Transport Phenomena.

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Lecture-54.

Analogy-Tutorial 1.

So we would continue with this analogy and most importantly the application of analogy in solving real-life problems in problems where we have a prototype in which heat transfer is taking place, we would like to know what is going to happen without doing the experiments if a mass transfer operation takes place over such a device. But before we do that, let us quickly look at the thicknesses of boundary layers, the momentum, the masses transfer and heat transfer boundary layers in light of the analogy that we have developed so far.

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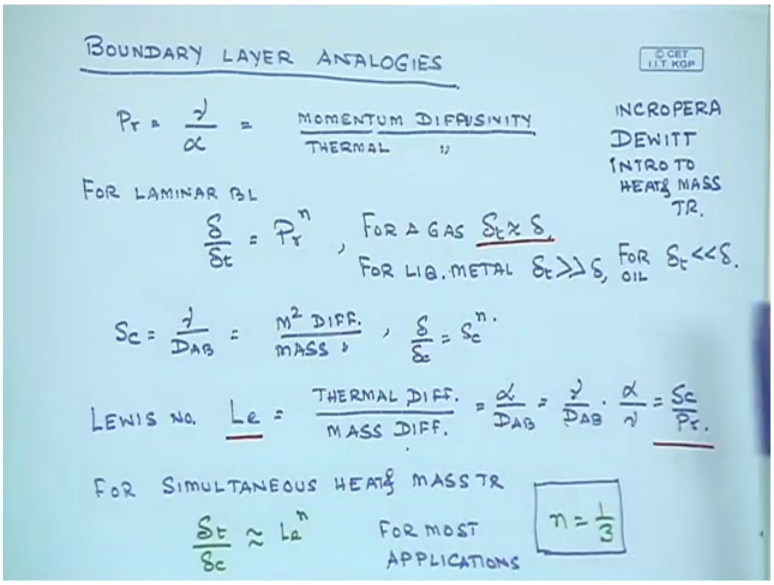

We are looking at the boundary layer analogies and we are looking at the significance of the numbers which we have come across. The 1st number that we have come across is Prandlt number which is the momentum diffusivity, so this is mu by rho divided by thermal diffusivity which is K by rho CP. So the Prandlt number is essentially then a ratio of momentum diffusivity by thermal diffusivity. And it has been, it can be seen from the results of Delta and Delta T that is hydrodynamic boundary layer thickness divided by the thermal boundary layer thickness is equal to Prandlt, Prandlt number to the power N.

And for a gas, since the gases are very well mixed, then for a gas Delta T, the thickness of the thermal boundary layer would be equal to the thickness of the hydrodynamic boundary layer. But for a liquid metal, since value of K is very large for the case of the liquid metal, the thermal conductivity of a liquid metal is so high that the effect of the surface, effect of the solid surface in contact with the liquid will be felt at a greater distance inside the liquid metal.

So if we have a solid plate and you have a liquid metal on top of it, says the thermal, since the thermal conductivity is for life, the thermal diffusivity is so large, the effect of the plate will diffuse to a greater depth into the liquid metal and therefore the point over here will sense, the point over here will sense in terms of temperature that solid plate exists over here whose temperature is different from, different that from the flowing liquid metal. But at this point, the hydrodynamically, the molecule, the liquid molecule at this point will not feel the presence of the solid plate.

So for a liquid metal, the thickness of the thermal boundary layer is going to be much more than the thickness of the hydrodynamic boundary layer. So the entire, the thickness of the thermal boundary layer is going to be large since the thermal diffusivity is large, the thermal diffusivity, diffusivity of a liquid metal is going to be large since the value of K is very large for the case of a liquid metal. Whereas the thickness of the hydrodynamic boundary layer does not depend on the value of K, so in cases of liquid metals when it flows over a hot surface, the thermal boundary layer, the temperature front, the temperature front will propagate, will penetrate more into the liquid metal as compared to the momentum momentum penetration of the, due to the presence of the solid plate.

