air side and the water molecules on the liquid side are the vapour just in contact with the air film is saturated. So the concentration of water at this picture can be taken equal to rho A sat. So the water film, the concentration would be rho A sat and the concentration over here far from the solid plate is rho A infinity.

Which, since it is a dry air, this rho A infinity would be equal to 0 and the concentration at the water film air interface, we will assume that it is the air film is saturated with water and therefore the concentration of water at this junction would be equal to rho A sat. And therefore the effective concentration difference, so the concentration difference which is used, which is the cause of evaporation would simply be equals rho A sat at the interface - rho A infinity at a point far from the interface. So once I have the concentration difference, this concentration difference can be multiplied by the area and multiplied by H in order to obtain the total, total mass flux or in other words evaporation rate from the waters film covered surface.

Thus what we have here then is rho AS to be equal to 0, I mean rho A infinity to be equal to 0 and the value of rho AS is provided which is 0.04361, it is in KG per metre cube, that is tha the concentration of a kind of concentration of air, concentration of water near the surface. Therefore the, when you put in the values, what you are going to get, M dot would simply be equals HM, we have evaluated it to be 2.12 into 10 to the power -2 metre per second, the area is 1 metre square and this is simply going to be 0.04361 KG per metre cube. So what you have then is M dot, these metre cubes will cancel and we will simply have the multiplication which is which is going to be 9.243 into 10 to the power - 4 and unit as you can see would be KG per second.

So this is the total amount of mass of water which goes from the film to the air or in other words this is the evaporation rate. So all the evaporation that you can think of is a result of mass transfer from the water surface, water air surface to a point far into the air wherein where the concentration of water is almost negligible. So once you have the mass product in there, then the rate, the next part, last part of the problem, so the 2nd part of the problem tells us to find out what is the mass transfer coefficient and the evaporation rate of water which we have evaluated, so this is my mass transfer coefficient and this is the rate of evaporation of water.

(Refer Slide Time: 26:59)

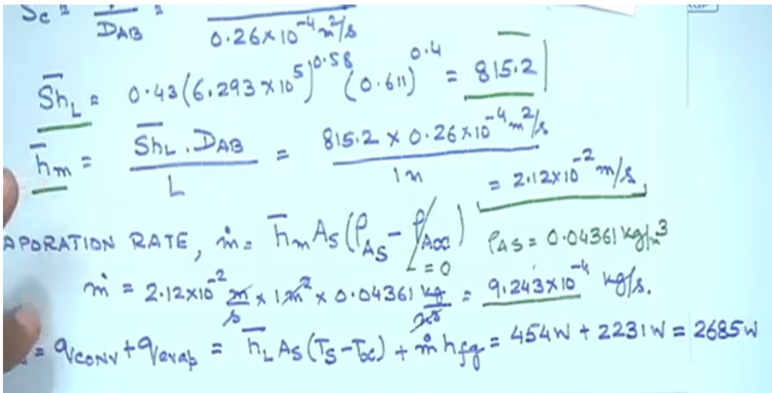


i) The rate of heat loss from the surface by convection

ii) The mass transfer coefficient and the evaporation rate of water

iii) The rate at which heat must be supplied to the surface for these conditions.

Air ($T_f = (T_s + T_\infty)/2 = (290 + 310)K/2 = 300$ K, 1 atm), $\gamma = 15.89 \times 10^{-6}$ m$^2$/s, k = 0.0263 W/m.K, Pr = 0.707; *For* Air-water mixture (300 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ m$^2$/s, Sat. water (Ts = 310 K): $\rho_{A,sat} = 0.04361$ kg/m$^3$, $h_{fg} = 2414$ kJ/kg.



$$Sc = \frac{\nu}{D_{AB}} = \frac{}{0.26 \times 10^{-4}\,m^2/s}$$

$$\overline{Sh}_L = 0.43(6.293 \times 10^5)^{0.58}(0.611)^{0.4} = \underline{815.2}$$

$$\bar{h}_m = \frac{\overline{Sh}_L . D_{AB}}{L} = \frac{815.2 \times 0.26 \times 10^{-4}\,m^2/s}{1\,m} = \underline{2.12 \times 10^{-2}\,m/s}$$

APORATION RATE, $\dot{m} = \bar{h}_m A_S(\rho_{AS} - \underbrace{\cancel{\rho_{A\infty}}}_{=0})$ $\quad \rho_{AS} = 0.04361\,kg/m^3$

$$\dot{m} = 2.12 \times 10^{-2}\,\frac{m}{s} \times 1\,m^2 \times 0.04361\,\frac{kg}{m^3} = \underline{9.243 \times 10^{-4}}\,kg/s.$$

$$= q_{conv} + q_{evap} = \bar{h}_L A_S (T_S - T_\infty) + \dot{m} h_{fg} = 454W + 2231W = 2685W$$

The 3rd part tells us to find out the rate at which heat must be supplied to the surface, to the substrate for these conditions. So what is the rate, what is Q which needs to be added, which needs to be supplied to the solid substrate which is in contact with a flowing air at a different temperature as well as there is a film of water which gets evaporated. So if you think of the, if I think of this as my entire control volume, that means the total is my control volume over here, so it would be heat in - heat out must be equal to 0. So for this control surface, E dot in must be equal to E dot out because nothing gets generated here, nothing gets generated in this and it is a steady-state, so the right-hand side is 0.

So therefore E dot in - E dot out is equal to 0, so what you have then is E dot in is equal to E dot out and we know that this E dot out has 2 components, one is loss due to convection and the other is loss or heat used for evaporation. Okay. So the total amount of heat that needs to be supplied to the subset must be equal to the loss due to convection and the loss because of the evaporation of the air, evaporation of the water film. So in part C, which is just a small part, the Q dot in, the amount of heat to be supplied to the substrate would simply be the sum of convection + Q of evaporation which we have evaluated.

So this is going to be H bar L AS, T of the surface - T infinity + M dot HFG where HFG is the latent heat of vaporisation. So this gives you the convective transport of heat and this gives you the heat required for phase change. And when you put the values and evaluate them, what you would get is 454 watts for this and this is going to come as 2231 watts and the sum total of this is 2685 Watts. So I would just bring to your notice one thing which is, since I have calculated M dot, I can be I can simply multiplied with the change, phase change, the latent heat of vaporisation and obtain what is the heat that is due to the that is going to be used to meet the evaporation of water and convection is known to me.

And if you if you compare these 2 numbers, one interesting point to note here is that, most of the heat requirement is for evaporation and only a small fraction of it is going to be utilised as convective heat transfer. So in any heat transfer process where both convection as well as change phase is taking place, the heat requirement is in almost all the cases, the heat requirement is dictated by the requirement of heat for phase change. So phase change requirement overshadows any convection which may be present in the system. So that is a general observation which you can make, there will be exceptions but this is just a general observation.

So what we saw in this problem is a case in which you have simultaneous evaporation and convection and the convection relation is provided to you through some experimental data and you are going to, you using that experimental correlation obtained for convective heat transfer and you have used the analogy to transform that expression to a mass transfer process, to relation, to a correlation for convective mass transfer. And then you simply evaluated what is the heat required for convection and what is the heat required for evaporation, sum them together and you get the total heat to be supplied by the heater to the substrate in order to meet the convection evaporation process. So we will look another problem in the next segment of the class.