So the thickness of the thermal boundary layer is going to be very large compared to thickness of the hydrodynamic boundary layer for the case of a liquid metal. On the other hand if you have the case of a oil, for example, for any oil, this thermal boundary layer is going to be very thin in compared to the hydrodynamic boundary layer. So we have 2 extremes, liquid metal, high thermal conductivity, Delta T is going to be very large compared to Delta, for oil, with a low K and a very and a large value of rho, the thickness of the thermal boundary layer is going to be less than the thickness of the hydrodynamic boundary layer. Similarly we can get Schmidt number which is nothing but the momentum diffusivity by mass diffusivity.

So momentum diffusivity by mass diffusivity and Delta Y Delta C, similar to Delta by Delta T, you can easily see that delta by Delta C is going to be Schmidt number to the power N. And by combining these, the Prandlt number and Schmidt number, a new number has also been proposed which is especially use for the case of simultaneous heat and mass transfer which is called the Lewis number. So Lewis number is as I said is used for simultaneous heat and mass transfer, it is defined as the thermal, ratio of the thermal diffusivity by mass diffusivity.

So thermal diffusivity by mass diffusivity, if you begin the momentum diffusivity in here, both are the numerator and the denominator, so what you see is this is equal to Schmidt number, so this part is Schmidt number and this part is 1 by Prandlt number. So this is nothing but the inverse of Prandlt number. So Lewis number LE is equal to Schmidt by Prandlt number. Okay. And as I as I mentioned before for the case of simultaneous heat and mass transfer, the thickness of the boundary layer, thermal boundary layer and the thickness of the concentration boundary layer, they are related by LE, Lewis number has the power N and for most of the applications the value of N is taken to be 1 by 3.

So this is something which we have, we already know, which are obvious, which can be obtained easily from the from the analogy, I have just jotted down these together so as to give you the idea of relative thicknesses, the relative importance of momentum transfer, heat transfer and mass transfer and how the boundary layer thicknesses of these 3 transport processes are going to relate to each other through the relevant similarity parameters. That is in terms of and lumber, Schmidt number and Lewis number. So when you talk about hydrodynamic and heat transfer, it is going to be Prandlt number, hydrodynamic and mass transfer, it is going to be Schmidt number and when you talk about heat, compare between heat transfer and mass transfer, it is going to be the Lewis number.

So the importance and significance of these numbers, the dimensionless numbers should be very clear to all of you and there are exceptional cases where this analogy in its current form will not be applicable, that I gave you the examples of, the examples of liquid metals in which case the thickness of the thermal boundary layer will be very large in comparison to the hydrodynamic boundary layer or heavy oil where it is simply going to be the reverse. The boundary, the thickness of the momentum boundary layer would be more as compared to the thickness of the heat transfer boundary layer.

Whatever I have taught in terms of the analogy, it is available in the book Incropera and Dewitt, so this it is there in Incropera and Dewitt that I have mentioned at the beginning as a text, and they textbook for this course. So the name of the book is introduction to heat and mass answer. So these analogies, the questions, the functional forms, all these are from Incropera and Dewitt, so you take a look at it, it is very well described in the book and I do not think you will have any problem with any problem in understanding. But if you do, please contact me, write to me and I will clarify any doubts that you may have.

So now we are going to go into the last part of the course, the last few classes where we are going to solve problems related to these analogies, related to simultaneous heat and mass transfer and we would see how the analogies will make our life a lot simpler. There would be even more topic which I would cover, probably at the last class is the error function. The error function will keep coming back in the case of momentum, heat and mass transfer or many other problems. So I would give you one or 2 examples of when the solution of a differential equation having a specific set of boundary conditions will be of error function type. So you simply only have to write the governing equation, write the initial and the boundary conditions and you can directly write what is going to be the solution for the process. Okay.

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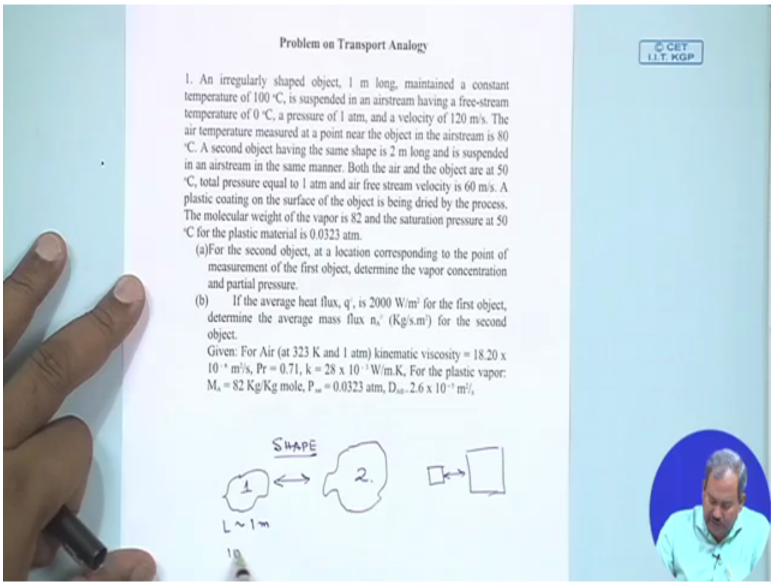

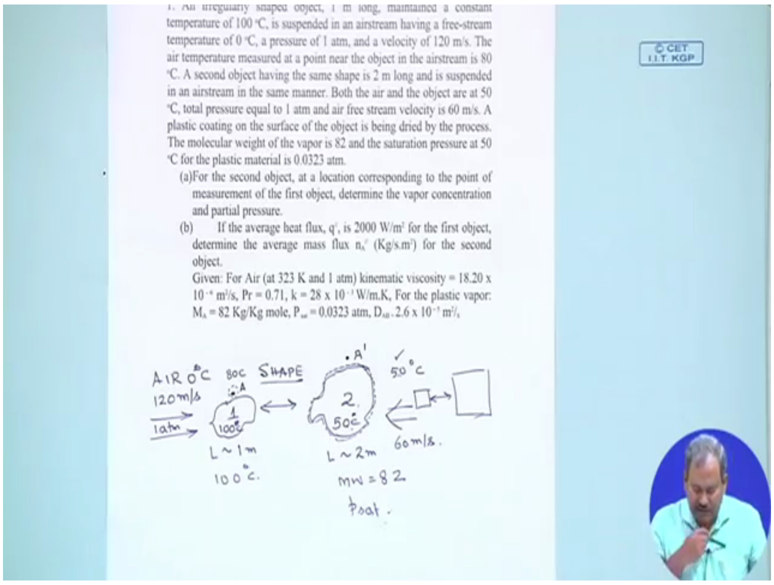

And the use of that through you will see that in the in the last class. But right now the problem that we are going to solve is, it is again a problem on transport analogy. What it tells is that we have an irregularly shaped object, it could be of any, it could be of any shape, so and we simply have the same shape, I mean these 2 are equivalent in terms of shape. I could not draw it properly but you can you can we can get what I try to mean is, they are irregular but whatever with the ship, the just the conversion factor, scale factor by which you have increased it. So this is object one and this is object 2 and you simply have made the size of the object a few times larger than that of the object normal one while keeping the shape intact.

So if this is, if the shape was just a square, you simply made it, you simply made it a bigger square. So this is what I mean when you keep the shape intact, but the sizes are more. So you have to, you have to pardon my drawing but what I, what I am trying to show you here is this is object 1, that 2 and let us assume the shapes are same. So the irregularly shaped objects which has an equivalent length to be equal to 1 metre and it is maintained at a constant temperature of 100 degrees centigrade. So this is maintained at 100 degrees centigrade and it is suspended in an air stream, so we have air flowing over it with air at 0 degrees centigrade which is, which is flowing over object 1.

And the pressure is given as 1 atmosphere and the velocity is 120 metre per second. So the velocity of the air stream is also given, then air temp, then what you do is you measure the air temperature at a point near the object in the air stream to be 80 degrees centigrade. So somewhere over here you measure the temperature of this point and you find this temperature to be 80 degrees centigrade. The 2nd object in this case the length scale is instead of 1 metre, it is 2 metres. It is also suspended in an air stream, now both the air and the object in this case, so one is at 100 degrees centigrade and it is it is interacting with air at 120, interacting with air at 0 degrees centigrade.

So this is from 100 to free stream temperature of 0 and the temperature near the surface is given as 80 degree, that is the only information experimental information that you have. And you know that the air is coming at 120 metre per second, its pressure is 1 atmosphere and so on. So you have measured the temperature at a point and found the temperature to be 80 degrees centigrade. Now in the 2nd case where the length scale, length is 2 metres, both the air and the object are at 50. So this is at 50 degrees centigrade and the air that blows, this is also at 50 degrees centigrade. So these 2 are separate things, so here you do and heat transfer experiment but here what you have is the mass transfer situation.

The air stream velocity here is 60 metre per second and what you, you have a plastic coating on the surface, a thin plastic coating on the surface that is that you are drying by putting the, putting the flow of air on object 2. So this plastic coating is to be is to be dried by passing an air which is that 50 degrees centigrade, the temperature of the object is also 50 degrees centigrade. The molecular weight of the vapour is, molecular weight of the wafer is 82, saturation pressure is provided, saturation pressure for the plastic material is provided, so P Sat is known to you. So what you have to do, you have to tell for a 2nd object at a location corresponding to the point of measurement of the 1st object, determine the vapour concentration and partial pressure.

So if this is your location which I call as A, there is a corresponding location over here which I call it as A prime. Your job is to find out without performing any experiment, without performing any mass transfer experiment, what is the vapour concentration and partial pressure. So at point A you measure the temperature, corresponding point is A prime on the scaled up object. In here, since both are, the air and the object and the rat same temperature, no heat transfer is taking place. But you have a plastic material on top of it, which is getting dried, which is getting a so slowly you are having some evaporation from here, in order for this plastic to be dried.

So you cannot measure the concentration, you cannot measure the vapour concentration because we do not have the tools to measure concentration. But we have tools to measure the temperature which we have done. Using analogy, what you have to find out is what is the, what is the vapour concentration and the partial pressure at point A prime. So this is a classic ideal example, I think you have understood the problem, now I am going to go and solve for it.

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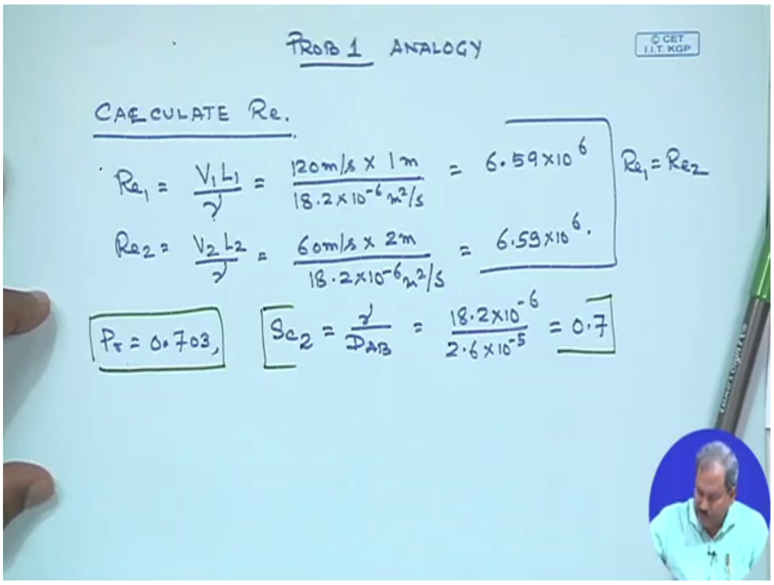

So object 1 and 2. The 1st step is to calculate Reynolds number. So what is the Reynolds number for the case of 1? Kinematic viscosity 120 metre per second where we have the heat transfer taking place, 1 metre is the length and nu has been provided in the problem to be this much of meter square per seconds, so it is 6.59 into 10 to the power 6. What is RE 2? So as you can see, these 2 are identical, so the Reynolds number, Reynolds numbers are same. The values of the Prandlt number which are shown over here, Prandlt number is 0.71 and the Schmidt number you can calculate, so and see what is its value.

So it is provided that Prandlt number is equal to 0.703, so this is for Schmidt number for the case of 2 where mass answer is taking place is kinematic viscosity by DAB. These values are provided in the problem, so look at the value of Schmidt number and Prandlt number, they are equal. Since they are equal, Reynolds numbers are equal, so if you go back to our fundamental equation and concentrate only on the heat and mass transfer part of it. So if you concentrate on the heat and mass transfer part of it, you would see that Prandlt number are the same for the case of heat transfer and mass transfer and at the same time the Prandlt number, the Prandlt number and Schmidt number are also same.

So whatever be and the velocity, so whatever be the solution for temperature, so the functional form of temperature and the functional form of concentration, both in dimensionless form must be equal to each other because your boundary conditions are met, the equality of the boundary conditions are met and this one, Re PR is equal to RE Sc since RE 1 is equal to RE 2 and Prandlt and Sc, Prandlt and SC are almost equal.

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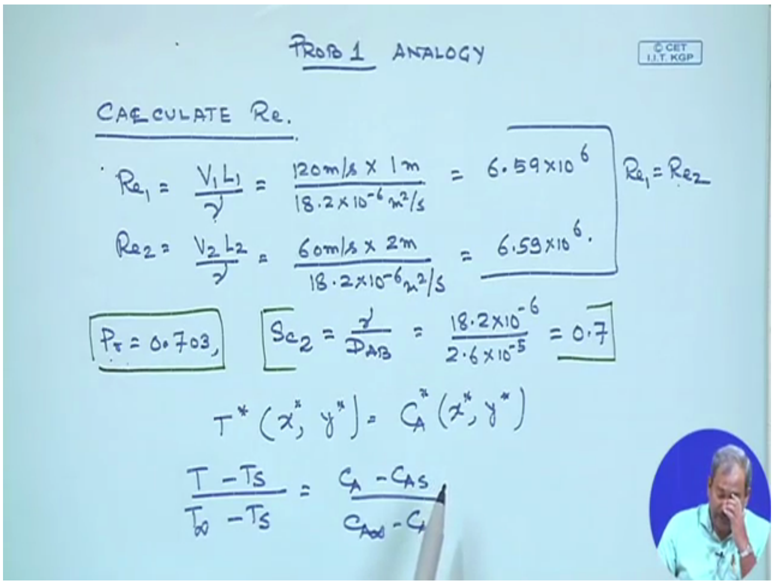

So therefore T star, the dimensionless temperature at given locations must be equal to CA Star X star Y star. And how we define T star is T - TS, surface temperature by T at infinite distance - TS is equal to CA - CAS by CA infinity - CAS. So the equality of dimensionless temperature and dimensionless concentration which results because the Reynolds numbers are and Prandlt number is equal to Schmidt number, so therefore the functional form of dimensionless temperature and dimensionless concentration which are defined it this way are the same. Now as was mentioned in the problem that location, location for measurement was identical for between object 1 and 2.

The problem was to the problem was to evaluate at a location corresponding to the point of measurement of the 1st object what would be the vapour concentration and partial pressure. So when it says corresponding to the point of measurement of the 1st object, it essentially tells you that the dimensionless values of X and Y which are used to denote the point of measurement, they are identical. So I can simply write this. So once based on the analogy and taking about the governing equations and the implications of the Prandlt number and Schmidt number and Reynolds number, so since I have written this, rest is very simple then.

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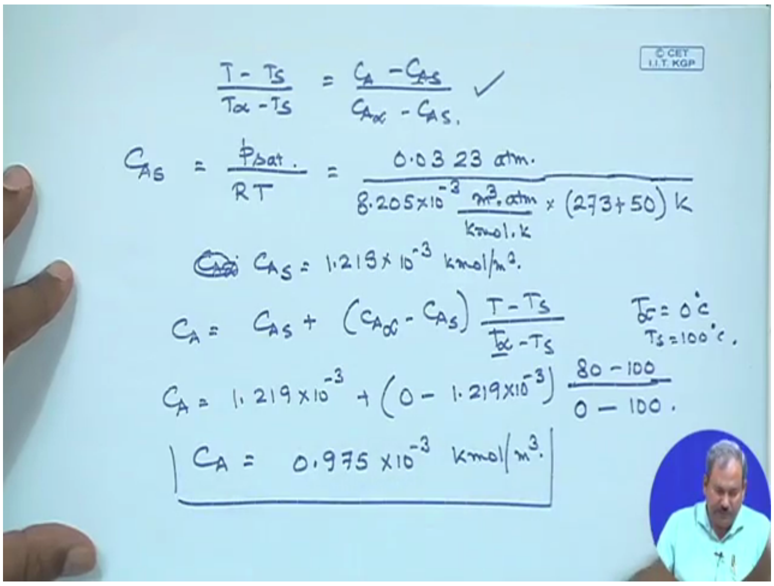

So T - TS by T infinity - TS equal to CA - CAS by CA infinity - CAS. Now I need to find out what is CAS, concentration of a species A at the solid surface which should be equal to PA saturation divided by RT using the ideal gas law. So this CAS is the concentration of species A at the surface, the P sat is provided as 0.0323 atmosphere, we use the corresponding value of R which is 8.205 metre cube atmosphere kilo moles Kelvin and the temperature in Kelvin scale, the temperature for the case of mass transfer, it is 50.

So this is the concentration sorry CAS, so the CAS would be equal to, so from here I can therefore write C A to be equals CAS + the denominator CA infinity - CAS times T - TS by T infinity - TS, just an expansion of that in terms of in terms of this. So your CA is, this is CA S, value of CA infinity at a point far from the far from the wall, temperature was measured to be 80 at that location, T surface is 100 and T infinity is 0 and TS is 100. So when you when you do that, you are going to get CA to be equals 0.975 into 10 to the power - 3, this is kilo moles per metre cube. So this T infinity, as you see in your, in the problem is 0 degrees centigrade and TS is 100 degree centigrade.

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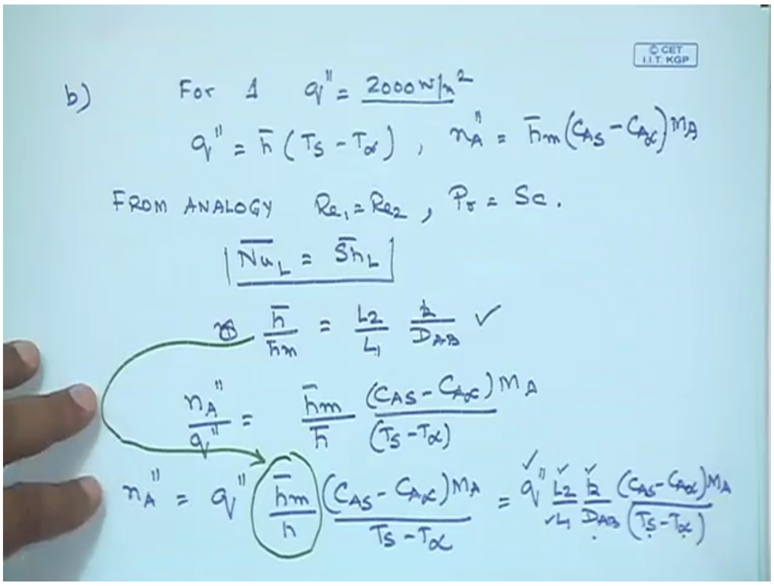

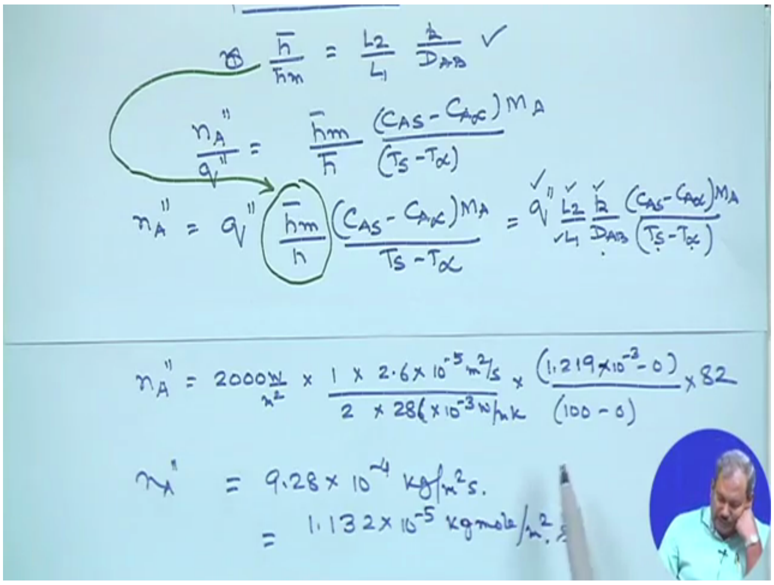

So this is the 1st part of the problem. The 2nd part of the problem, the part B tells you that if the average heat flux is 2000 Watts per metre square for the 1st object, what is the average mass flux for the 2nd object? So we have already calculated the vapour concentration over here, so now the next and I am sorry, the pressure, we have to also calculate the partial pressure. So partial pressure would simply be equals PA which is CA RT, the value of CA is already known, it would come to 0.0258 atmosphere. So an analogy essentially gives you value of concentration and the value of the partial pressure without having to solve any mass transfer equations, any complicated equation.

So results obtained in heat transfer experiments can simply be projected to obtain results of mass transfer experiments that you have not done. Okay. So this is one of the big advantages of this analogy and now we are going to find out, if the average heat flux is known for the 1^{st} object, what is going to be the average mass flux for the 2^{nd} object. So for this part, 2^{nd} part, we know that for 1 Q double prime is provided as 2000 Watts per metre square. We also know from Newton's law of cooling, this is H bar, average value of heat transfer coefficient, T of the surface - T infinity and N A double prime, the mass flux would simply be equal to H bar M, in terms of concentration, this is molar concentration, - CA infinity.

Since it is molar concentration, in order to change it to the mass form, I simply multiply it with the molecular weight. So this is mass flux, molar concentration, molecular weight and this is the average value of mass transfer, convective mass transfer coefficient. From analogy, since Reynolds number of 1 is equal to Reynolds number 2 and Prandlt number is equal to Schmidt number, I can simply write the average value of Nusselt, the average value of Nusselt number must be equal to the average value of Sherwood number, that is what is given by the analogy. So which I have discussed many times.

So if that is the case, then I simply divide this by this and what I would get is N and I can expand it a little bit more, so H bar by M, H bar M is equal to L 2 by L1, which is, L2 refers to the mass transfer case, L1 prefers to the heat transfer case times K by DAB. So this is the relation from which I can obtain the unknown values and then I divide NA by Q and what I would get is NA double prime by Q double prime is equal to H bar M by H bar. Simply dividing this by this. So therefore my NA double prime, the same expression over here.

And now in this H bar M by H, I bring this one over here. So in this, the value of Q double prime known to me to be 2000 Watts per metre square, L2, L1, the ratio would be 2, the value of K, DAB, CAS , CA infinity, TS, T infinity and MA are everything known to me. So I put this, I put this simply over here to complete the process, complete the calculation. This Q double prime is 2000 Watts per metre square, this H bar, H bar N, L2, L1, we put all the values in here that you have calculated, so after this, plug in all these values, you would get N A double prime to be 9.28 into 10 to the power - 4 KG per metre square per second or if you want express it in terms of KG moles by dividing it with the molecular weight, you get this value.

So essentially what you then what you see here is the advantage of using the analogy. Since the Prandlt number and Schmidt number equal and Reynolds numbers also equal since the length scale is, length scale and the velocity, they vary in such a way that length times velocity remains constant, Reynolds numbers are also equal. So the functional form of the dimensionless concentration and dimensionless temperature would be identical and by looking at the root laws which would give you the mass flux or the heat flux.

Namely Newton's law of cooling and something similar to Newton's law of cooling where the convective heat transfer coefficient H is replaced by H bar, that HM where HM is the convective mass transfer coefficient. And in the 1^{st} case for heat transfer, it is multiplied by temperature difference, in the 2nd case, mass transfer, it is multiplied by the concentration difference. So you can find out what is the ratio between mass flux by heat flux and on the right-hand side, whatever you have, the only unknown here is the heat transfer coefficient based on mass and heat transfer coefficient based on, heat transfer coefficient, it is simply convective heat transfer coefficient.

By analogy everything, this ratio is known to you, H bar M by H bar and this is simply, by one step plugging in the values, you would get the unknown mass flux which you would obtain on a similar surface where only mass transfer is taking place, no heat transfer takes place. So by performing supposedly a relatively easy experiment involving heat transfer, measuring temperature, you would be able to predict precisely what would be the mass flux from a similar object without doing the mass transfer experiment. So this specific problem highlights the utility of heat and momentum, heat and mass transfer analogy. So we would solve similar such problems which would also involve a little bit of thinking and modelling in the subsequent classes.